Big M or Two-phase Simplex (pick one)

Converge, how to write dual-problem (1)

(6.7, 6.10) + 6.5, 6.3, 6.2 ?

OCT. 25/17
Linear Prog.

MIDTERM ~ 6.10 (solutions to be posted)

Assignment 7 not due (Practice only)

$$\left.\begin{array}{l} \max\ z = \mathbb{c}\,\mathbb{x} \\ \text{s.t. } A\,\mathbb{x} \le \mathbb{b} \\ \quad \mathbb{x} \ge \emptyset \end{array}\right\} \quad \begin{array}{l} \min w = \mathbb{y}^T \mathbb{b} \\ \text{s.t. } A^T \mathbb{y} = \mathbb{c}^T \\ \quad \mathbb{y} \ge \emptyset \end{array}$$

$\mathbb{x}_{n\times 1}$ $\mathbb{y}_{m\times 1}$ $\mathbb{c}_{1\times n}$ $\mathbb{b}_{m\times 1}$ $A_{m\times n}$

weak: $Z(\mathbb{x}) \le W(\mathbb{y}) = \mathbb{y}^T \mathbb{b}$

$$\max_{\mathbb{x}} z(\mathbb{x}) \le \min_{\mathbb{y}} (\mathbb{y})$$

1) Big M or Two-phase

$$z(\mathbb{x}^*) = w(\mathbb{y}^*)$$

$$\mathbb{c}\,\mathbb{x}^* = (\mathbb{y}^*)^T \mathbb{b} = \boxed{\mathbb{c}_B B^{-1} \mathbb{b}}$$

$$(\mathbb{y}^*)^T = \boxed{\mathbb{c}_B B^{-1}}$$

$$z + (\mathbb{c}_B B^{-1} N - \mathbb{c}_N)\mathbb{x}_N = \mathbb{c}_B B^{-1} \mathbb{b}$$

$$\mathbb{x}_B + B^{-1} N \mathbb{x}_N = B^{-1} \mathbb{b}$$

$$(\mathbb{y}^*)^T = \mathbb{c}_B B^{-1}$$

$$w(y^*) = (\mathbb{y}^*)^T \mathbb{b} = \mathbb{c}_B B^{-1} b = z(\mathbb{x}^*)$$

$$z = \mathbb{c}\,\mathbb{x} + \emptyset s_j$$

$$a_{j1} x_1 + a_j x_2 + \dots a_j x_n + s_j = b_j$$

$$a_{s_1} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad a_{s_2} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} \quad a_{s_j} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}_j$$

Oct. 23/17

LINEAR PROG.

November 1st - Midterm 2

$$z - C_B X_B - C_N X_N = 0$$
$$B X_B + N X_N = b$$
$$X_B \geq 0, \quad X_N \geq 0$$

(OPTIMAL)? if > 0

x	x	x	
x	x	x	
x	x	x	

(FEASIBLE)? if > 0

$$\downarrow$$

$$z + (C_B B^{-1} N - C_N) X_N = C_B B^{-1} b$$
$$X_N + B^{-1} N X_N = B^{-1} b$$
$$X_B \geq 0, \quad X_N \geq 0$$

C_{x_2} $30 \to 43$ and $a_{x_2} \begin{bmatrix} 6 \\ 2 \\ 1.5 \end{bmatrix} \to \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$

$$\bar{C}_{x_2} = C_B B^{-1} \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} - 43$$

$$= \begin{bmatrix} 0 & 10 & 10 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} - 43 = 7 > 0$$

C_{x_2} $30 \to 43$ and $a_{x_2} \begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix} \to \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$

$$\bar{C}_{x_2} = C_B B^{-1} \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix} - 43$$

$$= \begin{bmatrix} 0 & 10 & 10 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix} - 43 \quad -3, \quad \bar{a}_{x_2} = \begin{bmatrix} -7 \\ -4 \\ 2 \end{bmatrix}$$

	x_2	x_4
Optimal	5	-5
	-2	3
	-2	2
	1.25	-0.5

$$B^{-1} a_{x_4} = \begin{bmatrix} 1 & 2 & 8 \\ 0 & 2 & -4 \\ 0 & -0.5 & 1.5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -0.5 \end{bmatrix}$$

ex. Max $z = x_1 + 4x_2$ (pg. 274) ↳ example 1 (sec. 6.2)

$$x_1 + 2x_2 \le 6$$
$$2x_1 + x_2 \le 8$$
$$x_1, x_2 \ge 0$$

$BV = \{x_2, s_2\}$

$$0 \; z = c_B x_B + C_N x_N$$
$$B x_B + N x_N = b$$
$$x_B \ge 0 , \; x_N \ge 0$$

How do you find B?

Section 6.5

		Max z				
min w		$x_1 \ge 0$	$x_2 \ge 0$	$\cdots$	$x_n \ge 0$	
$y_1 \ge 0$	y_1	a_{11}	a_{12}	$\cdots$	a_{1n}	$\le b_1$
$y_2 \ge 0$	y_2	a_{21}	a_{22}	$\cdots$	a_{2n}	$\le b_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$y_m \ge 0$	y_m	a_{31}	a_{32}	$\cdots$	a_{3n}	$\le b_m$
		$\ge c_1$	$\ge c_2$		$\ge c_n$	

$$\text{Max } z = cx \qquad\qquad \min w = b^T y$$
$$Ax \le b \quad \longleftrightarrow \quad A^T y = c^T$$
$$x \ge 0 \qquad\qquad y \ge 0$$

$$c = [c_1 \; \cdots \; c_n]$$

$$b = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad A_{m \times n} = [a_{ij}]$$

$$\text{Max } z = 2x_1 + 2x_2$$
$$x_1 + x_2 \le 2$$
$$-x_1 - x_2 \le -2$$
$$(x_1, x_2 \ge 0) \quad -2x_1 + x_2 \le -3$$
$$x - x_2 \le 1$$

$$\text{min } \omega = 2y_1 - 2y_2 - 3y_3 + y_4$$
$$y_1 - y_2 - 2y_3 + y_4 \ge 2$$
$$y_1 - y_2 + y_3 - y_4 \ge 1$$
$$y_1, y_2, y_3, y_4 \ge 0$$

	$x_1 \ge 0$	$x_2 \ge 0$	
y_1	1	1	≤ 2
y_2	-1	-1	≤ -2
y_3	-2	1	≤ -3
y_4	1	-1	≤ 1
	≥ 2	≥ 1	

Set $y_1' = y_1 - y_2$
$y_2' = -y_3$

$$\text{min } \omega = 2y_1' + 3y_2' + y_4$$
$$y_1' + 2y_2' + y \ge 2$$
$$y_1' - y_2' - y_3' \ge 1$$
$$y_1' \text{ urs}, \quad y_2' \le 0 \quad y_3' \ge 0$$

[IF x_2 urs

$$\text{Max } z = 2x_1 + x_2' - x_2''$$
$$x_1 + x_2' - x_2'' = 2$$
$$2x_1 - x_2 + x_2'' \ge 3$$
$$x_1 - x_2' + x_2'' \le 1$$
]