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Properties of Exponents

$a^{m/n} = \sqrt[n]{a^m}$

Example 16.

a) $(-3ab^4)(4ab^{-3})$

$= -12a^{(1+1)}b^{(4+(-3))}$

$= -12a^2b^1 \Rightarrow \boxed{-12a^2b}$

b) $(2xy^2)^3$

$= (2xy^2)(2xy^2)(2xy^2)$

$= 8x^{(1+1+1)}y^{(2+2+2)}$

$\boxed{= 8x^3y^6}$

b) Faster way:

$(2xy^2)^3$

$= 2^3x^3(y^2)^3$

$\boxed{= 8x^3y^6}$

c) $3a(-4a^2)^0$

$= 3a(1)$

$\boxed{= 3a}$

d) $\left(\dfrac{5x^3}{y}\right)^2$

$= \dfrac{5^2(x^3)^2}{y^2}$

$\boxed{= \dfrac{25x^6}{y^2}}$

Example 17

a) x^{-1}

$= 1/x$

b) $\dfrac{1}{3x^{-2}} \Rightarrow \dfrac{1}{3x^{-2}} = \boxed{\dfrac{x^2}{3}}$

$= \dfrac{1}{3(1/x^2)}$

$= \dfrac{1}{\frac{3}{x^2}} = 1 \div 3/x^2$

$\Rightarrow 1 \times \dfrac{x^2}{3}$

$\boxed{= \dfrac{x^2}{3}}$

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c) $\frac{12a^3b^{-4}}{4a^{-2}b}$

$= 3a^{(3-(-2))}b^{(-4-1)}$

$= 3a^5b^{-5}$

$\boxed{= \frac{3a^5}{b^5}}$

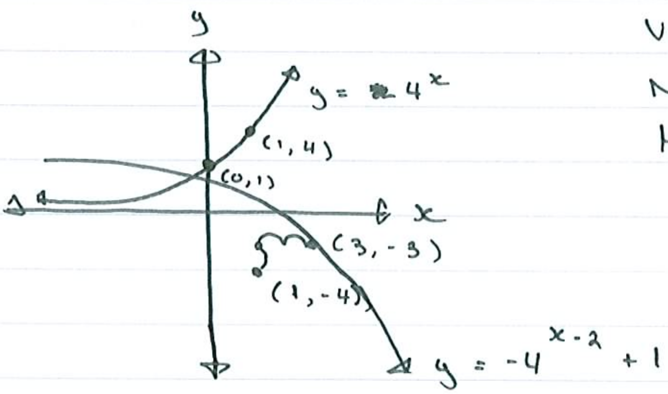
d) $\left(\frac{3x^2}{y}\right)^{-2}$

$= \left(\frac{y}{3x^2}\right)^2$

$= \frac{y^2}{3^2(x^2)^2} \Rightarrow \boxed{\frac{y^2}{9x^4}}$

Example 18

$f(x) = -4^{x-2} + 1$



Vertical reflection in x-axis

Vertical translation 1 unit up

No horizontal stretch / ref.

Horizontal shift 2 units right

Natural base of e

$\lim_{x \to 0} (1+x)^{1/x} = e$

Example 19

a) $\log_2 x = 3$

$\Rightarrow 2^3 = x$

$\boxed{x = 8}$

b) $\log_x 100 = 2$

$\Rightarrow x^2 = 100$

$\Rightarrow x = \pm 10$

(but you can't have a neg. base.)

$\boxed{x = 10}$

c) $\log_3 81 = x$

$\Rightarrow 3^x = 81$

$\Rightarrow 3^x = 3^4$

$\boxed{\Rightarrow x = 4}$

(because 3^x is a 1-1 graph)

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$\log_e x = \ln x, \quad x > 0$

(natural logarithmic function)

$\ln x = b$ if $e^b = x$

Inverse property

$\ln e^x = x$ and $e^{\ln x} = x$

Properties of Logarithms

1. $\ln xy = \ln x + \ln y$
2. $\ln x/y = \ln x - \ln y$
3. $\ln x^2 = 2 \ln x$

(1) PROOF:

let $a = \ln x$ and $b = \ln y$

$e^a = x$ and $e^b = y$

$xy = e^a \cdot e^b = e^{a+b}$

$\ln(xy) = \ln e^{a+b}$

$\ln xy = a + b = \ln x + \ln y$

Example 20

a) $\ln e$

let $x = \ln e$

$e^x = e^{\ln e}$

$e^x = e$

$x = 1$

$\boxed{\ln e = 1}$

b) $\ln 1$

let $x = \ln 1$

$e^x = e^{\ln 1}$

$e^x = 1$

$x = 0$

$\boxed{\ln 1 = 0}$

c) $\ln(1/e)$

$\Rightarrow \ln 1 - \ln e$

$\Rightarrow 0 - 1$

$\boxed{= -1}$

d) $\ln e^5$

$= 5 \ln e$

$= 5(1)$

$\boxed{= 5}$

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EXAMPLE 21

a) $\ln\left(\frac{1}{2}\right)$

$= \ln 1 - \ln 2$

$= 0 - \ln 2$

$\boxed{= -\ln 2}$

b) $\ln (x^2 y)^3$

$= 3 \ln (x^2 y)$

$= 3 [\ln x^2 + \ln y]$

$= 3 [2 \ln x + \ln y]$

$\boxed{= 6 \ln x + 3 \ln y}$

c) $\ln \dfrac{x^2 \cdot \sqrt[3]{(x^5 - e)^2}}{(x+1)^4}$

$= \ln \dfrac{x^2 (x^5 - e)^{2/3}}{(x+1)^4}$

$= \ln x^3 + \ln (x^5 - e)^{2/3} - \ln (x+1)^4$

$= 2 \ln x + \frac{2}{3} \ln (x^5 - e) - 4 \ln (x+1)$

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Ex. 22

a) $\ln(x+4) = \ln 23$

$e^{\ln(x+4)} = e^{\ln 23}$

$x+4 = 23$

$\boxed{x = 19}$

b) $e^{x} = 17$

$\ln e^{x} = \ln 17$

$\boxed{x = \ln 17}$

c) $8e^{2x} - 5 = 15$

$8e^{2x} = 20$

$e^{2x} = 5/2$

$\ln e^{2x} = \ln(5/2)$

$2x = \ln(5/2)$

$\boxed{x = \dfrac{\ln(5/2)}{2}}$

d) $\ln(x-1) + \ln(x+2) = \ln 13$

$\ln[(x-1)+(x+2)] = \ln 13$

$e^{\ln[(x-1)+(x+2)]} = e^{\ln 13}$

$[(x-1)+(x+2) = 13$

$x^2 + x - 2 = 13$

$x^2 + x - 15 = 0$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-15)}}{2(1)}$

$\boxed{x = \dfrac{-1 \pm \sqrt{61}}{2}}$

Extra Example

a) $e^{-x^2} = e^{-3x-4}$

$\ln e^{-x^2} = \ln e^{-3x-4}$

$-x^2 = -3x-4$

$0 = x^2 - 3x + 4$

$0 = (x-4)(x+1)$

$\boxed{x = 4, -1}$

b) $\dfrac{3(2^{x})}{3} = \dfrac{42}{3}$

$2^{x} = 14$

$\log_2 2^{x} = \log_2 14$

↳ $\dfrac{\log 14}{\log 2}$ $\left(\dfrac{\text{big}}{\text{small}}\right)$

$\boxed{\log_2 14 = x}$

c) $2e^{x} + 5 = 57$

$2e^{x} = -52$

$e^{x} = 26$

$\ln e^{x} = \ln 26$

$\boxed{x = \ln 26}$

d) $2(3^{2t-5}) - 4 = 11$

$2(3^{2t-5}) = 15$

$3^{2t-5} = 15/2$

$\log_3 3^{2t-5} = \log_3(15/2)$

$2t - 5 = \log_3(15/2)$

$\boxed{t = \dfrac{\log_3(15/2) + 5}{2}}$