MARCH 26/18
APPLIED ANAL.

- last time
  - confidence interval for μ $[\bar{x} - E, \bar{x} + E]$
  - hypothesis testing - null/alternative hypothesis
  - Type I / II errors
  - level of significance α
  - usually, we get α, set up experiment
  - σ is known, $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

    $H_0 : \mu \le \mu_0$

    $H_1 : \mu > \mu_0$

$H_0 : \mu \ge \mu_1$
$H_1 : \mu < \mu_0$

$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ and reject the null hypothesis if Z is "too negative"

reject that is $Z < -z_\alpha$

- e.g. the number of jagged metal Krusty-O's in boxes of cereal are normally distributed with a standard deviation of 5. Krusty claims that the average number per box is at least 40. We randomly select 100 boxes and get a sample mean of 38. Test the claim at a .01 level of significance

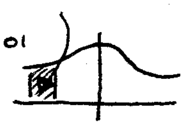
$H_0 : \mu \ge 40$
$H_1 : \mu < 40$

as σ is known, $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ and we reject the null hypothesis $Z < Z_{.01}$

$Z = \frac{38 - 40}{5/\sqrt{100}} = -4$; $Z_{.01} = 2.325$

As $Z < -Z_{.01}$, we reject the claim at a .01 level of significance

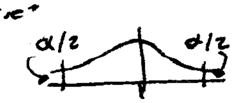
The last hypothesis of tests are called <u>one-handed tests</u>



The <u>two-handed test</u> is:

$H_0 : \mu = \mu_0$
$H_1 : \mu \ne \mu_0$

$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ and we reject that if Z is "too positive" or "too negative"



we reject if $Z > Z_{\alpha/2}$

or $Z < -Z_{\alpha/2}$

That is, reject if $|Z| > Z_{\alpha/2}$

- e.g. The height of Canadian women are normally distributed with $\sigma = 5$cm. Statistics Canada claims that the mean height is 166cm. We randomly select 25 women and get a sample mean of 164 cm

Test the claim at a .01 level of significance

$H_0 : \mu = 166$ / As σ is known, $Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ and reject that if $|Z| > Z_{.005}$

$H_1 : \mu \neq 166$

$Z = \frac{164 - 166}{5/\sqrt{25}} = -2$, $Z_{.005} = 2.575$

As $|Z| \not> Z_{.005}$, we cannot reject the claim at a .01 Level of Significance

If σ is unknown, we use s to approximate it

If σ is unknown, $n \geq 30$, $Z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$, and proceed as before

If σ is unknown, $n < 30$, $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$, and test it

$t > t_\alpha$, $t < -t_\alpha$, $|t| > t_{\alpha/2}$

- e.g. Krusty claims that the average box of Krusty O's contains at least 200 mg of vitamins. We randomly select n boxes. we get a sample mean of 197 mg and a sample standard deviation of 10 mg. Take the claim at a .05 level of sig: (i) $n = 25$ (ii) $n = 100$

$H_0 = \mu \geq 200$ (i) as σ is unknown, $n < 30$

$H_1 = \mu < 200$ Let $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ and reject H_0 if $t < -t_{.05}$

$t = \frac{197 - 200}{10/\sqrt{25}} = -1.5$, As $\nu = 24$, $t_{.05} = 1.711$

As $t \not< -t_{.05}$, we cannot reject the claim at a .05 level of significance

(ii) As σ is unknown, $n \geq 30$, $Z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ and reject that if $Z < Z_{.05}$

$Z = \frac{197 - 200}{10/\sqrt{100}} = -3$, $Z_{.05} = 1.645$

As $Z < -Z_{.05}$, we reject the claim at a .05 level of significance

Suppose we are performing a two-handed test.
Also assume σ is known, (but this doesn't matter)
We let $Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$, and reject if $|Z| > Z_{\alpha/2}$

$$\left|\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right| > Z_{\alpha/2}$$

$$|\bar{x}-\mu_0| > \frac{Z_{\alpha/2}\sigma}{\sqrt{n}} = E$$

We reject that if $\mu_0 \notin [\bar{x}-E, \bar{x}+E]$
That is, we reject H_0 if μ_0 is not in the $100(1-\alpha)\%$ confidence interval. Only for two-handed tests!

- e.g. Statistics Canada claims that the average height for Canadian men is 177 cm. We randomly select 25 men and get a sample mean at 178 and a sample standard deviation of 5. Test the claim at a .01 level of significance.

$H_0 = \mu = 177$ As σ is unknown, $\sigma < 30$

$H_1 = \mu \neq 177$ $E = \frac{t_{\alpha/2} S}{\sqrt{n}} = \frac{t_{.005}(5)}{\sqrt{25}} = 2.797$ $(\nu = 24)$

The 99% confidence interval for μ is:

$[\bar{x}-E, \bar{x}+E] = [178-2.297, 178+2.297] = [175.203, 180.797]$

As 177 lies in the confidence interval, we cannot reject the claim at a .01 level of significance.

MARCH 28 /18
APPLIED ANAL.

- Last time – hypothesis tests for μ
  - if σ is known, $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$

$H_0 : \mu \le \mu_0$ reject that if $Z > Z_\alpha$
$H_1 : \mu > \mu_0$

$H_0 : \mu \ge \mu_0$ reject that if $Z < -Z_\alpha$
$H_1 : \mu < \mu_0$

$H_0 : \mu = \mu_0$ reject that if $|Z| > Z_{\alpha/2}$
$H_1 : \mu \ne \mu_0$

- what if σ is unknown?
- two handed test : Can use Confidence interval!

Chapter 9 – Inferences Concerning Variances (9.1 – 9.3)

Normal populations we want to know about σ^2 (or σ)

We take a random sample of size n

Normally, we use S^2 to estimate σ^2 (or s to estimate σ).

If n is small, we can use the sample range, $\mathbb{R}$ = biggest − smallest

Then $\sigma \simeq \mathbb{R}/d_2$

- e.g. we take a random sample : 8, 23, 7, 14, 11, 12, 9

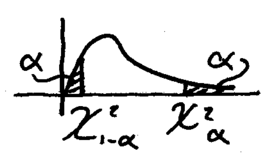
$n = 23 - 7 = 16$, $d_2 = 2.704$, $\sigma \simeq 16/2.704$

Let us discuss S^2

Let $\chi^2 = \frac{(n-1)S^2}{\sigma^2}$. This has chi-space distribution

with $\nu = n-1$

A table gives χ^2_α values when
$P_r(\chi^2 > \chi^2_\alpha) = \alpha$



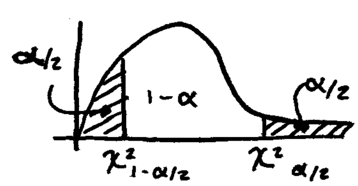
With probability $1-\alpha$:

$\chi^2_{1-\alpha/2} \le \chi^2 \le \chi^2_{\alpha/2}$

$\chi^2_{1-\alpha/2} \le \frac{(n-1)S^2}{\sigma^2} \le \chi^2_{\alpha/2}$

$\frac{1}{\chi^2_{\alpha/2}} \le \frac{\sigma^2}{(n-1)S^2} \le \frac{1}{\chi^2_{\alpha/2}}$

$\frac{(n-1)S^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{\alpha/2}}$



The $100(1-\alpha)\%$ confidence interval for σ^2 is :

$$\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2}} , \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} \right]$$

For σ :

$$\left[\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} , \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}} \right]$$

- e.g. We take a random sample of 21 boxes of Krusty O's and find that the max number of jagged metal Krusty O's per box is 30 with a sample standard deviation of 8. Find a 95% confidence interval for the standard deviation in all boxes.

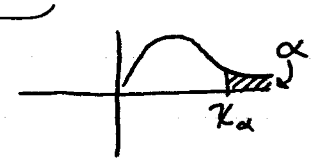
$$\left[\sqrt{\frac{20(8)^2}{\chi^2_{.025}}} , \sqrt{\frac{20(8)^2}{\chi^2_{.975}}} \right] = \left[\sqrt{\frac{20(8)^2}{34.170}} , \sqrt{\frac{20(8)^2}{9.591}} \right]$$

Hypothesis testing, for σ^2 (or σ):

$H_0 : \sigma^2 \le \sigma_0^2$ $\quad \chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$, and reject that if χ^2 is too large

$H_1 : \sigma^2 > \sigma_0^2$

if $\chi^2 > \chi^2_\alpha$ then reject.



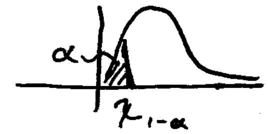
- e.g. A Pharmaceutical claims that the standard deviation in the amount of active ingredient in their pills is at most 5 micrograms. We randomly select 51 pills and get a sample standard deviation of 7 micrograms. Test the claim at a .05 level of significance.

$H_0 : \sigma^2 \le 5^2$ $\quad \chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$ and reject if $\chi^2 > \chi^2_{.05}$

$H_1 : \sigma^2 > 5^2$ $\quad \chi^2 = \frac{50(7^2)}{5^2} = 98$; As $\nu = 50$, $\chi^2_{.05} = 67.505$

$H_0 : \sigma^2 \geq \sigma_0^2$ $\quad \chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$; reject if χ^2 is "too-large"

$H_1 : \sigma^2 < \sigma_0^2$ $\quad$ reject if $\chi^2 < \chi^2_{1-\alpha}$



- e.g. A botanist needs leaves of various diameter. He insists upon a standard deviation of at least 10 cm. We randomly select 21 leaves and get a sample standard deviation of 9 cm. Test the claim that the leaves are satisfactory at a .01 level of significance

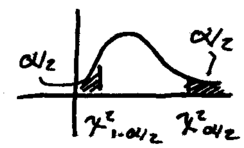
$H_0 : \sigma^2 \geq 10^2$ $\quad \chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$, reject that if $\chi^2 < \chi^2_{.99}$

$H_1 : \sigma^2 < 10^2$ $\quad \chi^2 = \frac{20(9)^2}{10^2} = 16.2$. As $\nu = 20$, $\chi^2_{.99} = 8.26$

As $\chi^2 \nless \chi^2_{.99}$ we cannot reject the claim at a .01 level of significance

$H_0 : \sigma^2 = \sigma_0^2$ $\quad \chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$; reject that if χ^2 is "too big" or "too small"

$H_1 : \sigma^2 \neq \sigma_0^2$



Reject that if $\chi^2 > \chi^2_{\alpha/2}$ or $\chi^2 < \chi^2_{1-\alpha/2}$

-e.g. Stats Canada claims that the standard deviation in the length of Canadian men is 5cm. We randomly select 101 men and get a sample standard deviation of 4 cm. Test the claim at a .05 level of significance.

$H_0 : \sigma^2 = \sigma^2$ $\quad \chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$; reject that if χ^2 is

$H_1 : \sigma^2 \neq \sigma^2$ $\quad$ either $> \chi^2$ ($\chi^2 > \chi^2_{.025}$ or $\chi^2 < \chi^2_{.975}$)

As $\nu = 100$, $\chi^2_{.025} = 129.561$

$\chi^2_{.975} = 79.222$

As $\chi^2 < \chi^2_{.975}$, we reject the claim at a .05 level of significance.