Basis and Dimension of a Vector Space

$S = \{x_1, \ldots, x_3\}$ is a basis for a subspace W if:

1) $x_1, x_2, \ldots, x_n$ span W
2) $x_1, \ldots, x_n$ are linearly independent

Determine whether the given set of vectors forms a basis for the given subspaces:

a) $S = \{\underbrace{(1, -1, 0)}_{x_1}, \underbrace{(0, 1, -1)}_{x_2}\}$ for the subspace of R^3

$W = \{(x, y, z) : x + y + z = \emptyset\}$

Step 1: Do the given vectors belong to W? Yes.

Step 2: $x + y + z = 0 \Rightarrow z = -(x+y)$

$x = c_1 x_1 + c_2 x_2$ arbitrary vector

$\underbrace{(x, y, -(x+y))}_{x} = c_1(1, -1, 0) + c_2(0, 1, -1)$ vector form of a system of linear equations

Step 3: $\underset{x\ y\ z}{(\emptyset, \emptyset, \emptyset)} = x(1, -1, \emptyset) + (x+y)(0, 1, -1)$ - linear independent

$x = 0 \quad x + y = 0 \quad c_1 + c_2 = \emptyset$

The set S is linearly independent

Find the basis for subspace W generated by

$S = \{\underbrace{(0,1,2,0)}_{x_1}, \underbrace{(0,1,0,0)}_{x_2}, \underbrace{(0,1,1,0)}_{x_3}\}$

$x = c_1 x_1 + c_2 x_2 + c_3 x_3$

x belongs to W $(x \in W)$

The set spans W

What is the $\dim W = ?$

$\emptyset = c_1 x_1 + c_2 x_2 + c_3 x_3$

$$A = \begin{array}{c}\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \\ \begin{array}{ccc} c_1 & c_2 & c_3 \end{array}\end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

c_3 - free variable

echelon form of A

$S_1 = \{\underbrace{(\emptyset, 1, 2, \emptyset)}_{x_1}, \underbrace{(\emptyset, 1, \emptyset, \emptyset)}_{x_2}\}$

x_3 can be expressed as a lin comb of x_1, x_2