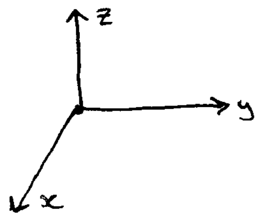
Sept. 4/18

- homework will be online (up to 4 tries per Q)
  - ↳ Practise questions will be posted before midterms
  - ↳ midterm at 7:00pm (Thursday, during lab time)

Sept. 6 /18
vectors

3 dimensional co-ordinate system

$\boxed{\mathbb{R}^3}$ NOTATION

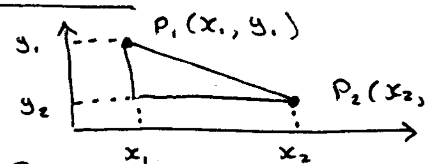


$P(a, b, c)$ — x-coord, y-coord, z-coord

- coordinate axes $\left\{ \begin{array}{l} x\text{-axis} \\ y\text{-axis} \\ z\text{-axis} \end{array} \right.$
- coordinate planes $\left\{ \begin{array}{l} xy\text{-plane} \\ xz\text{-plane} \\ yz\text{-plane} \end{array} \right.$

Distance between two points in $\mathbb{R}^3$

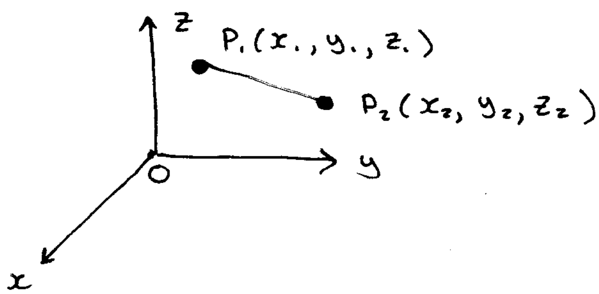
Remember 2 dim :



$|P_1P_2|$ = distance between P_1 and P_2

$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

3 - dim :



$$|P_1P_2| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

Example: Which surfaces in $\mathbb{R}^3$ are represented by the Following equations?

(1) $y = 5$

(2) $x = y$

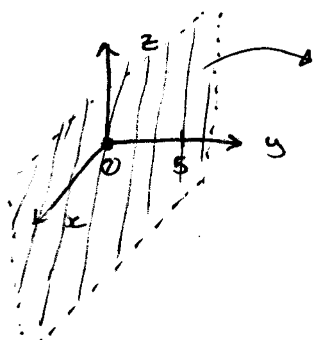
(3) $z = x^2$

(4) $x^2 + y^2 = 9$

(5) $x^2 + y^2 + z^2 = 25$

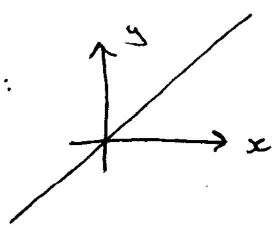
Solution: (1) $y = 5$

we have to Find all points in $\mathbb{R}^3$ (x, y, z) satisfying the constraint $y = 5$.

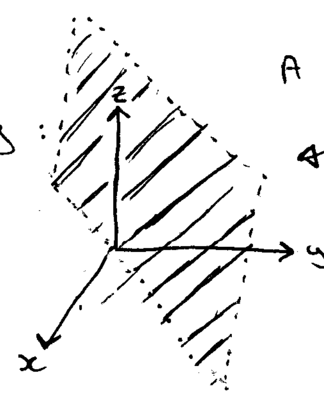


A plane (Parallel to x-z plane, intersecting $y = 5$)

(2) in 2-dim:

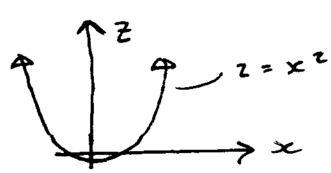


$x = y$:

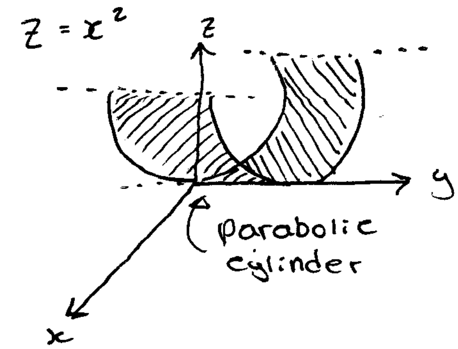


A plane

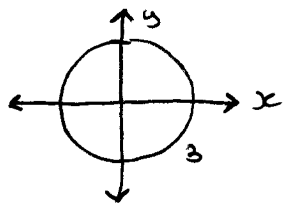
(3) in 2-dim:



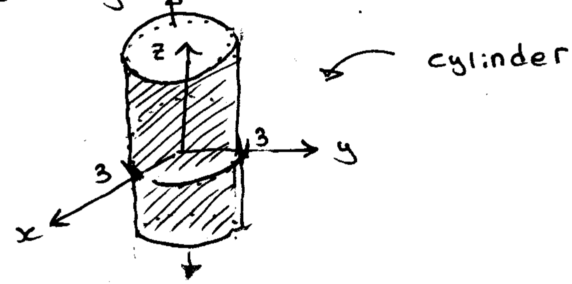
$z = x^2$



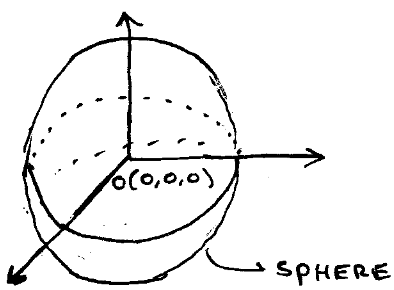
(4) in 2-dim:



$x^2 + y^2 = 9$



(5) $x^2 + y^2 + z^2 = 25$



$$\begin{aligned} |OP| &= \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} \\ &= \sqrt{x^2+y^2+z^2} \\ &= \sqrt{25} = 5 \end{aligned}$$

Example: Which surfaces in $\mathbb{R}^3$ are represented by the following equations?

(1) $z = x^2 + y^2$

(2) $y = z + x^2$

Solution: we will try to use the "trace method":

we will take 2-dim cross sections of the surface and we will visualize the surface based on this information.

1^{st}: cross-section with x-y plane

= intersection with plane $z = \emptyset$

$$\begin{cases} z = x^2+y^2 \\ z = \emptyset \end{cases} \sim \emptyset = x^2 + y^2 \sim \begin{matrix} x = \emptyset \\ y = \emptyset \end{matrix} \quad \begin{matrix} (\emptyset, \emptyset) \\ \text{POINT} \end{matrix}$$

2nd : cross-section with xz-plane ($y = 0$)

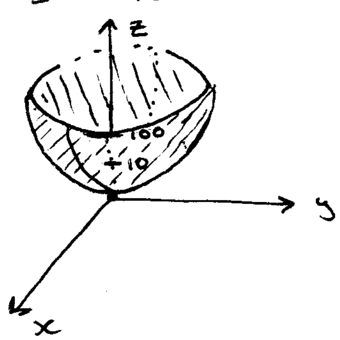
$$\begin{cases} z = x^2 + y^2 \\ y = 0 \end{cases} \rightsquigarrow z = x^2 \quad \text{parabola}$$

3rd : cross-section with yz ($x = 0$)

$$\begin{cases} z = x^2 + y^2 \\ x = 0 \end{cases} \rightsquigarrow z = y^2 \quad \text{parabola}$$

4th : Cross-section with $z = k$ → your Favourite number

$$\begin{cases} z = x^2 + y^2 \\ z = k \end{cases} \rightsquigarrow k = x^2 + y^2 \longrightarrow \begin{array}{ll} k > 0 & : \text{circle of radius } \sqrt{k} \\ k < 0 & : \text{no intersection} \end{array}$$



(2) $y = z + x^2$

1st cross-section with $x = 0$ (yz-plane)

$$\begin{cases} y = z + x^2 \\ x = 0 \end{cases} \rightsquigarrow y = z \quad (\text{a line})$$

2ND cross-section with $z = 0$ (xy-plane)

$$\begin{cases} y = z + x^2 \\ z = 0 \end{cases} \rightsquigarrow y = x^2 \quad (\text{parabola})$$

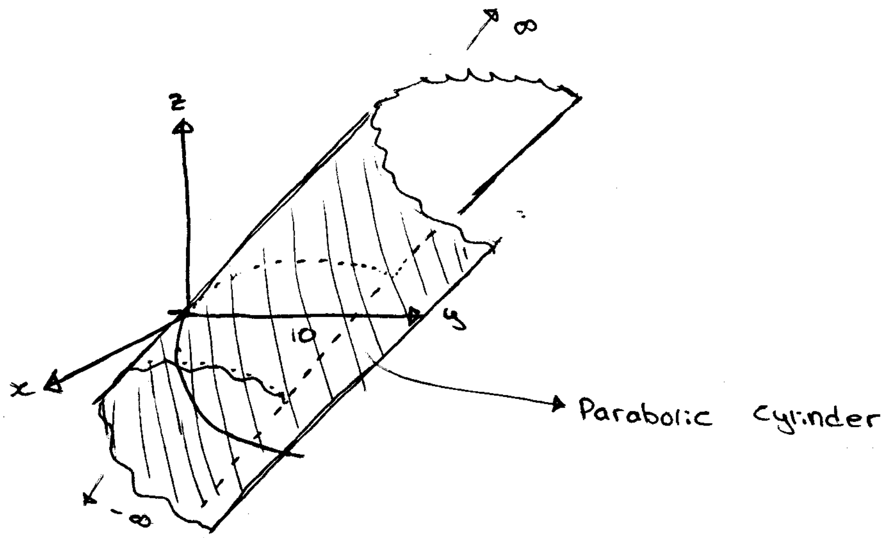
3RD cross-section with $y = 0$ (xz plane)

$$\begin{cases} y = z + x^2 \\ y = 0 \end{cases} \rightsquigarrow 0 = z + x^2$$

$z = -x^2$ (downward parabola)

4th cross-section with $z = k$ ~ your favourite number

$$\begin{cases} y = z + x^2 \\ z = k \end{cases} \rightsquigarrow y = k + x^2 \quad (\text{parabola})$$


∞
z
y
10
x
-∞
Parabolic Cylinder