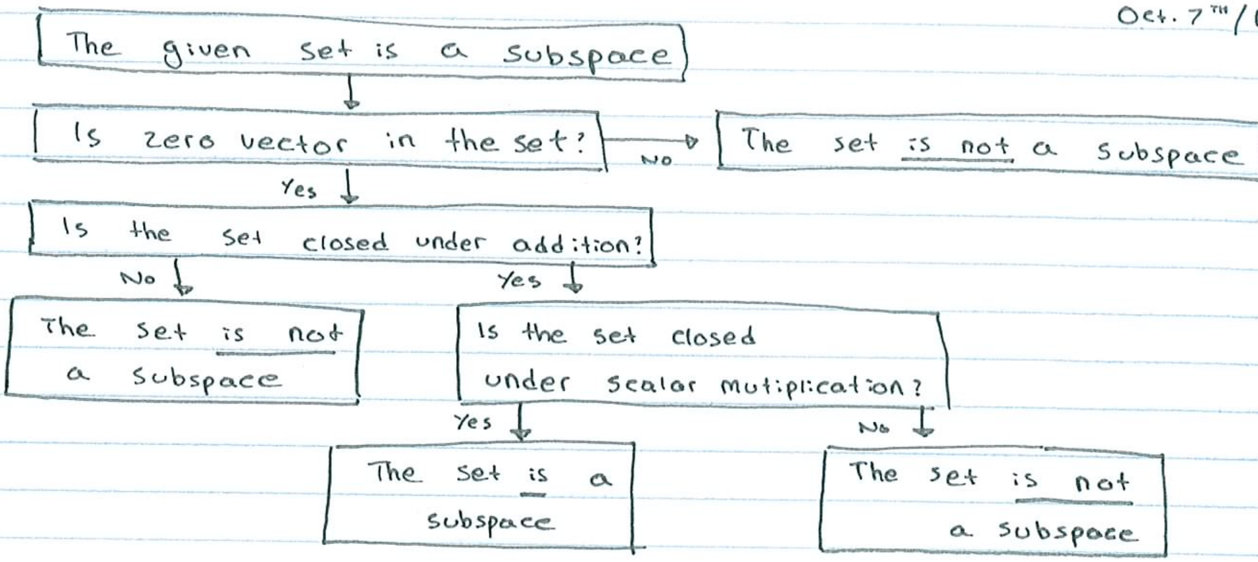
Oct. 7^{TH}/16



Consider the Following example:

If:

$$(7,4,-3) = c_1(1,-2,-5) + C_2(2,5,6)$$

The same would be:

$$(c_1, -2c_1, -5c_1) + (2C_2, 5C_2, 6C_2) = (7,4,-3)$$

$$\begin{aligned} 1c_1 + 2C_2 &= 7 \\ -2c_1 + 5C_2 &= 4 \\ -5c + 6C_2 &= -3 \end{aligned} \Rightarrow \begin{array}{ccc} x_1 & x_2 & x \\ \end{array} \left(\begin{array}{cc|c} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{array}\right) \text{ - Augmented Matrix} \Rightarrow$$

$$\begin{aligned} R_2 &= 2R_1 + R_2 \\ R_3 &= 5R_1 + R_3 \end{aligned} \Rightarrow \left(\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 9 & 18 \\ 0 & 16 & 32 \end{array}\right) \xrightarrow{R_2 = \left(\frac{1}{9}\right) R_2} \left(\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 1 & 2 \\ 0 & 16 & 32 \end{array}\right) \xrightarrow{R_3 = (-16) R_2 + R_3} \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$$

Extra Example:

$x = (1, 0, 3)$ $\qquad x_1 = (0, 2, 4)$ $\qquad x_2 = (1, -1, 1)$

$\qquad x_3 = (-2, 0, 6)$

$$x = c_1 x_1 + C_2 x_2 + C_3 x_3$$

$$A = \left(\begin{array}{ccc|c} 0 & 1 & -2 & 1 \\ 2 & -1 & 0 & 0 \\ 4 & 1 & 6 & 3 \end{array}\right) \xrightarrow{\text{SWAP ROWS.}} \left(\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

ECHELON FORM.

$$\begin{aligned} 2c_1 - 1C_2 &= 0 \\ c_1 &= \tfrac{1}{2} C_2 \end{aligned}$$

$$\begin{aligned} C_2 - 2C_3 &= 1 \\ C_2 &= 1 + 2C_3 \end{aligned}$$

$$\therefore \; x = \left(\tfrac{1}{2} + C_3\right) x_1 + \left(1 + 2C_3\right) x_2 + C_3 x_3$$

Determine whether the given subset of R^n is a subset.

1. is the zero vector in the set?
2. is the set closed under addition?
3. is the set closed under scalar multiplication?

1. The set of all vectors in R^3 such that $x_3 = 0$

$S = \{(x_1, x_2, 0\}$

$x_1 = x_2 = 0$

$0 = (0, 0, 0)$ belongs to S.

$x = (x_1, x_2, 0)$, $y = (y_1, y_2, 0)$

$\boxed{1)}$ $x + y = (x_1 + y_1, x_2 + y_1, 0, 0) = (\underbrace{x_1 + y_1}_{c}, \underbrace{x_2 + y_2}_{d})$

$\boxed{2}$ $cx = (cx_1, cx_2, c0) = (cx_1, cx_2, 0)$

The given set $S = \{x_1, x_2, 0)\}$ is a subset or R^3

$S = \{(x_1, 1, x_2)\}$ is this set a subspace of R^3

$0 = \{(0, 0, 0)\}$ Zero vector does not belong to S.

The set S is not a subspace.

$x_1 = x_2 = 0$ $(0, 1, 0)$

$\boxed{3)}$ The set of all vectors in R^2 such that

$W = \{(x_1, x_2) : x_1 + 2x_2 = 0\}$

zero vector belongs to the given set

$x = \{(x_1 + x_2) : x_1 + 2x_2 = 0\}$

$y = \{(y_1 + y_2) : y_1 + 2y_2 = 0\}$

$x + y = (x_1 + y_1, x_2 + y_2)$

$= (x_1 + y_1) + (2(x_2 + y_2)) = 0$

$\underbrace{(x_1 + 2x_2)}_{0} + \underbrace{(y_1 + 2y_2)}_{0} = 0$ The set is closed under addition

$cx = (cx_1, cx_2)$

$cx_1 + 2cx_2 = 0 \rightarrow c\underbrace{(x_1 + 2\cancel{c}x_2)}_{0} = 0$

$(0,0) = 0$

$y_1 + 2x_2 = 0$

3) The vector space R^2 is not a subspace of R^3 because R^2 is not even a subset of R^3.
→ Vectors in R^3 have 3 components, whereas vectors in R^2 have only two components.

4) The set of all vectors in R^4 such that $x_1 + x_2 = x_3 + x_4$

$S = \{(x_1, x_2, x_3, x_4) : x_1 + x_2 = x_3 + x_4\}$

$0 = (0, 0, 0, 0)$ - belongs to s. $\quad x = (x_1, x_2, x_3, x_4)$

$0 + 0 = 0 + 0 \quad y = (y_1, y_2, y_3, y_4)$

$x_1 + x_2 = x_3 + x_4$

$y_1 + y_2 = y_3 + y_4$

$x + y = (x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4)$ belongs to S

R^4

$W = \{(x, y, z, t)\} : x + y + z + t = 2\}$ x, y, z, t any real numbers.

$0 = (0, 0, 0, 0) \quad 0 + 0 + 0 + 0 \neq 2$

does not belong to w

the w is not a subspace.

$w = \{(x, y, z) : x \geq y\}$

$0 = (0, 0, 0) \quad 0 \geq 0 \quad x = (x_1, y_1, z_1) \quad x_1 \geq y_1$

belongs to $w \quad y = (x_2, y_2, z_2) \quad x_2 \geq y_2$

$x + y = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

The set is closed under addition

$$\begin{array}{r} x_1 \geq y_1 \\ + \; x_2 \geq y_2 \\ \hline x_1 + x_2 \geq y_1 + y_2 \end{array}$$

$cx = \{cx, cy, cz\}$

$cx_1 \geq cy_1$

is correct for $c \geq 0$ / For $c < 0$

$cx_1 < cy_1$

The set does not close under scalar multiplication.

Next exercise:

The set of all solutions to the homogeneous system

$$\begin{aligned} 3x + 7y + z &= 0 \\ -x \quad\quad + z &= 0 \\ x - y + z &= 0 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 3 & 7 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \xrightarrow{R_{13}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 3 & 7 & 1 & 0 \end{array}\right) \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 + R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 4 & -2 & 0 \end{array}\right) \xrightarrow{\quad} \dots$$

$$\xrightarrow{(1/2)R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right) \xrightarrow{R_3 = R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 5 & 0 \end{array}\right)$$

$$\begin{aligned} x + y + z &= 0 \\ y + 2z &= 0 \\ 5z &= 0 \end{aligned} \qquad \begin{aligned} x &= 0 \\ y &= 0 \end{aligned}$$

Determine if the set of all matrices of the form $\left\{\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}\right\} \to S$ is a subspace of $M_{2\times 2}$

The zero matrix is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ belongs to S.

$$\begin{pmatrix} a_1 & b_1 \\ 0 & c_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ 0 & c_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ 0 & c_1 + c_2 \end{pmatrix}$$

the set is closed under addition

$$c\begin{pmatrix} a_1 & b_1 \\ 0 & c_1 \end{pmatrix} = \begin{pmatrix} ca_1 & cb_1 \\ 0 & cc \end{pmatrix}$$

The set S is closed under multiplication

The set $S = \left\{\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}\right\}$ is a subspace of $M_{2\times 2}$.

Let H be the set of all points inside and on the unit circle in the xy-plane.

$H = \{(x, y) : x^2 + y^2 \leq 1\}$

$0 = \{(0, 0)\}$ $\quad 0 + 0 \leq 1$

$x = (x_1, y_1)$ $\quad x_1^2 + y_1^2 \leq 1$ $\quad x + y = (x_1 + y_1, x_2 + y_2)$

$y = (x_2, y_2)$ $\quad x_2^2 + y_2^2 \leq 1$

~~$x_1 = 0.5$ $y_1 = 0.5$ $x_2 = 0.5$ $y_2 = 0.5$~~

$x_1 = 0.5 \quad y_1 = 0.5 \quad 0.5^2 + 0.5^2 = 0.25 + 0.25 = 0.5 \leq 1$

$x_2 = 0.3 \quad y_2 = 0.4 \quad 0.3^2 + 0.4^2 = \underset{0.09}{\cancel{0.09}} + 0.16 = 0.25 \leq 1$

$(0.5 + 0.5)^2 + (0.3 + 0.4)^2 \geq 1^2 + 0.7^2 = 1 + 0.49 = 1.49 \geq 1$

$x_1^2 + 2x_1 y_1 + y^2 + x_2^2 + 2x_2 y_2 + y_2^2 \leq 1$