SEPT. 11/17

LINEAR PROG.

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 3 \\ 1 & 2 & 1 & 4 \end{array}\right] \end{array} \sim \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$0x_1 + 0x_2 + 0x_3 = 0$

$BV = \{x_1, x_2\}$

$NBV = \{x_3\}$

$$-2\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 4 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & -2 \end{array}\right]$$

$0 = 0x_1 + 0x_2 \neq -2$ (NOT POSSIBLE)

$\therefore$ No solution

→ Gauss-Jordan Elimination Review:

- two rows can be exchanged at any time in the process

Section 2.4 (Linear Dependence / Independence)

ex. $C_1[1, 0] + C_2[0, 1] = 0$

$\Rightarrow [C_1, 0] + [0, C_2] = [0, 0]$

$\Rightarrow [C_1, C_2] = [0, 0]$ LI

$\Rightarrow C_1 = 0$ AND $C_2 = 0$ (linearly independent)

ex. $C_1[1, 2] + C_2[3, 6] = [0, 0]$

$[C_1, 2C_1] + [3C_2, 6C_2] = [0, 0]$ LD

$C_1 + 3C_2 = 0 \xrightarrow{\text{then}} C_1 = 3$

$2C_1 + 6C_2 = 0 \xrightarrow{\text{then}} C_2 = -1$ (linearly dependent)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

rank $(A'|b') = 3$
rank $(A|b) = 3$
(Letting A' be the Final result, and A the original, rank A' = rank A)

ex. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $m = 3$
(row number)
→ think instead as number of equations

rank $(A'|b') = 2$

ex. $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 4 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 1 \end{array}\right]$ $m = 2$

-2

rank $(A'|b') = 2$

If rank $A = m$, then V is a linearly independent set of vectors

If rank $A < m$, then V is a linearly dependent set of vectors

If rank $A > m$, then solution doesn't exist (?)

→ If Rank $(A|b) \neq$ Rank (A) THEN there is no solution

Square matrix
where $A_{m \times n}$, $m = n$

$D = \begin{bmatrix} * & & 0 \\ & \ddots & \\ 0 & & * \end{bmatrix}$; $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$

$AB = BA = I$
$B = A^{-1}$

ex. $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$; $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 5/2 & 1/2 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$$

$$[A^{-1}A \mid A^{-1}I] \longrightarrow [I \mid A^{-1}]$$

$$\left[\begin{array}{cc|cc} 1 & 5/2 & 1/2 & 0 \\ 0 & 1 & -1 & 2 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

$$A \cdot A^{-1} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

↳ Try to create inverse of matrix in Excel

Determinants

$$\det \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = 6 - 5 = 1 \neq 0$$

rank $(A) = 2$

$\det A_{2\times2} \neq 0 \iff A^{-1} \exists$

$Ax = b$ has only one solution

$$\begin{array}{cccccc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} & a_{33} \end{array}$$

$A_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$

$A_{12} = \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$

$A_{13} = \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

$$\det A = (-1)^{1+1} a_{11} \det(A_{11}) + (-1)^{1+2} a_{12} \det(A_{12}) + (-1)^{1+3} a_{13} \det(A_{13})$$

Sept. 13/17

LINEAR PROGRAMMING

LP - linear programming

Example 1 : Giapetto's Woodcarving

Toys	Sell($)	Raw material cost ($)	Labor($)	Finishing	Carpentry	Demand/wk
Soldier	27	10	14	2	1	40
Train	21	9	10	1	1	∞
Company				100/wk.	80 wk.	

Profit = revenues - costs

$= 27x_1 + 21x_2 - (10x_1 + 9x_2) - (14x_1 + 10x_2)$

$= (27-10-14)x_1 + (21-9-10)x_2$

$= 3x_1 + 2x_2$

Objective Function : $z = 3x_1 + 2x_2$

Constraint 1 : $2x_1 + x_2 \leq 100$

Constraint 2 : $x_1 + x_2 \leq 80$

Constraint 3 : $x_1 \leq 40$

Sign restrictions : $x_1 \geq 0$

$x_2 \geq 0$

Subject to

Max : $Z_{max} = 3x_1 + 2x_2$

where $x_1 = 30$

$x_2 = 30$

$(2 \times 30) + 30 = 90 < 100$ ✓

$30 + 30 = 60 < 80$ ✓

$(30, 30) \in S$

Assignment to be posted on mycourselink.