NOV. 25/16

THERE WILL BE CLASS DEC. 5TH /16.

EXAMPLE 1:

$$\int_0^1 \frac{dx}{x+1} \Rightarrow \ln|x+1| \Big|_0^1$$

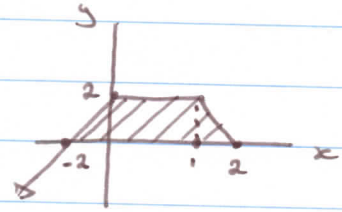
$$\Rightarrow \ln 2 - \ln 1 \Rightarrow \ln 2 - \varnothing$$

$$\boxed{\Rightarrow \ln 2}$$

EXAMPLE 2:

Let

$$f(x) = \begin{cases} x+2 & ; \ x \leq \varnothing \\ 2 & ; \ \varnothing \leq 1 \\ 4-2x & ; \ x > 1 \end{cases}$$



Determine

$$\int_{-2}^{2} f(x)\,dx \Rightarrow \int_{-2}^{0} (x+2)\,dx + \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx$$

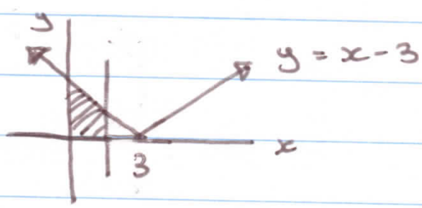
$$\Rightarrow (\tfrac{1}{2})x^2 + 2x \Big|_{-2}^{0} + 2x \Big|_0^1 + 4x - (\cancel{\tfrac{1}{2}})\cancel{2}x^2 \Big|_1^2$$

$$\Rightarrow \left[\varnothing - (\tfrac{1}{2})(-2)^2 + 2(-2)\right] + \left[2(1) - \varnothing\right] + \left[4(2) - (2)^2 - 4(1) - (1)^2\right]$$

$$\boxed{\Rightarrow 5}$$

EXAMPLE 3:

$$\int_0^1 \sqrt{x^2 - 6x + 9}\,dx$$



$$\Rightarrow \int_0^1 \sqrt{(x-3)^2}\,dx$$

$$= \int_0^1 |x-3|\,dx$$

$$\Rightarrow \int_0^1 (-x+3)\,dx$$

$$\Rightarrow \left[-\tfrac{1}{2}x^2 + 3x\right] \Big|_0^1$$

$$= -\tfrac{1}{2}(1)^2 + 3(1) - \varnothing$$

$$= 2\tfrac{1}{2}$$

EXAMPLE 4 :

The velocity of a particle moving along in a straight line is given by $v(t) = 4t + 1$ m/s. Given that the particle is at position $s = 2$ meters @ time $t = 1$, Find an expression for s in terms of t.

$v(t) = 4t + 1$

$\int v(t)\,dt = \int (4t+1)\,dt$

$s(t) = 2t^2 + t + c$

$s(1) = 2(1)^2 + (1) + c = 2$

$c = -1$

$\therefore\ s(t) = 2t^2 + t - 1$

Nov. 28/16

Integration Problems

Example 1

a) Yes - f is integrable on $[-1, 2]$

b) $c \in (-1, 2)$

$$\int_{-1}^{2} (1 + x^2)\,dx$$
$$= \left[x + \frac{x^3}{3}\right]\Big|_{-1}^{2}$$
$$= 2 + 8/3 - (1 - 1/3)$$
$$= 6$$

$$f(c)(b-a) = 6$$
$$f(c)(2-(-1)) = 6$$
$$f(c) = 2$$
$$1 + C^2 = 2$$
$$C^2 = 1$$
$$C = \pm 1$$
$$\boxed{C = 1}$$

Example 2

$$\int_{1}^{2} \frac{4 + u^2}{u^3}\,du$$
$$= \int_{1}^{2} \left(\frac{4}{u^3} + \frac{u^2}{u^3}\right) du$$
$$= \int_{1}^{2} (4u^{-3} + u^{-1})\,du$$
$$= \left[4(-1/2)u^{-2} + \ln|u|\right]_{1}^{0}$$
$$= -2^{-1/2}(1/4) + \ln 2$$
$$\boxed{= 1\tfrac{1}{2} + \ln 2}$$

Example 3

$$\int_{0}^{b} (x+1)\sqrt{x^2 + 2x + 4}\,dx = 56/3$$

let $u = x^2 + 2x + 4$

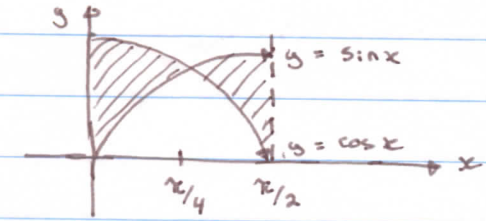
$du/dx = 2x + 2 = 2(x+1)$

$(1/2)\,du = (x+1)\,dx$

$$\int \sqrt{u}\,(1/2)\,du$$
$$= (2/3)u^{3/2}(1/2) + C$$
$$= 1/3(x^2 + 2x + 4)^{3/2}\Big|_{0}^{b} = 56/3$$
$$= 1/3(b^2 + 2b + 4)^{3/2} - 1/3(4)^{3/2} = 56/3$$
$$= 1/3(b^2 + 2b + 4) = 64/3$$
$$= b^2 + 2b + 4 = 64^{2/3}$$
$$= b^2 + 2b - 12 = 0$$

$\boxed{b = -1 + \sqrt{13}}$ (by quadratic formula)

Example 4



$$\int_0^{\pi/4} \cos x\,dx - \int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \sin x\,dx - \int_{\pi/4}^{\pi/2} \cos x\,dx$$

$$\Rightarrow \sin x\Big|_0^{\pi/4} + \cos x\Big|_0^{\pi/4} - \cos x\Big|_{\pi/4}^{\pi/2} - \sin x\Big|_{\pi/4}^{\pi/2}$$

$$\Rightarrow \sin \pi/4 - \sin 0 + \cos \pi/4 - \cos 0 - \cos \pi/2 + \cos \pi/4 - \sin \pi/2 + \sin \pi/4$$

$$+(1/\sqrt{2}) + 0 + (1/\sqrt{2}) - (1) - 0 + (1/\sqrt{2}) - 1 + (1/\sqrt{2})$$

$$\boxed{\Rightarrow 2\sqrt{2} - 2}$$

Example 5

$$\int_{1/2}^{-1} \frac{\cos(x^{-2})}{x^3}\,dx$$

Let $u = x^{-2}$

$du/dx = -2/x^3$

$-1/2\,du = dx/x^3$

$\int \cos u(-1/2\,du)$

$= -1/2 \sin u + C$

$= -1/2 \sin(x^{-2})$

$= -1/2(\sin 1 - \sin 4)$

Example 6

$\int x(2x+5)^8\,dx \Rightarrow$

Let $u = 2x+5 \Rightarrow x = \frac{u-5}{2}$

$du/dx = 2$

$1/2\,du = dx$

$\int\left(\frac{u-5}{2}\right)u^8(1/2\,du)$

$= \int\left(\frac{u^9}{4} - \frac{5u^8}{4}\right)du$

$= (1/10)\cdot\left(\frac{u^{10}}{4}\right) - (1/9)\cdot\left(\frac{5u^9}{4}\right) + C$

$\Rightarrow \left(\frac{(2x+5)^{10}}{40} - \frac{5(2x+5)}{36} + c\right)$

Example 7

$\int_0^{\pi/2}[\sin x]^3\,dx$

$\Rightarrow \int_0^{\pi/2}(\sin x)^2(\sin x)\,dx$

$\Rightarrow \int_0^{\pi/2}(1-\cos^2 x)(\sin x)\,dx$

$\Rightarrow \int_0^{\pi/2}\sin x\,dx - \int_0^{\pi/2}\cos^2 x \sin x\,dx$

Let $u = \cos x$

$du/dx = -\sin x$

$-du = \sin x\,dx$

$-u^3/3 + c$

$-\cos x\Big|_0^{\pi/2} + \frac{\cos^3 x}{3}\Big|_0^{\pi/2}$

$\Rightarrow -\cos \pi/2 + \cos 0 + \frac{(\cos \pi/2)^3}{3} - \frac{\cos 0}{3}$

$\Rightarrow 1 - 1/3 \Rightarrow \boxed{2/3}$

EXAMPLE 8

$$\int \frac{e^{2x}}{2-e^{2x}}\,dx$$

Let $u = 2 - e^{2x}$

$du/dx = -2e^{2x}$

$-\frac{1}{2}\,du = e^{2x}\,dx$

$$\int \frac{-\frac{1}{2}\,du}{u}$$

$\Rightarrow -\frac{1}{2}\ln|u| + C$

$\Rightarrow \boxed{-\frac{1}{2}\ln|2-e^{2x}| + C}$

EXAMPLE 9

$$\int (x+2)(x+1)^{1/4}\,dx$$

Let $u = x+1$

$du/dx = 1 \qquad x = u - 1$

$du = dx$

$$\int (u-1+2)\,u^{1/4}\,du$$

$$\int \left(u^{5/4} + u^{1/4}\right) du$$

$= \frac{4}{9}u + \frac{4}{5}u^{5/4} + C$

$\Rightarrow \boxed{\frac{4}{9}(x+1)^{9/4} + \frac{4}{5}(x+1)^{5/4} + C}$