Sept. 30/18

Find those values of k, for which $\begin{vmatrix} k & k \\ 4 & 2k \end{vmatrix} = 0$

$$\det\begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -1 \\ 1 & 5 & -2 \end{pmatrix} = \left|\begin{array}{ccc|cc} 1 & -2 & 3 & 1 & -2 \\ 2 & 4 & -1 & 2 & 4 \\ 1 & 5 & -2 & 1 & 5 \end{array}\right| \ldots$$

$2k^2 - 4k = 0 \longrightarrow k(2k-4) = 0$

$k = 0 \quad k = 2$

$$\ldots = (1)(4)(-2) + (2)(-1)(1) + (3)(2)(5) \ldots$$

$$\ldots - (1)(4)(3) - (5)(-1)(1) - (-2)(2)(-2) = 9$$

Evaluate the determinate of the following matrices:

$\det I = 1$

$\det 3 = 3$

$$\det\begin{pmatrix} 2 & 1 & 1 \\ 3 & 0 & -1 \\ 4 & 5 & 2 \end{pmatrix} = (-1)^{1+2}(1)\begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} + \ldots$$

$$\ldots + (-1)^{3+2}(5)\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = \ldots$$

$$= -\left[3(2) - (-1)(4)\right] - 5\left[2(-1) - 3(1)\right] = \ldots$$

$$\ldots = -10 + 25 = 15$$

Evaluate the determinate of a given matrix by inspection

$$\det\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} = 24 = 1 \times 4 \times 6$$

$$= 24$$

$$\det\begin{pmatrix} 2 & 0 & 0 \\ 8 & 4 & 0 \\ 3 & 7 & -1 \end{pmatrix} = 2(4)(-1) = (-1)^{1+1}(2)\begin{vmatrix} 4 & 0 \\ 7 & -1 \end{vmatrix} = 2 \cdot \left[4(-1) - 7(0)\right] = -24$$

$$\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} = (1)(3)(5) = 15$$

$$\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 8 \\ -2 & -6 & -4 \end{vmatrix} = (2)\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 8 \\ 1 & 3 & 2 \end{vmatrix} \overset{R_3 = R_3 - R_1}{=} \begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 8 \\ 0 & 0 & 0 \end{vmatrix} = \emptyset$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 7 \\ 1 & 4 & 13 \end{vmatrix} \xrightarrow[R_3 - R_1]{R_2 - R_1} \begin{vmatrix} 1 & 2 & 3 \\ \emptyset & 1 & 4 \\ \emptyset & 2 & 10 \end{vmatrix} \xrightarrow{R_3 := -2R_2 + R_3} \begin{vmatrix} 1 & 2 & 3 \\ \emptyset & 1 & 4 \\ \emptyset & \emptyset & 2 \end{vmatrix} \cancel{\xrightarrow{R_3 = \frac{1}{2}R_3}} \cancel{\begin{vmatrix} 1 & 2 & 3 \\ \emptyset & 1 & 4 \\ \emptyset & \emptyset & 1 \end{vmatrix}}$$

Dont reduce further for determinant

$$\det = (1)(1)(2) = 2$$

(I guess you can swap columns ...)

$$\begin{array}{c} \quad\quad C_2 \;\; C_3 \\ \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} \end{array} = -\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} = (-)(1)(3)(2)(4) = -24$$

$$\boxed{\det A^{-1} = \frac{1}{\det A}}$$

if $\det A = 5$

$\det A^{-1} = 1/5$

$$\boxed{\det A = \det A^T}$$

$$\det \begin{array}{c} c_1 \quad c_2 \quad c_3 \\ \begin{pmatrix} s & s+1 & s+2 \\ t & t+1 & t+2 \\ u & u+1 & u+2 \end{pmatrix} \end{array} \ldots$$

$$\ldots \xrightarrow[C_3 - C_1]{C_2 - C_1} \begin{pmatrix} s & 1 & 2 \\ t & 1 & 2 \\ u & 1 & 2 \end{pmatrix} = 2\det\begin{pmatrix} s & 1 & 1 \\ t & 1 & 1 \\ u & 1 & 1 \end{pmatrix} = \emptyset$$

Is the matrix A invertible

$$A = \begin{pmatrix} 5 & 6 & 1 \\ 0 & 2 & 3 \\ 4 & 6 & 9 \end{pmatrix}$$

$$\det\begin{pmatrix} 5 & 6 & 1 \\ 0 & 2 & 3 \\ 4 & 6 & 9 \end{pmatrix} = (1)^{1+1}(5)\begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} + (-1)^{3+1}(4)\begin{vmatrix} 6 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= 5\,\Big|\underbrace{(2)(9)}_{18} - \underbrace{3(6)}_{-18}\Big| + 4(6(3) - 2(1)) = 64 \neq \emptyset$$

A is invertible.

$$\begin{aligned} x + y - z &= 1 \\ 2x \qquad - 3z &= 0 \\ 2y + z &= 1 \end{aligned}$$

$$\det A = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & -3 \\ 0 & 2 & 1 \end{vmatrix} \xrightarrow{R_2 = -2R_1 + R_2} \begin{vmatrix} 1 & 1 & -1 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{vmatrix} \xrightarrow{R_3 = R_2 + R_3} \begin{vmatrix} 1 & 1 & -1 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Cramer's rule cannot be used because $\det A = 0$ and the system has no unique solution.

$$\begin{aligned} 3x - y + z &= 2 \\ 2x + y - z &= 1 \\ x + 5y - 3z &= 3 \end{aligned}$$

$$\det A = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 5 & -3 \end{vmatrix} = -\det \begin{pmatrix} 1 & 5 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{pmatrix} \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 - 2R_1} \begin{pmatrix} 1 & 5 & 3 \\ 0 & -9 & 5 \\ 0 & -16 & 10 \end{pmatrix}$$

$$\ldots \; -(1)^{1+1}(1)\begin{vmatrix} -9 & 5 \\ -16 & 10 \end{vmatrix} = \left[-9(10) - (-16)(5)\right] = 10 \neq 0$$

The system has a unique solution

$$x = \frac{\det A_1}{\det A} \qquad y = \frac{\det A_2}{\det A} \qquad z = \frac{\det A_3}{\det A}$$

$$\det A_1 = \det \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \\ 3 & 5 & -3 \end{pmatrix} = 6$$

$$\det B = \det \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 3 & 3 \end{pmatrix} = 19$$

$$\det C = \det \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & 5 & 3 \end{pmatrix} = 17$$