Lecture The Ratio and Root Tests (Section 9.6)

Thm (The Ratio Test)

1. The series $\sum_{n=1}^{\infty} a_n$ converges absolutely if

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$$

2. The series diverges absolutely if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$

3. If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$ The test is inconclusive

- $\sum a_n$ conv. absolutely if $\sum |a_n|$ conv.
- Absolute conv. test

  $\sum a_n$ conv. absolutely then it is convergent

Examples

1. $\sum_{n=1}^{\infty} \frac{5^n}{(n+1)!}$

Let $a_n = \frac{5^n}{(n+1)!}$. Then $a_n = \frac{5^{n+1}}{(n+2)!}$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+2)!}}{\frac{5^n}{n!}} = \lim_{n\to\infty} \frac{5(n+1)!}{(n+2)!}$$

$$\lim_{n\to\infty} \frac{5\cdot 1\cdot 2\cdot 3\cdots n(n+1)}{1\cdot 2\cdot 3\cdots n(n+1)(n+2)} \Rightarrow \lim_{n\to\infty} \frac{5}{n+2} = 0 < 1$$

So by the ratio test the series conv.

2. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 4^{n+1}}{3^n}$ Let $a_n (-1)^n \frac{n^2 4^{n+1}}{3^n}$. Then

$$a_{n+1} = (-1)^{n+1} \frac{(n+1)^2 \cdot 4^{n+2}}{3^{n+1}}$$

2

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}(n+1)^2\cdot 4^{n+2}}{3^{n+1}}}{\frac{(-1)^n n^2\cdot 4^{n+1}}{3^n}}\right|$$

$$= \lim_{n\to\infty}\left| - \frac{(n+1)^2}{n^2}\cdot\frac{4}{3}\right| = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^2\frac{4}{3}$$

$$= \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2\cdot\frac{4}{3} = \frac{4}{3} > 1$$

Hence, the series div. by the Ratio Test

(3) $\sum_{n=1}^{\infty}\frac{n!}{(3n)!}$ Let $a_n = \frac{n!}{(3n)!}$. Then $a_{n+1} = \frac{(n+1)!}{(3n+3)!}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{(n+1)!}{(3n+3)!}}{\frac{n!}{(3n)!}} = \lim_{n\to\infty}\frac{\frac{1\cdot2\cdot3\cdots n\,(n+1)}{1\cdot2\cdot3\cdots3n\,(3n+1)(3n+2)(3n+3)}}{\frac{1\cdot2\cdot3\cdot n}{1\cdot2\cdot3\cdots3n}}$$

$$\lim_{n\to\infty}\frac{n+1}{(3n+1)(3n+2)\underbrace{(3n+3)}_{=3(n+1)}} = \lim_{n\to\infty}\frac{1}{(3n+1)(3n+2)3}$$

$= 0 < 1$

Hence the series conv. by the Ratio Test.

(4) Determine for which x the series $\sum_{n=1}^{\infty}\frac{1}{n}(x-1)^n$ conv.

<u>Sol.</u>

Let $a_n = \frac{1}{n}(x-1)^n$. Then $a_{n+1} = \frac{1}{n+1}(x-1)^{n+1}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{n+1}(x-1)^{n+1}}{\frac{1}{n}(x-1)^n}\right| = \lim_{n\to\infty}\frac{n}{n+1}|x-1| = |x-1|$$

By the Ratio Test, the series conv. if $|x-1|<1$ and div. if $|x-1|>1$

$$\lim_{n\to\infty}\frac{n}{n+1}\cdot\frac{1/n}{1/n} = \frac{1}{0+1} = 1$$

<u>Conv:</u> $|x-1| < 1 \iff -1 < x-1 < 1 \iff 0 < x < 2$

<u>Div:</u> $|x-1| > 1 \iff x > 2$ or $x < 0$

When $|x-1| = 1 \iff x = 0$ or $x = 2$

<u>Case $x=0$</u> $\sum_{n=1}^{\infty} \frac{1}{n}(x-1)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Conv. by the alternating series test

<u>Case $x=2$</u> $\sum_{n=1}^{\infty} \frac{1}{n}(x-1)^n = \sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic Series (divergent)

<u>Thm</u> (The Root Test)

1. The series $\sum_{n=1}^{\infty} a_n$ converges absolutely if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$
2. The series $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$
3. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$ The test is inconclusive.

Examples:

1. $\sum_{n=0}^{\infty} \frac{3^n}{n^n}$ Let $a_n = \frac{3^n}{n^n}$. Then

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{3^n}{n^n}} = \lim_{n\to\infty} \frac{3}{n} = 0 < 1$$

Hence the series conv. by the root test

2. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{e}{3}\right)^n$

Let $a_n = (-1)^n \left(\frac{e}{3}\right)^n$

Then $\lim \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{(-1)^n \left(\frac{e}{3}\right)^n}$
$= \frac{e}{3} < 1$

Then the series conv. absolutely by the Root Test. and conv. by the absolute ~~test~~ conv. test.

Examples

3 $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$

Let $a_n = \left(\frac{n+1}{n}\right)^n$ so

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{n+1}{n}\right)^n} = \lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} \frac{n/n + 1/n}{n/n} = 1$$

$\therefore$ The root test is inconclusive.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty} \left(1 + 1/n\right)^n \Rightarrow e \neq 0$$

$$y = \lim_{x\to\infty} \left(1 + 1/x\right)^x$$

$$\ln y = \lim_{x\to\infty} \ln\left(\left(1 + 1/x\right)^x\right) = \lim_{x\to\infty} x \ln\left(1 + 1/x\right)$$

$$= \lim_{x\to\infty} \frac{\ln(1 + 1/x)}{1/x}$$

$$= \lim_{x\to\infty} \frac{\left(\ln|1 + 1/x|\right)'}{(1/x)'}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1+1/x}\left(\cancel{-1/x^2}\right)}{\left(\cancel{-1/x^2}\right)} = 1$$

Hence the series div. by the n^{th} term divergence test.

n^{th} term div. test

If $\lim_{n\to\infty} a_n \neq 0$

then

$\sum a_n$ div.

Sequences (Section 9.1)

① $\lim_{n\to\infty} n^2(1-\cos(1/n))$

$\hookrightarrow (\infty)(1-1) \Rightarrow \infty\cdot 0$ (indeterminate)

$\lim_{x\to\infty} x^2(1-\cos(1/x)) \Rightarrow \lim_{x\to\infty} \frac{1-\cos(1/x)}{1/x^2} \quad \left(=\frac{0}{0}\right)$

$\Rightarrow \lim_{x\to\infty} \frac{\sin(1/x)(-1/x^2)}{(-2/x^3)}$

$\Rightarrow \lim_{x\to\infty} \frac{\sin(1/x)}{-2/x} = \lim_{x\to\infty} \frac{\cos(1/x)(-1/x^2)}{-2/x^2} = 1/2$

$\Rightarrow \lim_{x\to\infty} n^2(1-\cos(1/n)) = 1/2$

② $\lim_{n\to\infty} \sin^2(\pi/2\, n)$ where $\sin(\pi/2)=1$

$\therefore$ DNE.

$\sin(2\pi/2)=0$

$\sin(3\pi/2)=-1$

③ $\lim_{n\to\infty} \sin^2(\pi/2(2n+1)) = 1$

$\sin(4\pi/2)=0$

$\sin(\pi/2\, n) = \begin{cases} 0 \\ -1 \\ 1 \end{cases}$

④ $\lim_{n\to\infty} \sqrt[n]{2} = \lim_{n\to\infty} 2^{1/n} = 2^0 = 1$

⑤ $\lim_{n\to\infty} \sqrt[n]{n} = \lim_{n\to\infty} n^{1/n} = \infty^0$ (indeterminate)

$y = \lim_{x\to\infty} x^{1/x} \Rightarrow \ln y = \lim_{x\to\infty} \ln(x^{1/x})$

$\Rightarrow \lim_{x\to\infty} \frac{\ln x}{x}$

$\Rightarrow \ln y = 0 \qquad \Rightarrow \lim_{x\to\infty} \frac{1/x}{1} = 0$

$y = e^0 = 1$

$\Rightarrow \lim_{x\to\infty} x^{1/x} = 1$

$\Rightarrow \lim_{n\to\infty} n^{1/n} = 1$

⑥ $\lim_{n\to\infty} \frac{1}{n}(1-\cos n)$

By squeeze theorem:

$\Rightarrow -1 \le \cos n \le 1$

$\Rightarrow 0 \le 1-\cos n \le 2$

$\Rightarrow 0 \le \frac{1}{n}(1-\cos n) \le 2/n$

$\searrow \; 0 \; \swarrow$

likely an exam question.

* 7 Show that the sequence

$$a_n = \frac{2^n}{n!}$$

is bounded and non-increasing, and compute its limit.

NOTE THAT:

Non-increasing

$a_n \geq a_{n+1} \ \forall n$

Bounded

$N \leq a_n \leq M \ \forall n$

First, prove either: $\boxed{a_n - a_{n+1} \geq 0}$ or $\boxed{\dfrac{a_n}{a_{n+1}} \geq 1}$

Solution:

$$\frac{a_n}{a_{n+1}} = \frac{\frac{2^n}{n!}}{\frac{2^{n+1}}{(n+1)!}} = \frac{(n+1)!}{2 \cdot n!} = \frac{(1 \cdot 2 \cdot 3 \cdot 4 \cdots n)(n+1)}{2 \cdot (1 \cdot 2 \cdot 3 \cdot 4 \cdots n)}$$

$$= \frac{n+1}{2} \geq 1$$

Hence $a_n \geq a_{n+1} \ \forall n$

So, $\{a_n\}$ is non-increasing

we have $0 \leq a_n \ \forall n$

$$a_n = \frac{2^n}{n!} = \frac{2 \cdot 2 \cdots 2}{1 \cdot 2 \cdots (n-1)\, n}$$

$$= \underset{\substack{\| \\ 2}}{\frac{2}{1}} \cdot \underset{\substack{\leq \\ 1}}{\frac{2}{2}} \cdot \underset{\substack{< \\ 1}}{\frac{2}{3}} \cdots \underset{\substack{< \\ 1}}{\frac{2}{n-1}} \cdot \underset{\substack{< \\ 1}}{\frac{2}{n}}$$

So, $\{a_n\}$ is bounded $\leq 2 \ \forall n$

$$0 \leq a_n \leq 2 \cdot 1 \cdot 1 \cdots 2/n = 4/n$$

$$0$$

$= \lim a_n = 0$ by the squeeze thm.

8 $\dfrac{n!}{(2n)!}$

Lecture Taylor Polynomials and Approximation

Polynomials $P(x) = a_0 + a_1 x^1 + a_2 x^2 + \dots a_n x^n, \quad a_n \neq 0$

Degree: $n = P$

Def'n: (n^{th} taylor polynomial and Maclaurin Series)

If f has n derivatives at c

Then

$$P_n(x) = f(c) + \frac{f'(c)}{1!}(x-c) + \frac{f^{(2)}(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is the n^{th} Taylor polynomial of f at $x = c$, and

$$Q_n(x) = \frac{f(0)}{1!} + \frac{f'(0)}{1!}x + \frac{f^{(2)}(0)}{2!}x^2 + \dots \frac{f^{(n)}(0)}{n!}x^n$$

is the n^{th} Maclaurin Polynomial of f.

1. $f(x) = e^x$

$f(x) = e^x \qquad f(0) = 1$

$f'(x) = e^x \qquad f'(0) = 1$

$f^{(2)}(x) = e^x \qquad f^{(2)}(0) = 1$

$\vdots \qquad \vdots$

$f^{(n)}(x) = e^x \qquad f^{(n)}(0) = 1$

$$Q_n(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots \frac{x^n}{n!}$$ (n^{th} Maclaurin polynomial of F) (n^{th} Taylor polynomial of $x = 0$)

2. $f(x) = \ln x$ at $x = 1$

$f(x) = \ln x$

$f'(x) = x^{-1} \qquad f'(1) = 1$

$f''(x) = -x^{-2} \qquad f^{(2)}(1) = -1!$

$f'''(x) = 2x^{-3} \qquad f^{(3)}(1) = 2!$

$f^{(4)}(x) = (-2)(-3)x^{-4} \qquad f^{(4)}(1) = -3!$

$f^{(5)}(x) = (2)(3)(4)x^{-5} \qquad \vdots$

$f^{(n)}(1) = (-1)^{n+1}(n-1)!$

$$\ln(x) = \emptyset + \frac{1}{1!}(x-1) - \frac{1}{2!}(x-1)^2 + \frac{2!}{3!}(x-1)^3 \cdots$$

$$\cdots + (-1)^{n+1}\frac{(n-1)!}{n!}(x-1)^n$$

$$= x-1 - \tfrac{1}{2}(x-1)^2 + \tfrac{1}{3}(x-1)^3 - \cdots + (-1)^{n+1}\frac{1}{n}(x-1)^n$$

n^{th} Taylor Polynomial For $f(x) = \ln x$ at $x=1$

(3) $f(x) = \sin(x)$

$$\begin{cases} f(x) = \sin x \\ f'(x) = \cos x \\ f^{(2)}(x) = -\sin x \\ f^{(3)}(x) = -\cos x \end{cases}$$

$$\begin{cases} f^{(4)}(x) = \sin x \\ f^{(5)}(x) = \cos x \\ f^{(6)}(x) = -\sin x \\ f^{(7)}(x) = -\cos x \end{cases}$$

$$f^{(4n)}(x) = \sin x$$
$$f^{(4n+1)}(x) = \cos x$$
$$f^{(4n+2)}(x) = -\sin x$$
$$f^{(4n+3)}(x) = -\cos x$$

At $x = \emptyset$

$$f^{(4n)}(\emptyset) = \sin\emptyset = \emptyset$$
$$f^{(4n+1)}(\emptyset) = \cos\emptyset = 1$$
$$f^{(4n+2)}(\emptyset) = -\sin(\emptyset) = \emptyset$$
$$f^{(4n+3)}(\emptyset) = -\cos(\emptyset) = -1$$

Maclaurin Polynomials For $n = 7, 8, 11$

$$Q_7 = \emptyset + \frac{x}{1!} + \frac{\emptyset x^2}{2!} - \frac{x^3}{3!} + \frac{\emptyset x^4}{4!} + \frac{x^5}{5!} + \frac{\emptyset x^6}{6!} \cdots$$

$$\cdots - \frac{x^7}{7!} \Rightarrow \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

Q_8 = (Same as above)

$n = 11$

$$Q_{11}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}$$

<u>Thm</u> (Taylor thm) :

If f has $(n+1)$ derivatives on an interval I containing c, then for each x in I There is z between x and c, such that :

$$f(x) = P_{n/c}(x) + R_n(x)$$

where $$R_n(x) = \frac{f^{(n+1)}(z)\,(x-c)^{n+1}}{(n+1)!}$$

That is

$$f(x) = \underbrace{f(c) + \frac{f'(c)}{1!}(x-c) + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \dots}_{P_{n,c}(x)}$$

$$+ \underbrace{\frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}}_{R_n(x)}$$

Example :

Approximate $e^{0.1}$ so that the error is less than 0.001.

Solution :

Let $f(x) = e^x$ then the n^{th} Maclaurin Polynomial of f $$Q(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}$$

By Taylor Thm for $c = 0$ and $I = [0,1]$ $(x = 0.1)$

we have $$|f(0.1) - Q(0.1)| = |R_n(0.1)|$$

We have n such that $|f(x) - P_{n/c}(x)| = |R_n(x)|$

$|R_n(0.1)| < 0.001$

We have $|R_n(0.1)| = \left|\frac{e^z}{(n+1)!}(0.1)^{n+1}\right|$

$$\leq \frac{e^1}{(n+1)!}\frac{1}{10^{n+1}}$$

$$< \frac{3}{(n+1)!}\frac{1}{10^{n+1}}$$

We want n such that $\frac{3}{(n+1)!}\frac{1}{10^{n+1}} < \frac{1}{10^3}$

This holds for $n = 2$.

Hence, we get :

$$|\underset{e^{0.1}}{\underset{\parallel}{f(0.1)}} - Q_2(0.1)| < \frac{1}{10^3} = 0.001$$

$$Q_2(x) = 1 + x + \frac{x^2}{2} \Rightarrow Q_2(0.1) = 1 + 0.1 + \frac{0.1^2}{2}$$

$$= 1.105$$

Hence $e^{0.1} \approx 1.105$

Lecture: • Taylor Polynomials and Approximation (Sec. 9.7)
• Power Series (Section 9.8)

Maclaurin Polynomials of $f(x)$

$$Q_n(x) = f(0) + \frac{f'(0)}{1!} + \frac{f^{(2)}(0)x^2}{2!} + \frac{f^{(3)}(0)x^3}{3!} + \dots + \frac{f^{(n)}(0)x^n}{n!}$$

Taylor Polynomials of $f(x)$ at $x = c$

$$P(x) = P_n(x) = f(c) + \frac{f'(c)}{1!}(x-c) + \frac{f^{(2)}(c)(x-c)^2}{2!} + \dots + \frac{f^{(n)}(c)(x-c)^n}{n!}$$

Taylor's Thm

$f(x) = P_n(x) + R_n(x)$

where:

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)}(x-c)^{n+1}$$

where z is between x and c.

Example:

Determine the degree of the Taylor Polynomial at $c=1$ that should be used to approximate $\ln(1.2)$ so that the error is less than 0.001.

Solution:

We need to find

$|R_n(x)| \leq 0.001$

For $x = 1.2$, $c = 1$, $f(x) = \ln(x)$

Note: $\ln 1.2 = f(1.2) = P_n(1.2) + R_n(1.2)$

$$|R_n(1.2)| = \left| \frac{f^{n+1}(z)\,(1.2-1)^{n+1}}{(n+1)!} \right| = \frac{|f^{(n+1)}(z)|\,(0.2)^{n+1}}{(n+1)!}$$

$1 \le z \le 1.2 \;\rightarrow\; 1 \le z \Rightarrow 1/z \le 1 \Rightarrow \frac{1}{z^n} \le 1$

$f(x) = \ln x$

$f'(x) = 1/x = x^{-1}$

$f^2(x) = -x^{-2}$

$f^3(x) = 2x^{-3}$

$f^4(x) = -2 \cdot 3\, x^{-4}$

$f^{(n)}(x) = (-1)^{n+1}(n-1)\, x^{-n}$

$f^{n+1}(z) = (-1)^{n+2}(n)\, z^{-(n+1)}$

$|f^{(n+1)}(z)| = \left|(-1)^{n+2} n!\, \frac{1}{z^n}\right| = n!\,\frac{1}{z^{n+1}} \le n!$

$|R_n(1.2)| \le \frac{n!}{(n+1)!}(0.2)^{n+1}$

$\left[\text{NOTE: } \frac{n!}{(n+1)!} = \frac{1}{n+1}\right]$

$|R_n(1.2)| \le \frac{n!}{(n+1)!}(0.2)^{n+1} = \frac{1}{n+1}(0.2)^{n+1}$

we want n such that

$|R_n(1.2)| \le \frac{1}{n+1}(0.2)^{n+1} \le 0.001$

(this holds for $n=3$)

Def'n: (Power Series)

A power series (centered at $x=0$) is an infinite series of the form

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots$$

A Power Series centered at $x=c$ is an infinite series of the form

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c)^1 + a_2(x-c)^2 + \dots$$

Thm (Convergence of Power Series)

Let $\sum_{n=0}^{\infty} a_n(x-c)^n$ be a power series.

There is $R \geq 0$ (R could be ∞) such that the series converges on the interval $(R+c, R-c)$ (+ converges absolutely) and diverges on $(-\infty, -R+c) \cup (R-c, \infty)$

R is the <u>radius of convergence</u> of the series.

The <u>interval of convergence</u> of the series is the set of all x for which the series converges.

Examples

1. $\sum_{n=1}^{\infty} \frac{2^n}{n!} x^n$

Let $a_n = \frac{2^n}{n!} x^n$

Then $a_{n+1} = \frac{2^{n+1}}{(n+1)!} x^{n+1}$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\frac{2^{n+1}}{(n+1)!} x^{n+1}}{\frac{2^n}{n!} x^n} \right| = \lim_{n\to\infty} \left|\frac{2x}{n+1}\right|$$

$$= \lim_{n\to\infty} \frac{2|x|}{n+1}$$

$\hookrightarrow = 0 \leq 1$ (which means it's convergent, for every x.)

OR.

The series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ conv. on $(-\infty, \infty)$.

<u>Radius of conv.</u> : $R = \infty$

<u>Interval — —</u> : $(-\infty, \infty)$

2 $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n} x^n$

Let $a_n = (-1) \dfrac{2^n x^n}{3^n}$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left|(-1)^n \frac{2^n x^n}{3^n}\right|} = \lim_{n\to\infty} \sqrt{\frac{2^n}{3^n} |x|^n}$$

$$= \lim_{n\to\infty} \frac{2}{3}|x| = \frac{2}{3}|x|$$

The series converges if $\frac{2}{3}|x| < 1$

$\therefore \; -3/2 < x < 3/2$

The series converges if $\frac{2}{3}|x| > 1$

$\therefore \; x > 3/2$ or $x < -3/2$

Radius of conv: $R = 3/2$

For $x = -3/2$ we have $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n} (-3/2)^n$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n} (-1)^n \frac{3^n}{2^n}$$

$$= \sum_{n=0}^{\infty} 1 \quad (\text{diverges})$$