LINEAR COMBINATION

x_0 IS CALLED A LIN. COMB OF $x_1, \dots, x_n$ IF THERE EXIST SCALARS OF $c_1, \dots, c_n$ SUCH THAT

$$\underbrace{x_0}_{\text{PARTICULAR VECTOR}} = c_1 x_1 + \dots + c_n x_n$$

1. Determine whether $x_0 = (2, -6, 3)$ is a lin. comb. of vectors $x_1 = (1, -2, -1)$, $x_2 = (3, -5, 4)$

$x_0 = c_1 x_1 + c_2 x_2$ - vector equation

$$(2, -6, 3) = c_1 (1, -2, -1) + c_2 (3, -5, 4)$$
$$(2, -6, 3) = (c_1 + 3c_2, -2c_1 + (-5)c_2, -c_1 + 4c_2)$$

$$A = \left(\begin{array}{cc|c} 1 & 3 & 2 \\ -2 & -5 & -6 \\ -1 & 4 & 3 \end{array}\right) \xrightarrow[R_3 = R_1 + R_3]{R_2 = R_2 + 2R_1} \left(\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 1 & -2 \\ 0 & 7 & 5 \end{array}\right) \xrightarrow{R_3 = -7R_2 + R_3} \longrightarrow \dots$$

$$\dots \underbrace{\left(\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 19 \end{array}\right)}_{\text{ECHELON FORM}} \longrightarrow \text{SYSTEM IS INCONSISTENT.}$$

$0c_1 + 0c_2 = 19$

So x_0 IS NOT A LINEAR COMBINATION OF x_1, x_2 (c_1, c_2 does not exist)

2. Determine whether $x_0 = (-7, 7, 11)$ is a lin. comb. of vectors $x_1 = (1, 2, 1)$, $x_2 = (-4, -1, 2)$, $x_3 = (-3, 1, 3)$

$$x_0 = c_1 x_1 + c_2 x_2 + c_3 x_3$$

$$A = \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left(\begin{array}{ccc|c} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \end{array}\right) \end{array} \longrightarrow \text{reduce to echelon form}$$

$$\left(\begin{array}{ccc|c} 1 & -4 & -3 & -7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

c_3 is a free variable

$c_2 + c_3 = 3 \rightarrow c_2 = 3 - c_3$

$c_1 - 4c_2 = 7 \rightarrow c_1 + 4c_2 + 3c_3$

(3) Write the vector equation that is equivelent to the linear system

$$4x_1 + x_2 + 3x_3 = 9$$
$$1x_1 + 7x_2 - 2x_3 = 2$$
$$8x_1 + 6x_2 - 5x_3 = 15$$

Standard form.

$$AX = B$$

$$\begin{pmatrix} 4 & 1 & 3 \\ 1 & -7 & -2 \\ 8 & 6 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 15 \end{pmatrix}$$

Matrix form

$$C_1x_1 + C_2x_2 + C_3x_3 = x_0$$

$$x_1\begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} + x_2\begin{pmatrix} 1 \\ -7 \\ 6 \end{pmatrix} + x_3\begin{pmatrix} 3 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 15 \end{pmatrix}$$

(4) Determine whether $x_0 = (3, -7, -3)$ is a ~~se~~ lin. comb. of vectors formed from the columns of the matrix

$$A = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{pmatrix}$$
$$x_1 \quad x_2 \quad x_3$$

$$x_0 = c_1x_1 + c_2x_2 + c_3x_3$$

$$\left(\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & 7 \\ -2 & 8 & -4 & -3 \end{array}\right) \xrightarrow{R_3 + (R_1 \cdot 2)} \left(\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & 7 \\ 0 & 0 & 0 & 3 \end{array}\right)$$

$\therefore$ c_1, c_2, c_3 does not exist.

x_0 is not a linear combination or x_1, x_2, x_3

⑤ The subspace W from R^n. The set $S = \{x_1, \dots, x_n\}$ where x_i belongs to W.
The set S spans W if every vector from W can be expressed as a lin. comb. of $x_1 \dots x_n$

$x = c_1 x_1 + \dots + c_n x_n$

arbitrary vector

W is generated by S

$S = \{x_1, \dots, x_n\}$ is a spanning set for W

$W = \text{Span}\{x_1, \dots, x_n\}$

Does the set $S = \{(1,2), (-1,1)\}$ span R^2?

$(x, y) = c_1(1,2) + c_2(-1,1)$ - vector form of a linear system.

$x = c_1 - c_2 \longrightarrow c_1 - c_2 = x$

$y = 2c_1 + c_2 \qquad 2c_1 + c_2 = y$

$$A = \left(\begin{array}{cc|c} 1 & -1 & x \\ 2 & 1 & y \end{array}\right) \xrightarrow{R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & -1 & x \\ 0 & 3 & y - 2x \end{array}\right)$$

$3c_2 = y - 2x$

$\to c_2 = \frac{y - 2x}{3}$

$c_1 - c_2 = x$

$c_1 = x - c_2$

$c_1 \Rightarrow \frac{x + y}{3}$

$$(x, y) = \left(\frac{x + \cancel{3}y}{3}\right)(1,2) + \left(\frac{y - 2x}{3}\right)(-1,1)$$

$\frac{x+y}{3}$ and $\frac{y-2x}{3}$ are defined for any x, y

This means that c_1, c_2 exist for any x, y
(The set S spans R^2)

6 Does the set $S = \{\underbrace{(1,2,3)}_{x_1}, \underbrace{(-1,0,1)}_{x_2}, \underbrace{(0,1,2)}_{x_3}\}$ span R^3?

$x = c_1 x_1 + c_2 x_2 + c_3 x_3$

(x, y, z)

$$A = \left(\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 2 & 0 & 1 & y \\ 3 & 1 & 2 & z \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 0 & x \\ 0 & 2 & 1 & y-2x \\ 0 & 0 & 0 & z-2y-x \end{array}\right)$$

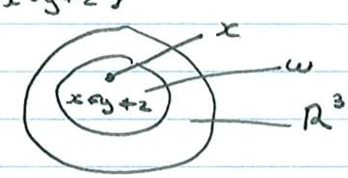
So does S span R^3? NO. $\boxed{z - 2y - x = 0}$ only.

IT meant that the solution of the system exists for x, y, z.

7 Show that $S = \{(1,1,0), (1,0,1)\}$ spans the subspace of R^3. $W = \{(x,y,z)\} : x = y+z\}$

$(y+z, y, z) = c_1(1,1,0) + c_2(1,0,1)$

arbitrary vector taken from W.



$$A = \left(\begin{array}{cc|c} 1 & 1 & y+z \\ 1 & 0 & y \\ 0 & 1 & z \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & y+z \\ 0 & 1 & -z \\ 0 & 0 & 0 \end{array}\right)$$

$(y+z, y, z) = y(1,1,0) + z(1,0,1)$

where y and z are any real nombers.

$\boxed{c_2 = z}$ $\boxed{c_1 = y}$ $c_1 + c_2 = y+z \rightarrow c_1 = y+z - c_2$

$= y+z-z = y$

The set S spans W.