EXAMPLE 9

$\int (x+2)(x+1)^{1/4}\,dx$

let $u = x+1 \longrightarrow x = u-1$

$\frac{du}{dx} = 1 \Rightarrow du = dx$

$\int (u-1+2)\,u^{1/4}\,du$

$= \int (u^{5/4} + u^{1/4})\,du$

$\Rightarrow (4/9)\,u^{9/4} + 4/5\,u^{5/4} + C$

$\boxed{\Rightarrow (4/9)(x+1)^{9/4} + (4/5)(x+1)^{5/4} + C}$

EXAMPLE 10

$\int \frac{\ln\sqrt{x}}{x}\,dx$

$= \int \frac{1/2 \ln x}{x}\,dx$

$= \int \frac{1}{2}\,u \cdot du$

$= \left(\frac{1}{2}\right)\cdot(1/2)\,u^2 + C$

$\boxed{= (1/4)(\ln x)^2 + C}$

let $u = \ln x$

$\frac{du}{dx} = 1/x$

$\Rightarrow du = dx/x$

EXAMPLE 11

$\int \cot x \ln|\sin x|\,dx$

$= \int u\,du$

$= (1/2)\,u^2 + C$

$\boxed{= (1/2)(\ln|\sin x|)^2 + C}$

let $u = \ln|\sin x|$

$\frac{du}{dx} = \frac{1}{\sin x}(\cos x)$

$\Rightarrow \cot x \quad \left(\text{or } \frac{\cos x}{\sin x}\right)$

$du = (\cot x)\,dx$

EXAMPLE 12

$\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$

$= \int \frac{du}{\sqrt{u}}$

$= (2)u^{(1/2)} + C$

$\boxed{= 2\sqrt{1+\tan t} + C}$

let $u = 1+\tan t$

$du/dt = \sec^2 t$

$du = \frac{1}{\cos^2 t}\cdot dt$

EXAMPLE 13

$\int x^2 \underbrace{(x-1)}_{u}^{100} \underbrace{dx}_{du}$ let $u = (x-1) \longrightarrow x = u+1$

$du/dx = 1$ $x^2 = 2u+1$

$du = dx$

$= \int (u^2 + 2u + 1)\, u^{100}\, du$

$= \int (u^{102} + 2u^{101} + u^{100})\, du$

$\Rightarrow (1/103)\, u^{103} + (2/102)\, u^{102} + (1/101)\, u^{101} + C$

$\Rightarrow (1/103)(x-1)^{103} + (1/51)(x-1)^{102} + (1/101)(x-1)^{101} + C$