Sept. 18/17

Linear Prog.

Two-Variable LP Problems

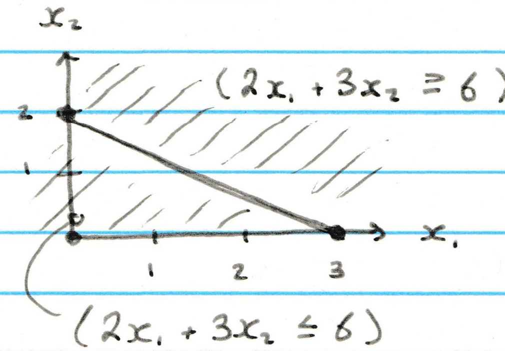
$2x_1 + 3x_2 = 6$

Set $x_1 = 0$, $x_2 = 6/3 = 2$

$x_2 = 0$, $x_1 = 6/2 = 3$

Let $x_1, x_2 = 0$

$0 + 0 < 6$



From Giapetto Example:

$2x_1 + x_2 \leq 100$

$x_1 + x_2 \leq 80$

$x_1 \leq 40$

$x_1 \geq 0$

$x_2 \geq 0$

1. $2x_1 + x_2 = 100$

$x_1 = 0$, $x_2 = 100$

$x_2 = 0$, $x_1 = 50$

$(0,0)$, $0 < 100$

$S_1 = \{(x_1, x_2) : 2x_1 + x_2 \leq 100\}$

$S_2 = \{(x_1, x_2) : x_1 + x_2 \leq 80\}$

$S_3 = \{(x_1, x_2) : x_1 \leq 40\}$

$S_4 = \{(x_1, x_2) : x_1 \geq 0\}$

$S_5 = \{(x_1, x_2) : x_2 \geq 0\}$

$S = S_1 \cap S_2 \cap S_3 \cap S_4 \cap S_5$

Binding Constraints:

$$\begin{cases} 2x_1 + x_2 = 100 \\ x_1 + x_2 = 80 \end{cases}$$

$\Rightarrow x_1 = 80 - x_2$

$2(80 - x_2) + x_2 = 100$

$160 - 2x_2 + x_2 = 100$

$160 - x_2 = 100$

$\therefore x_2 = 60;\ x_1 = 20$

- Feasible region of any LP has only a finite number of "extreme points"
- Any LP that has an optimal solution has an extreme point that is optimal.

$$\min Z = 50x_1 + 100x_2$$
s.t. $7x_1 + 2x_2 = 28$
$2x_1 + 12x_2 = 24$
$x_1 \geq 0, \; x_2 \geq 0$

$$50x_1 + 100x_2 = C$$
$$x_2 = -500/100\,x_1 + C/50$$
↳ $-1/2$ slope

Chapter 4:

Example 1 →

x_1 = number of belts produced / week (deluxe)
x_2 = number of belts produced / week (regular)

$\max z = 4x_1 + 3x_2$
s.t. $x_1 + x_2 \leq 40$
$2x_1 + x_2 \leq 60$
$x_1, x_2 \geq 0$

Let $S_1 = 40 - x_1 - x_2 \geq 0$
$S_2 = 60 - 2x_1 - x_2 \geq 0$

$\max z = 4x_1 + 3x_2 + 0S_1 + 0S_2$
s.t. $x_1 + x_2 + S_1 = 40$
$2x_1 + x_2 + S_2 = 60$
$x_1, x_2, S_1, S_2 \geq 0$

→ Standard Form

$\min z = 50x_1 + 100x_2$
s.t. $7x_1 + 2x_2 - e_1 = 28$
$2x_1 + 12x_2 - e_2 = 24$
$x_1, x_2 \geq 0$

→ Standard Form

Set $e_1 = 7x_1 + 2x_2 - 28 \geq 0$
$e_2 = 2x_1 + 12x_2 - 24 \geq 0$

↳ Midterm to cover Assignment 2, 3, 4

Sept. 20/12

Linear prog

$$\text{Max } z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

$$\begin{aligned} \text{s.t. } & a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ & a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ & a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \\ & x_1, x_2, \dots, x_n \geq 0 \end{aligned}$$

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}_{m \times n} \qquad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

$$C = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \qquad n \geq m \qquad \begin{aligned} \max z &= C^T \mathbf{x} \\ \text{st. } A\mathbf{x} &= \mathbf{b} \\ \mathbf{x} &\geq 0 \end{aligned}$$

ex. $x_1 + x_2 = 3$ $\qquad n = 3 \qquad n - m = 1$

$-x_2 + x_3 = -1$ $\qquad m = 2$

Set $x_3 = 0$: $x_2 = 1$, $x_1 = 2$ $\therefore$ Solution: $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ is a b.s. (bfs)

because all are non-negative

$$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\{x_1, x_2\} \in BV$

$\{x_3\} \in NBV$

Set $x_2 = 0$; $x_1 = 3$; $x_3 = -1$

$\therefore$ Solution $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ is a b.s., but it is not a bfs (contains negatives)

$$\begin{array}{cc} x_1 & x_3 \end{array}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

ex.

$$x_1 + 2x_2 + x_3 = 1$$
$$2x_1 + 4x_2 + x_3 = 3$$

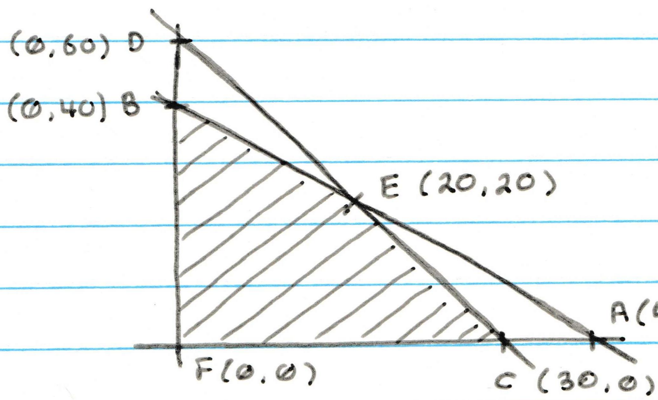
$n = 3$; $m = 2$ $\quad n - m = 1$

Set $x_3 = 0$

$$\begin{array}{ccc} x_1 & x_2 & b \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 4 & 3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 1 \end{array}\right]$$

$\therefore$ no b.s. For
$BV = \{x_1, x_2\}$
$NBV = \{x_3\}$

Leather Limited Problem



$$\max z = 4x_1 + 3x_2$$

s.t. $x_1 + x_2 + S_1 = 40$

$2x_1 + x_2 + S_2 = 60$

$x_1, x_2, S_1, S_2 \geq 0$

$n = 4$ $\quad n - m = 2$ — Set two variables to zero. (NBU)

$m = 2$

$\therefore$ when $S_1 = S_2 = 0$; $x_1 = x_2 = 20$ (E)

$x_2 = S_2 = 0$; $x_1 = 30$, $S_1 = 10$ (c)

$x_2 = S_1 = 0$; $x_1 = 40$, $S_2 = -20$ (not a bFS, $S_2 < 0$)

$x_1 = S_2 = 0$; $x_2 = 60$, $S_1 = -20$ (not a bFS, $S_1 < 0$)

$x_1 = S_1 = 0$; $x_2 = 40$, $S_2 = 20$ (B)

$\underbrace{x_1 = x_2 = 0}_{NBV}$; $\underbrace{S_1 = 40 , S_2 = 60}_{BV}$ (F)

Course #: ________ Experiment/Assignment #: ________

Date: ________ Course Instructor: ________

Title: ________

Author: ________ / ________
Student# Name (print)

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	bhs	
Row 0	1	-60	-30	-20	0	0	0	0	0	$Z = 0$
Row 1		8	6	1	1	0	0	0	48	$s_1 = 48$
2		4	2	1.5	0	1	0	0	20	$s_2 = 20$
3		2	1.5	0.5	0	0	1	0	8	$s_3 = 8$
4		0	1	0	0	0	0	1	5	$s_4 = 5$

$n = 7$

$m = 4$

$n - m = 3$ zero

Set $x_1 = x_2 = x_3 = 0$

$NBV = \{x_1, x_2, x_3\}$

$BV: \{s_1, s_2, s_3, s_4\}$

$X = [x_1, x_2, x_3, s_1, s_2, s_3, s_4]^T$

$Z = c_1 x_1 + c_2 x_2 + \dots c_n x_n$

$c_{j_0} = \max\{c_1, \dots c_n\}$, x_{j_0} NBV $\rightarrow$ BV

$c_1 = 60$, $j_0 = 1 \leftarrow x_1$: NBV $\rightarrow$ BV

x_1 must satisfy

$8x_1 + s_1 = 48 \rightarrow s_1 = 48 - 8x_1 \geq 0, \; x_1 \leq 48/8$

$4x_1 + s_2 = 20 \rightarrow s_2 = 20 - 4x \geq 0, \; x_1 \leq 20/4$

$2x_1 + s_3 = 8 \rightarrow s_3 = 8 - 2x \geq 0, \; x \leq 8/2$

$s_4 \geq 0$

$\theta_{i_0} = \min\left\{\frac{48}{8}, \frac{20}{4}, \frac{8}{2}\right\}$

$= \{6, 5, 4\} = 4 =$

$\theta_{i_0} = \min\left\{\frac{b_i}{a_{ij_0}}, i \dots\right\}$

$= \min\left\{\frac{b_1}{a_{11}}, \frac{b_2}{a_{21}}, \frac{b_3}{a_{31}}\right\}$

$= \left\{\frac{b_3}{a_{31}}\right\}$ $\quad i_0 = 3$