

Sept. 30 / 18

Find those values of μ , for which $\begin{vmatrix} \mu & \mu \\ 4 & 2\mu \end{vmatrix} = 0$

$$\det \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -1 \\ 1 & 5 & -2 \end{pmatrix} = \left(\begin{array}{ccc|cc} 1 & -2 & 3 & 1 & -2 \\ 2 & 4 & -1 & 2 & 4 \\ 1 & 5 & -2 & 1 & 5 \end{array} \right) \dots$$

$$2\mu^2 - 4\mu = 0 \rightarrow \mu(2\mu - 4) = 0$$

$$\mu = 0 \quad \mu = 2$$

$$\dots = (1)(4)(-2) + (2)(-1)(1) + (3)(2)(5) \dots$$

$$\dots - (1)(4)(3) - (5)(-1)(1) - (-2)(2)(-2) = 9$$

Evaluate the determinate of the following matrices:

$$\det I = 1$$

$$\det 3 = 3$$

$$\det \begin{pmatrix} 2 & 1 & 1 \\ 3 & 0 & -1 \\ 4 & 5 & 2 \end{pmatrix} = (-1)^{1+2}(1) \begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} + \dots$$

$$\dots + (-1)^{3+2}(5) \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = \dots$$

$$= -[3(2) - (-1)(4)] - 5[2(-1) - 3(1)] = \dots$$

$$\dots = -10 + 25 = 15$$

Evaluate the determinate of a given matrix by inspection

$$\det \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} = 24 = 1 \times 4 \times 6$$

$$= 24$$

$$\det \begin{pmatrix} 2 & 0 & 0 \\ 8 & 4 & 0 \\ 3 & 7 & -1 \end{pmatrix} = 2(4)(-1) = (-1)^{1+1}(2) \begin{vmatrix} 4 & 0 \\ 7 & -1 \end{vmatrix} = 2 \cdot [4(-1) - 7(0)] = 24$$

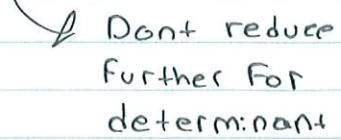
$$\det \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} = (1)(3)(5) = 15$$

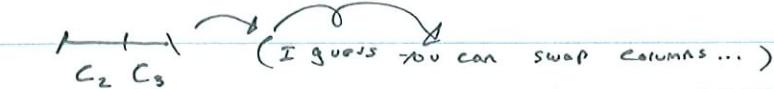
(2)

$$\left| \begin{array}{ccc} 1 & 3 & 2 \\ 4 & 1 & 8 \\ -2 & -6 & -4 \end{array} \right| = (2) \left| \begin{array}{ccc} 1 & 3 & 2 \\ 4 & 1 & 8 \\ 1 & 3 & 2 \end{array} \right| \stackrel{\substack{R_3 \leftrightarrow \\ R_3 - R_1}}{=} \left| \begin{array}{ccc} 1 & 3 & 2 \\ 4 & 1 & 8 \\ 0 & 0 & 0 \end{array} \right| = \emptyset$$

$$\left| \begin{array}{ccc} 1 & 2 & 3 \\ 1 & 3 & 7 \\ 1 & 4 & 13 \end{array} \right| \xrightarrow{R_2 - R_1} \left| \begin{array}{ccc} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 2 & 10 \end{array} \right| \xrightarrow{-2R_2 + R_3} \left| \begin{array}{ccc} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{array} \right| \xrightarrow{R_3 = \frac{1}{2}R_3} \left| \begin{array}{ccc} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{array} \right|$$

$$\det = (1)(1)(2) \\ = 2$$

 Don't reduce further for determinant



$$\left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{array} \right| = - \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{array} \right| = (-)(1)(3)(2)(4) \\ = -24$$

$$\boxed{\det A^{-1} = \frac{1}{\det A}} \quad \text{if } \det A = 5 \\ \det A^{-1} = \frac{1}{5} \\ \boxed{\det A = \det A^T}$$

$$\det \begin{pmatrix} s & s+1 & s+2 \\ t & t+1 & t+2 \\ u & u+1 & u+2 \end{pmatrix} \dots$$

$$\dots \xrightarrow{c_2 - c_1} \left(\begin{array}{ccc} s & 1 & 2 \\ t & 1 & 2 \\ u & 1 & 2 \end{array} \right) \xrightarrow{c_3 - c_1} \left(\begin{array}{ccc} s & 1 & 1 \\ t & 1 & 1 \\ u & 1 & 1 \end{array} \right) = 2 \det \left(\begin{array}{ccc} s & 1 & 1 \\ t & 1 & 1 \\ u & 1 & 1 \end{array} \right) = 0$$

Is the matrix A invertible?

$$A = \begin{pmatrix} 5 & 6 & 1 \\ 0 & 2 & 3 \\ 4 & 6 & 9 \end{pmatrix}$$

$$\det \begin{pmatrix} 5 & 6 & 1 \\ 0 & 2 & 3 \\ 4 & 6 & 9 \end{pmatrix} = (1)^{1+1}(5) \left| \begin{array}{cc} 2 & 3 \\ 6 & 9 \end{array} \right| + (-1)^{3+1}(4) \left| \begin{array}{cc} 5 & 1 \\ 0 & 3 \end{array} \right| \\ = 5 \left| \begin{array}{cc} (2)(9) - 3(6) \\ 18 - 18 \end{array} \right| + 4(6(3) - 2(1)) = 64 \neq 0$$

A is invertible.

(3)

$$x + y - z = 1$$

$$2x - 3z = 0$$

$$2y + z = 1$$

$$\det A = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & -3 \\ 0 & 2 & 1 \end{vmatrix} \xrightarrow{\substack{R_2 = -2R_1 + R_2 \\ R_3 = R_2 + R_3}} \begin{vmatrix} 1 & 1 & -1 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{vmatrix} \xrightarrow{\substack{R_3 = R_3 + R_1}} \begin{vmatrix} 1 & 1 & -1 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Cramer's rule cannot be used because $\det A = 0$
and the system has no unique solution.

$$3x - y + z = 2$$

$$2x + y - z = 1$$

$$x + 5y - 3z = 3$$

$$\det A = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 5 & -3 \end{vmatrix} = -\det \begin{pmatrix} 1 & 5 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{pmatrix} \xrightarrow{\substack{R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 3R_1}} \begin{pmatrix} 1 & 5 & 3 \\ 0 & -9 & 5 \\ 0 & -16 & 10 \end{pmatrix}$$

$$\dots - (1) \text{ " } (1) \begin{vmatrix} -9 & 5 \\ -16 & 10 \end{vmatrix} = [-9(10) - (-16)(5)] = 10 \neq 0$$

The system has a unique solution

$$x = \frac{\det A_1}{\det A}$$

$$y = \frac{\det A_2}{\det A}$$

$$z = \frac{\det A_3}{\det A}$$

$$\det A_1 = \det \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \\ 3 & 5 & -3 \end{pmatrix} = \cancel{\text{...}} \quad 6$$

$$\det A_2 = \det \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 3 & 3 \end{pmatrix} = \cancel{\text{...}} \quad 19$$

$$\det A_3 = \det \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & 5 & 3 \end{pmatrix} = 17$$