Sept. 19/16

Extra Example

a) $\lim_{x \to 2} \frac{3x-7}{x^2-4} = \frac{3(2)-7}{(2)^2-4} \Rightarrow \frac{1}{0}$ $\boxed{\text{DNE}}$ (Does not exist)

b) $\lim_{x \to 4} \frac{(x^2-3x-4)^2}{x-4} = \frac{(4^2-3(4)-4)^2}{(4)-4} = \frac{0}{0}$ MORE WORK.

$\lim_{x \to 4} \frac{\cancel{(x-4)}^2 (x+1)^2}{\cancel{(x-4)}}$

$\lim_{x \to 4} (x-4)(x+1)^2 \Rightarrow (4-4)(4+1)^2 \Rightarrow \boxed{0}$

Extra Example

Sept. 21/16

a) $\lim_{x \to 0} \frac{\sin 4x}{3x} \times \frac{4/3}{4/3}$

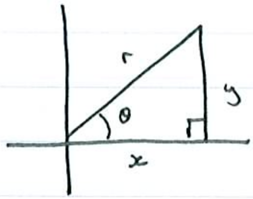
$= \lim_{x \to 0} \left[\frac{\sin 4x}{4x} \times \frac{4}{3} \right]$

$= (1)(4/3)$

$\boxed{= (4/3)}$

Way to Remember

Pythagorean Identities



b) $\lim_{x \to 0} (x \cot 3x)$

$= \lim_{x \to 0} \left(x \cancel{x} \cdot \frac{\cos 3x}{\sin 3x} \right)$

$= \lim_{x \to 0} \left(\frac{x}{\sin x} \cdot \cos 3x \right)$

$= \lim_{x \to 0} \left[\frac{x}{\sin 3x} \cdot \frac{3}{3} \cdot \cos 3x \right]$

$= \lim \left[\frac{3x}{} \cdot \frac{1}{} \cdot \cos 3x \right]$

$x^2 + y^2 = r^2 \quad (\div r^2)$

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$

$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

$\boxed{(\cos\theta)^2 + (\sin\theta)^2 - 1}$

$x^2 + y^2 = r^2 \quad (\div x^2)$

Sept. 23 /16

Extra examples (ex. 1)

Given, $f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8-2x & \text{if } x < 4 \end{cases}$

Determine

$\lim_{x \to 4} f(x)$

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} (8-2x) \Rightarrow 0$

$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (\sqrt{x-4}) \Rightarrow 0$

$\therefore$ the $\lim_{x \to 4} f(x) = \boxed{0}$

Example 2

$f(x) = \begin{cases} \sqrt{x-3} & \text{if } x > 4 \\ 8-2x & \text{if } x < 4 \end{cases}$

Determine

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} 8-2x \Rightarrow 0$

$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \sqrt{x-3} \Rightarrow 1$

$\therefore \lim_{x \to 1} f(x)$ $\boxed{\text{DNE}}$

Example 3

$$\lim_{x \to 4} \frac{5x^2-4}{x+1} = \frac{5(4)^2-4}{(4)+1} = \boxed{\frac{76}{5}}$$

Example 4

$$\lim \frac{x^2-4}{} = \frac{(2)^2-4}{} = \frac{0}{}$$ MORE

$$\lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(2x+3)}$$

Side

$2x^2 - x - 6$

$2x^2 - 4x + 3x - 6$

$2x(x-2) + 3(x-2)$

$$\lim_{x \to 2} \frac{(x+2)}{(2x+3)}$$

$$= \frac{2+2}{2(2)+3} \Rightarrow \boxed{4/7}$$

Example 5

$$\lim_{x \to 3} \frac{x^2+4x-12}{x^2-9} = \frac{(3)^2+4(3)-12}{(3)^2-9} = \frac{9}{0} \quad \boxed{\text{DNE}}$$

Example 6

$$\lim_{x \to 1} \frac{x^3-1}{x^2-1} = \frac{0}{0}$$ MORE WORK

$$\lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$$

$$\lim_{x \to 1} \frac{(x^2+x+1)}{(x+1)} \Rightarrow \frac{(1)+(1)+(1)}{(1+1)}$$

$$\Rightarrow \boxed{3/2}$$

Side

Factor x^3-1

Since (sub $x=1$ in) $1^3-1=0$

we have $x^3-1 = (x-1)$

$$\begin{array}{r} x^2 + x + 1 \\ x-1 \overline{) x^3 + 0x^2 + 0x - 1} \\ (-)\ x^3 - x^2 \\ \hline 0 \quad x^2 + 0x \\ -x^2 - x \\ \hline x - 1 \\ -x - 1 \\ \hline 0R \end{array}$$

(Polynomial division)

(x^2+x+1)

Example 7

$$\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} \Rightarrow \frac{0}{0}$$ MORE WORK

$$\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} \cdot \frac{\sqrt{x+3} + \sqrt{3}}{\sqrt{x+3} + \sqrt{3}}$$

$$\lim_{x \to 0} \frac{\cancel{x} + \cancel{3} - \cancel{3}}{\cancel{x}(\sqrt{x+3} + \sqrt{3})} \Rightarrow \frac{1}{(\sqrt{x+3} + \sqrt{3})} \Rightarrow \frac{1}{2(\sqrt{3})} \Rightarrow \boxed{\frac{\sqrt{3}}{6}}$$

$$\cdot \left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

Example 8

Side

$$\frac{x^2-4-x+2}{(x-2)(x^2-4)}$$

$= \varnothing/\varnothing$

Determine

$$\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$$

$$\lim_{x\to 2}\left(\frac{1}{x+2}-\frac{4}{(x-2)(x+2)}\right)$$

$$\lim_{x\to 2}\left(\frac{1}{x-2}\right)\left(1-\frac{4}{x+2}\right) \Rightarrow \frac{1}{x-2}\left(\frac{x+2-4}{x+2}\right)$$

$$\Rightarrow \frac{1}{\cancel{x-2}}\left(\frac{\cancel{x-2}}{x+2}\right)$$

$$\lim_{x\to 2}\frac{1}{x+2} = \boxed{1/4}$$

Example 9

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} \quad \text{if } f(x) = 2x^2-4$$

$$\lim_{\Delta x\to 0}\frac{2(x+\Delta x)^2-4-(2x^2-4)}{\Delta x}$$

$$\lim_{\Delta x\to 0} 2(x^2+{}^{2x}\Delta x+\Delta x \quad \Rightarrow \quad \lim_{\Delta x\to 0}\frac{2(x^2+2x\Delta x+\Delta x^2)-2x^2}{\Delta x}$$

$$\Rightarrow \lim_{\Delta x\to 0}\frac{\cancel{2x^2}+4x\Delta x+2\Delta x^2-\cancel{2x^2}}{\Delta x}$$

$$\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}(4x+2\Delta x)}{\cancel{\Delta x}}$$

$$\lim_{\Delta x\to 0}(4x+2\Delta x) = \boxed{4x}$$

Example 9

$$\lim \frac{\tan t}{} \Rightarrow \frac{\sin t}{}\cdot\frac{1}{}$$

$$\lim_{x \to 0} \frac{\sin 5x}{\sin 7x}$$

$$\lim_{x \to 0} \left[\frac{\sin 5x}{\sin 7x} \times \frac{\cancel{\sin 7}\,7x}{\cancel{\sin}\,5x} \times \frac{5}{7} \right]$$

$$\lim_{x \to 0} \left[\frac{\sin 5x}{5x} \times \frac{\cancel{\sin}\,7x}{\sin 7x} \times \frac{5}{7} \right]$$

$$= (1)(1)(5/7) = \boxed{5/7}$$

Example 12

$$\lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{1 - \cos^2 x} \Rightarrow \frac{\cancel{1 - \cos x}}{(1 + \cos x)\cancel{(1 - \cos x)}} \Rightarrow \frac{1}{(1 + \cos x)}$$

$$\Rightarrow \boxed{\frac{1}{2}}$$

Example 13

$$\lim_{x \to 0} \left[\sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x}\right) \right] = 0$$

$$-1 \le \sin\left(\frac{\pi}{x}\right) \le 1$$

$$-\sqrt{x^3 + x^2} \le \sqrt{x^3 + x^2} \sin\left(\pi/x\right) \le \sqrt{x^3 + x^2}$$

$$\underbrace{\lim_{x \to 0} \left(-\sqrt{x^3 + x^2}\right)}_{0} \le \lim_{x \to 0} \sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x}\right) \le \underbrace{\lim_{x \to 0} \sqrt{x^3 + x^2}}_{0}$$

$f(x) = \sqrt{\sin x} \qquad (0 \le x < 2\pi)$

Continuous everywhere in its domain

$\sin x \ge 0$

$0 \le x \le \pi$

Thus, f is continuous on $[0, \pi]$

Intermediate Value Theorem