Sept. 25/18

Larson Textbook

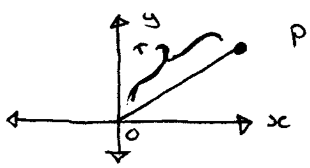
Chapter 11 : Vectors and Geom. of Lines and Planes

Chapter 12 : Vector valued Funct.

Chapter 13 : Functions in several variables

Cylindrical and Spherical Coordinates in $\mathbb{R}^3$

Review : In 2-dimensions



$P(x,y) = P(r,\theta)$ ← polar coordinates

r = distance from O to P

θ = angle with Ox - axis $\theta \in [0, 2\pi]$

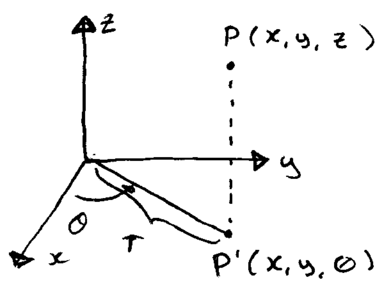
$P(x,y) \longleftrightarrow P(r,\theta)$

rectangular coordinates — Polar coordinates

$x/r = \cos\theta \rightarrow x = r\cos\theta$ $\qquad r^2 = x^2+y^2 \qquad r = \sqrt{x^2+y^2}$

$y/r = \sin\theta \rightarrow y = y\sin\theta$ $\qquad \tan\theta = y/x$

Cylindrical Coordinates in $\mathbb{R}^3$

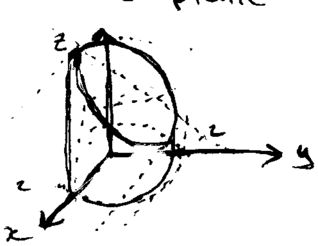


$P(x,y,z) = \boxed{P(\underbrace{r,\theta}_{\text{Polar coordinate for } x,\,y}, z)}$ ← cylindrical coordinates

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{aligned}}$$

Ex. Find a parameterization for the curve C of intersection between the cylinder $\underbrace{x^2+y^2=4}_{\rightarrow\, r=2}$ and the plane $z = 4 - y$.

Sol



$$\left.\begin{aligned} x &= f(t) \\ y &= g(t) \\ z &= h(t) \end{aligned}\right\} t = \text{parameter}$$

Use cylindrical coordinates for the points (x, y, z) on curve C.

$$C: \quad x = 2\cos\theta \to r$$
$$y = 2\sin\theta$$
$$z = 4 - y \Rightarrow 4 - 2\sin\theta$$

Parameterization of C:

$$\boxed{\begin{aligned} x &= 2\cos\theta \\ y &= 2\sin\theta \\ z &= 4 - 2\sin\theta \end{aligned}}$$

θ = parameter

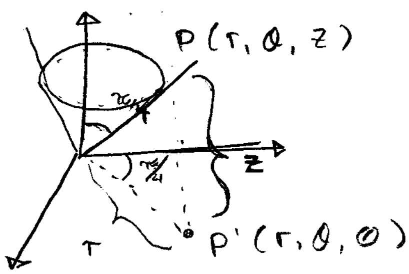
$\theta \in [0, 2\pi]$

<u>Ex</u>: Describe the surfaces in $\mathbb{R}^3$ with equations:
(given in cylindrical coordinates)

(1) $z = r$

(2) $z = r^2$

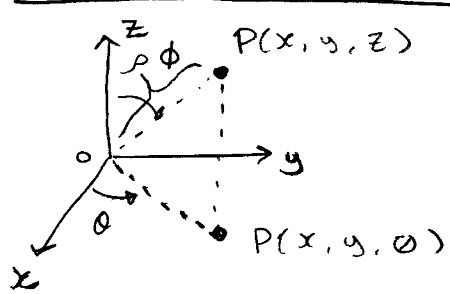
<u>Solution</u> (1) $z = r \longrightarrow z = \underbrace{\sqrt{x^2+y^2}}_{r} \longrightarrow$ positive cone



(2) $z = r^2 \longrightarrow z = \underbrace{x^2 + y^2}_{r^2} \longrightarrow$ PARABOLOID

<u>Spherical Coordinates</u>

$\longrightarrow P(\rho, \theta, \phi)$



$\Rightarrow$

ρ = distance from O to P

θ = as in polar / cylindrical coord

ϕ = angle between OP and OZ-axis

where $\rho \geq 0$

$\theta \in [0, 2\pi]$

$\phi \in [0, \pi]$

$P(x, y, z)$ $\longleftrightarrow$ $P(\rho, \theta, \phi)$

rectangular coord — spherical coord

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

→ $z/\rho = \cos\phi$

$z = \rho\cos\phi$

→ $r/\rho = \sin\phi$

$r = \rho\sin\phi$ ⟶ $x = r\cos\theta = \rho\sin\phi\cos\theta$
$y = r\sin\theta = \rho\sin\phi\sin\theta$

Then ; $x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

<u>Ex.</u> Identify the surfaces with following constraints (given in spherical coordinates)

(1) $\rho = 2$, $0 \le \phi \le \pi/2$, $0 \le \theta \le \pi/2$

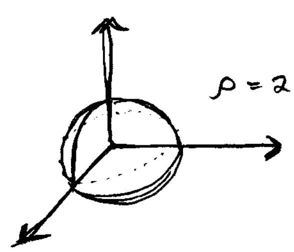
(2) $\phi = \pi/3$

(3) $\rho^2(\sin^2\phi\cos^2\theta + \cos^2\phi) = 4$

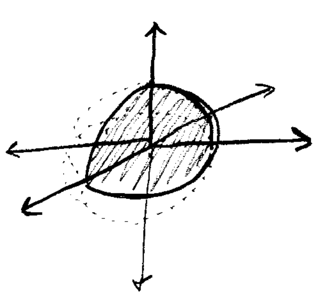
(4) $\rho = 2\cos\phi$

<u>Sol'n</u>

(1)



$\rho = 2$ (sphere of radius 2)



$\rho = 2$
$0 \le \phi \le \pi/2$

Final answer:

(1/8) of sphere $\rho = 2$

$0 \le \phi \le \pi/2$

$0 \le \theta \le \pi/2$