- last time - parity check code.
  - Hamming code.
  - experiment, outcome, sample space.
  - discrete / continuous.
  - event.
  - intersection $E \cap F$.
  - mutually exclusive.

Let E and F be events. Then their <u>union</u>, $E \cup F$, is the event that E or F (or both) will occur.

It is the set of all outcomes lying in at least one of E or F.

- e.g. $E = \{1,3,5\}$, $F = \{2,4,6\}$, $G = \{3,6\}$.

  $E \cup F = \{1,2,3,4,5,6\}$, $E \cup G = \{1,3,5,6\}$, $F \cup G = \{2,3,4,6\}$.

$E \cup E = E$, $E \cup S = S$, $E \cup \emptyset = E$.

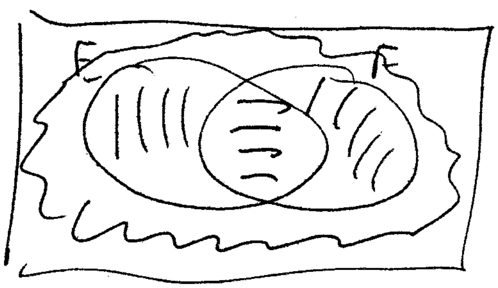
Let E be an event. Then its complement, $\overline{E}$ is the event that E does not occur. It is the set of all outcomes in S that are not in E.

- e.g. E, F, G as above. $\overline{E} = F$, $\overline{F} = E$, $\overline{G} = \{1, 2, 4, 5\}$.

$\overline{S} = \phi$, $\overline{\phi} = S$, $\overline{(\overline{E})} = E$.

We can illustrate using a Venn diagram. The sample space is indicated with a rectangle. Events are circles inside it. The intersections are the overlapping regions.

- e.g.



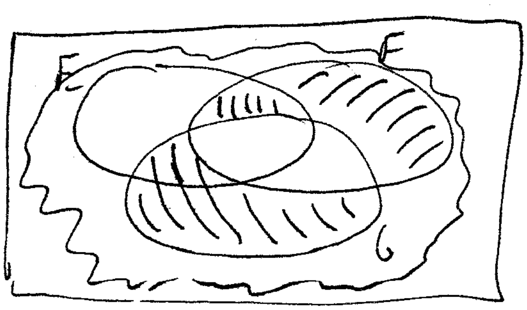
$\equiv = E \cap F$

$||| = E \cap \overline{F}$

$/// = \overline{E} \cap F$

$= \overline{E} \cap \overline{F} = \overline{E \cup F}$.

- e.g.



$|||| = E \cap F \cap \overline{G}$.

$/// = \overline{E} \cap F \cap \overline{G}$

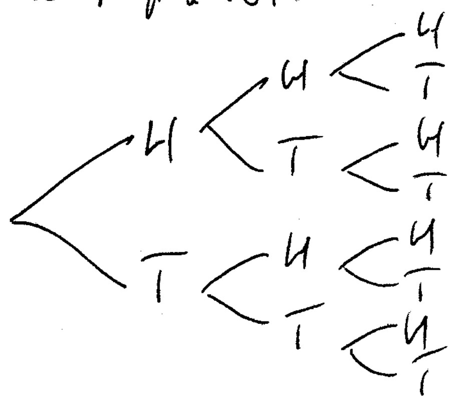
$\backslash\backslash\backslash = G \cap \overline{F}$.

$= \overline{E} \cap \overline{F} \cap \overline{G} = \overline{E \cup F \cup G}$.

$E \cap F = F \cap E$.

A multistage experiment can be illustrated with a <u>tree diagram</u>.

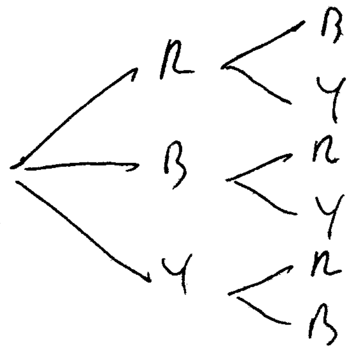
- e.g. we flip a coin 3 times.



HHH
HHT
HTH
HTT
THH
THT
TTH
TTT.

- e.g. a drawer contains a red sock, a blue sock, and a yellow sock.

  we draw 2 socks at random, without replacement.



RB
RY
BR
BY
YR
YB.

MULTIPLICATION RULE: Suppose an experiment has k stages.

Also, suppose there are n_i outcomes at stage i, no matter what happened before. Then the total number of possible outcomes is $n_1 n_2 \cdots n_k$.

-e.g. Flip a balanced coin 9 times. Total # outcomes: $2^9 = 512$.

-e.g. our Xena fan club has 10 members. We must select a president, VP, treasurer. How many ways? $10 \cdot 9 \cdot 8 = 720$.

-e.g. We have 6 Buffy action figures to line up on a shelf.

# of ways? $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

Let n be a positive integer. Then

$$n! := n(n-1)(n-2)\cdots(2)(1)$$

"n factorial"

$0! = 1$.

-e.g. $5! = 5(4)(3)(2)(1) = 120$.

A <u>permutation</u> is an ordered list of elements drawn without replacement from a set.

Suppose we have a set of size n, and we want a permutation of length k. The number of possibilities is:

$$P(n,k) = {}_nP_k = n(n-1)(n-2)\cdots(n-k+1)$$

$$= \frac{n(n-1)\cdots(n-k+1)(n-k)(n-k-1)\cdots(1)}{(n-k)(n-k-1)\cdots(1)}$$

$$= \frac{n!}{(n-k)!}$$

- e.g. $P(8,3) = 8\cdot 7\cdot 6 = \frac{8!}{5!} = 336$.

- e.g. our Xena fan club has 14 female, 12 male members

 we must select a pres, VP, treasurer.

 (i) How many ways?

 (ii) How many ways result in at least one man getting a job?

 (iii) How many ways result in a woman being president?

 (iv) How many ways result in Bob getting a job?

 (i) $P(26,3)$ (ii) $P(26,3) - P(14,3)$.

 (iii) $14\,P(25,2)$ (iv) $P(26,3) - P(25,3)$

- e.g. at a track meet, there are 15 male, 15 female competitors.

 we must award 1st through 5th place ribbons to each gender.

 (i) How many ways? (ii) How many ways result in Jane coming 4th?

 (i) $P(15,5)\,P(15,5)$ (ii) $P(15,5)\,1\,P(14,4)$

- e.g. a license plate has 4 letters followed by 3 digits

(i) # of possible plates? (ii) # of possible plates with no repetition?

(i) $26^4 10^3$ (ii) $P(26,4)\,P(10,3)$

03/02/18

- last time - union, complement.
  - Venn diagram.
  - tree diagram.
  - multiplication rule.
  - permutation. $P(n,k) = n(n-1)\cdots(n-k+1) = \frac{n!}{(n-k)!}$.

A combination is a subset of a particular size drawn from a set. Repetition is not allowed, and order does not matter.

$\#\text{perms} = (\#\text{combs})(\#\text{of orderings})$

$P(n,k) = (\#\text{combs})(k!)$

The number of combinations is $\binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{(n-k)!k!}$

"n choose k"

- e.g. $\binom{6}{2} = \frac{6!}{4!2!} = \frac{720}{24 \cdot 2} = 15$.

$\binom{n}{1} = n$

$\binom{n}{n-1} = n.$

$\binom{n}{k} = \binom{n}{n-k}.$

$\binom{n}{n} = 1 = \binom{n}{0}.$

- e.g. in poker, we draw a 5-card hand randomly from the deck.

  (i) how many hands? (ii) how many hands contain 3 cards of one rank, but another ("full house").

  (i) $\binom{52}{5}$ (ii) $13\binom{4}{3}12\binom{4}{2}$

- e.g. we have 1000 batteries, 40 of them are dead. How many ways are there to pick 12 batteries and get 3 dead ones?

  $\binom{40}{3}\binom{960}{9}.$

- e.g. our Xena fan club has 30 members. We must select a pres, VP, treasurer, and an advisory committee of 5. How many ways to assign the positions?

  $P(30,3)\binom{27}{5} = \binom{30}{5}P(25,3)$

Let E be an event. Then its probability, $Pr(E)$,

is a number with $0 \leq Pr(E) \leq 1$ indicating the

likelihood of that event occurring. The higher, the

more likely.

We write $N(E)$ for the number of outcomes in E.

If all outcomes for an experiment are equally likely, then

for any event E, $Pr(E) = \frac{N(E)}{N(S)}$

- e.g. we roll 2 balanced dice, find the prob. that we get a

total of 10.

$S = \{11, 12, \ldots, 16, 21, 22, \ldots, 66\}$.

$E = \{46, 55, 64\}$.

$Pr(E) = \frac{N(E)}{N(S)} = \frac{3}{36} = \frac{1}{12}$.

- e.g. In Lotto 6/49, we randomly select 6 numbers from

1 to 49. 6 winning numbers are randomly selected.

Find the prob of (i) matching all 6 numbers

(ii) matching 4 numbers

(c) $\dfrac{1}{\binom{49}{6}}$ (ii) $\dfrac{\binom{6}{4}\binom{43}{2}}{\binom{49}{6}}$

- e.g. in poker, find the probability of getting "two pairs".

QQ77K.

$$\frac{\binom{13}{2}\binom{4}{2}^2 44}{\binom{52}{5}}$$

- e.g. find the prob. of getting a "flush".

$$\frac{4\binom{13}{5} - 4\cdot 10}{\binom{52}{5}}$$

rule out straight flush
- all one suit, all in sequence.

- e.g. find the prob. of getting exactly 2 kings, at least 2 queens, no clubs

$$\frac{\binom{3}{2}\left(1+\binom{3}{2}11(31)\right)}{\binom{52}{5}}.$$

- e.g. a license plate consists of 6 letters. Find the prob. that a randomly selected plate (i) has no repeated letters (ii) has at least one Q.

(i) $\dfrac{P(26,6)}{26^6}$ (ii) $\dfrac{26^6 - 25^6}{26^6}$