EXAM REVIEW

1) $x_1 - x_2 - x_3 = 3$

$2x_1 - 2x_2 + 3x_3 = 5$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 3 \\ 2 & -2 & 3 & 5 \end{array}\right) \xrightarrow{R_2 - 2R_1} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left(\begin{array}{ccc|c} 1 & -1 & -1 & 3 \\ \emptyset & \emptyset & 5 & -1 \end{array}\right) \end{array}$$

x_2 is the free variable.

echelon form

$5x_3 = -1$

$x_3 = -1/5$

$x_1 = 3 + x_2 + x_3$

$\Rightarrow 3 + x_2 - 1/5$

$x_1 \Rightarrow 14/5 + x_2$

Vertex Form:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 14/5 + x_2 \\ x_2 \\ -1/5 \end{pmatrix} = \begin{pmatrix} 14/5 \\ \emptyset \\ -1/5 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \\ \emptyset \end{pmatrix}$$

where x_2 is any real number

$\therefore$ the system has infinitely many solutions

Matrix form:

$$\begin{pmatrix} 1 & -1 & -1 \\ 2 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

x_2 is the free variable.

Vector Form:

$$\underbrace{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}_{x_1} x_1 + \underbrace{\begin{pmatrix} -1 \\ -2 \end{pmatrix}}_{x_2} x_2 + \underbrace{\begin{pmatrix} -1 \\ 3 \end{pmatrix}}_{x_3} x_3 = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

2) $2x_1 + 3x_2 - 2x_3 = 5$
$x_1 - 2x_2 + 3x_3 = 2$
$4x_1 - x_2 + 4x_3 = 1$

$$\left(\begin{array}{ccc|c} 2 & 3 & -2 & 5 \\ 1 & -2 & 3 & 2 \\ 4 & -1 & 4 & 1 \end{array}\right) \xrightarrow[1 \leftrightarrow 2]{\text{ROW SWAP}}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 2 & 3 & -2 & 5 \\ 4 & -1 & 4 & 1 \end{array}\right) \xrightarrow[R_3 - 4R_1]{R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 2 \\ 0 & 7 & -8 & 1 \\ 0 & 7 & -8 & -7 \end{array}\right) \xrightarrow{R_3 - R_2}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 2 \\ 0 & 7 & 8 & 1 \\ 0 & 0 & 0 & -8 \end{array}\right)$$

$0x_1 + 0x_2 + 0x_3 \neq -8$
$\therefore$ The system has no solution.

3)
$$\begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \left(\begin{array}{c} ? \\ \text{(printing error?)} \end{array} \right) \end{array} \longrightarrow \begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \left(\begin{array}{ccc|c} 3 & 2 & 1 & 1 \\ 0 & 5 & 3 & 2 \\ 0 & 0 & 9 & 3 \end{array}\right) \end{array}$$

echelon form

via Back substitution

$9x_3 = 3$
$\boxed{x_3 = 1/3}$

$5x_2 + 3x_2 = 2$
$5x_2 = 2 - 3(1/3)$
$\boxed{x_2 = 1/5}$

$3x_1 + 2x_2 + x_3 = 1$
$3x_1 + 2(1/5) + (1/3) = 1$
$3x_1 = 1 - 2/5 - 1/3$
$\boxed{x_1 = .4/9}$

$\therefore$ The system has a unique solution.

In vector form:
$(x_1, x_2, x_3) = (-4/9, 1/5, 1/3)$

$AX = B$

$$x = \underbrace{x_0}_{\substack{\text{Particular}\\ \text{Solution}\\ \text{to } AX=B}} + \underbrace{\sum_{i=1}^{s} t_i x_i}_{\text{Solution to the corresponding homogeneous system}}$$

4) $x_1 - x_2 - x_3 - x_4 = 0$

$x_1 - x_2 + x_3 - x_4 = 0$

$x_1 + x_2 - x_3 + x_4 = 0$

$$\left(\begin{array}{cccc|c} 1 & -1 & -1 & -1 & 0 \\ 1 & -1 & 1 & -1 & 0 \\ 1 & 1 & -1 & 1 & 0 \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 - R_1} \left(\begin{array}{cccc|c} 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{array}\right) \xrightarrow[(2 \leftrightarrow 3)]{\text{Row swap}}$$

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$
$$\left(\begin{array}{cccc|c} 1 & -1 & -1 & -1 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{array}\right)$$

x_4 is a free variable.

$x_3 = 0$

$x_2 = 0$

$x_1 = x_4 \quad (x_1 - x_2 - x_3 - x_4 = 0)$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_4 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Row-reduced ech.

echelon Form

↳ no solution

↳ unique solution

↳ infinite many solution

Standard Form

matrix Form

Vector Form

$A = \begin{pmatrix} 3 & -5 \\ 2 & 7 \end{pmatrix}_{2\times 2}$ $B = \begin{pmatrix} -1 & 0 \\ 3 & -4 \end{pmatrix}_{2\times 2}$ $C = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{pmatrix}_{2\times 3}$

$A + C$ does not exist

if $C = 3$ and $D = 4$

$CA + DB = \begin{pmatrix} 9 & -15 \\ 6 & 21 \end{pmatrix} + \begin{pmatrix} -4 & 0 \\ 12 & -16 \end{pmatrix} = \begin{pmatrix} 5 & -15 \\ 18 & 5 \end{pmatrix}$

$\underset{1\times 3}{A} = (1, 2, 3)$ $\underset{3\times 1}{B} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$

$AB = (1\times 3)\cdot(3\times 1)$
$= (1\times 1)$

$AB = (1, 2, 3)\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = (1)(3) + (2)(4) + (3)(5) = 26_{(1\times 1)}$

$BA = (3\times 1)\cdot(1\times 3)$
$= (3\times 3)$

$BA = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}(1, 2, 3) = \begin{pmatrix} 3(1) & 3(2) & 3(3) \\ 4(1) & 4(2) & 4(3) \\ 5(1) & 5(2) & 5(3) \end{pmatrix}$

$= \begin{pmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \\ 5 & 10 & 15 \end{pmatrix}$

$A = (3, -5)_{1\times 2}$ $B = \begin{pmatrix} 2 & 7 & 5 & 6 \\ -1 & 4 & 2 & 3 \end{pmatrix}_{2\times 4}$

$AB = (1\times 2)\cdot(2\times 4)$

$AB = (1\times 4)$

$BA = (2\times 4)\cdot(1\times 2)$

$BA =$ DNE.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

TRANSPOSE

$A^{-1} \cdot A = I$

To find inverse: (A^{-1})

$$A = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}$$

$$(A|I) = \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|cc} 1 & 0 & 4 & -1 \\ 0 & 1 & -3 & 1 \end{array}\right)$$

A^{-1}

$$(A|I) = (I|A^{-1})$$

Find a non-zero vector, such that

$$\underbrace{\begin{pmatrix} 1 & -2 & 1 & -1 \\ 2 & -3 & 4 & -3 \\ 3 & -5 & 5 & 4 \end{pmatrix}}_{A} \quad (x = 0)$$

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = 0$$

Dec. 2ND /16

$A_{n \times n}$ $\quad \det A$

$$\begin{matrix} a_{11} & a_{22} & a_{313} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = 1,2 \text{ - minor}$$

$$C_{12} = (-1) M_{12}$$

$$\det A = \sum_{i=1}^{n} a_{ij} C_{ij}$$

$$\det A = \begin{vmatrix} 0 & 0 & 0 \\ - & - & - \\ - & - & - \end{vmatrix} = 0$$

multiplied

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ & \ddots & & \vdots \\ 0 & & \ddots & a_{nm} \end{vmatrix} = (a_{11}) \cdots (a_{nm})$$

$$\det A = \begin{vmatrix} a_{n} & \ddots & 0 \\ 0 & \ddots & a_{nm} \end{vmatrix} = (a_{n}) \cdots (a_{nm})$$

Row operations for determinants:

Swapping rows makes det. (−)

$$\begin{vmatrix} -2 & 7 & 2 \\ 5 & 7 & 9 \\ 5 & 14 & 11 \end{vmatrix} \xrightarrow[R_3 - 2R_1]{R_2 - R_1} \begin{vmatrix} -2 & 7 & 2 \\ 7 & 0 & 7 \\ 7 & 0 & 7 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & -1 & -3 \\ 2 & -2 & 0 & -4 \\ 5 & 4 & 7 & -15 \\ 4 & 0 & 1 & -10 \end{vmatrix} = 2 \begin{vmatrix} 1 & 0 & -1 & -3 \\ 1 & -1 & 0 & -2 \\ 5 & 4 & 7 & -15 \\ 4 & 0 & 1 & -10 \end{vmatrix} \xrightarrow{\substack{R_2 - R_1 \\ R_3 - 5R_1 \\ R_4 - 4R_1}} 2 \begin{vmatrix} 1 & 0 & -1 & -3 \\ 0 & -1 & 1 & 1 \\ 0 & 4 & 12 & 0 \\ 0 & 0 & 5 & 2 \end{vmatrix}$$

$$\rightarrow (2)(4) \begin{vmatrix} 1 & 0 & -1 & -3 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 5 & 2 \end{vmatrix} \xrightarrow{R_3 + R_2} (8) \begin{vmatrix} 1 & 0 & -1 & -3 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 5 & 2 \end{vmatrix} \xrightarrow{R_4 - (5/4) R_3}.$$

$$\rightarrow (8) \begin{vmatrix} 1 & 0 & -1 & -3 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 3/4 \end{vmatrix} \rightarrow 8 \left[1(-1)(4)(3/4) \right] = 24 ?$$

$$\begin{vmatrix} 2 & -3 & 1 \\ 4 & 0 & 2 \\ 3 & -1 & -3 \end{vmatrix} \; C_1 + 2C_3 \; \begin{vmatrix} 3 & -3 & 1 \\ 0 & 0 & -2 \\ -3 & -1 & -3 \end{vmatrix} = (-1)^{2+3}(-2) \begin{vmatrix} 3 & -3 \\ -3 & -1 \end{vmatrix}$$

$$= 2[3(-1) - (-3)(-3)] = -24$$

$$\begin{vmatrix} 2 & -3 & 1 \\ 4 & 0 & 2 \\ 3 & -1 & -3 \end{vmatrix} \xrightarrow{C_1 + 2C_3} \begin{vmatrix} 3 & -3 & 1 \\ 0 & 0 & -2 \\ -3 & -1 & -3 \end{vmatrix} \longrightarrow - \begin{vmatrix} 3 & -3 & 1 \\ -3 & -1 & -3 \\ 0 & 0 & -2 \end{vmatrix}$$

$$\xrightarrow[R_2 + R_3]{} \begin{vmatrix} 3 & -3 & 1 \\ 0 & -4 & -2 \\ 0 & 0 & 2 \end{vmatrix} = 3(-4)(2) = -24$$

Use the det to determine whether system has unique sol.

$$x + y + z = 1$$
$$x - y + 2z = 0$$
$$2y - z = 1$$

$$A_{3\times 3} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & -1 \end{pmatrix} \xrightarrow{R_2 - R_1} \begin{vmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 2 & -1 \end{vmatrix}$$

$$\xrightarrow[R_3 + R_2]{} \begin{vmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\therefore A^{-1}$ does not exist

the system has no unique sol.

(infinitely many)

$$\det A^{-1} = \frac{1}{\det A}$$

$$AB \neq BA$$

$$\det AB = (\det A) \cdot (\det B) = \det BA$$
$$\det BA = (\det B) \cdot (\det A) = \det AB$$

A and $A^{-1} = ?$

$$(A \mid I) \xrightarrow[\text{operations}]{\text{row}} (I \mid A^{-1})$$

row-reduced

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & -2 & 4 \\ 5 & -1 & -4 \end{pmatrix}$$

adj A Cof A

$\hookrightarrow$ adj $A = (\text{Cof } A)^T$

$A^{-1} = \left(\frac{1}{\det A}\right) \text{adj } A$

$$C_{11} = (1)^{1+1} \begin{vmatrix} -2 & 4 \\ -1 & -4 \end{vmatrix} = 12$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 4 \\ 5 & -4 \end{vmatrix} = 20$$

$$\text{Cof } A = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$$

$$\ldots \begin{pmatrix} 12 & 20 & 10 \\ 10 & -14 & 16 \\ 16 & -4 & -2 \end{pmatrix} \longrightarrow \text{adj } A = \begin{pmatrix} 12 & 10 & 16 \\ 20 & -14 & -4 \\ 10 & 16 & -2 \end{pmatrix}$$

$\det A = 92$

$A^{-1} = 1/92$

$$A^{-1} = \begin{pmatrix} 12 & 10 & 16 \\ 20 & -14 & -4 \\ 10 & 16 & -2 \end{pmatrix} \left(\frac{1}{92}\right)$$

$(1/\det A)$

$\hookrightarrow$ (adj A)

$\left(A^{-1} = \text{adj } A \cdot \left(1/\det A\right)\right)$

$x + y - z = 1$

$2x \quad - 3z = 0$

$2y + z = 1$

$$\det A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & -3 \\ 0 & 2 & 1 \end{pmatrix} = (-1)^{1+1}(1) \begin{vmatrix} 0 & -3 \\ 2 & 1 \end{vmatrix} + (1)^{1+1}(2) \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = \ldots = 0$$

(we cannot apply Cramers rule)

$$3x - y + z = 2$$
$$2x + y - z = 1$$
$$x + 5y - 3z = 3$$

$$x = \frac{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \\ 3 & 5 & -3 \end{vmatrix}}{10} = 3/5$$

$$y = \frac{\begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 3 & -3 \end{vmatrix}}{10} = 3/2 \quad (?)$$

$$z = \frac{\begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & 5 & 3 \end{vmatrix}}{10} = 17/10$$

$$\boxed{X = A^{-1} B}$$

SUBSPACE

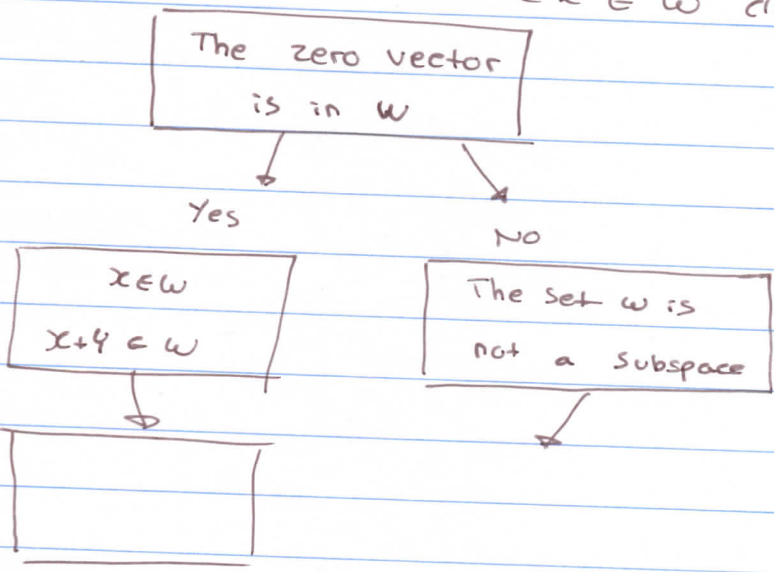
Set W

Whether W is a subspace of R^n

$x \in W$ 1. $x + y \in W$ Closure under addition

$y \in W$ 2. c (any scalar), $x \in W$

$cx \in W$ closure under multiplication



Is the given set a subspace of R^2

1) $U = \{(x, y),\ x + y = 1\}$ $x + y = (x_1 + y_1,\ x_1 + y_2)$

$0 = \{0, 0\}$ $0 + 0 \neq 1$

U is not a subspace of R^2

2) $V = \{(x_1, x_2);\ x_1 + x_2 = 0\}$

$x = (x_1, x_2):\ x_1 + x_2 = 0$ $(x \in V,\ y \in V)$

$y = (y_1, y_2):\ y_1 + y_2 = 0$

$(x_1 + y_2) + (x_2 + y_2) = 0$

$\quad$ " 0 $\quad$ " 0

$\therefore$ the set V is a subspace of R^2

$cx = (cx_1,\ cx_2)$

$x \in V$ $c_1 x_1 + c_2 x_2 = 0$ $x = (x_1, x_2 : x_1 + x_2 = 0\}$

$c(x_1 + x_2) = 0$

" 0

3) $W = \{(x,y) : (x+1)^2 = (x-1)^2\}$ is W a subspace of R^3

$x^2 + 2x + 1 = x^2 - 2x + 1$

$x = -x \longleftrightarrow x = 0$

$W = \{(0, y)\}$ $(0,0)$

$x \in W$ $x = (0, x)$ $x + y = (0, x+y)$

$y \in W$ $y = (0, y)$ $cx = (c0, cy) = (0, cy) \in W$

$x = (0, y) \in W$

4) $W = \{(x,y) : x+y = 0$ or $x - y = 0\}$

$x = (1,1) \in W$ $(1+1 \neq 0$

$(1-1 = 0$

$y = (1,-1) \in W$ $(1-1 = 0$

$(1+1 \neq 0$

$x + y = (1,1) + (1,-1) = (2,0)$

$2 + 0 = 2$ $x+y \notin W$

$2 + 0 = 2$ W is not a subspace

5) $W = \{(x,y) : |x-y| = 0\}$

$|x-y| = 0$ $x = y$

$W = \{(x,y) : x = y\}$ $x = (x_1, x_2, x_1 = x_2) \in W$

$x + y = \{(x_1+y_1, x_2+y_2) : x_1+y_1 = x_2+y_2\} \in W$ $y = (y_1, y_2, y_1 = y_2) \in W$

$cx = (cx_1, cx_2 : cx_1 = cx_2)$

$x_1 = x_2$

$x = \{(x_1, x_2) : x_1 = x_2\} \in W$

Linear combination

$y = c_1 x_1 + c_2 x_2 + \dots + c_k x_k$

if $c_1, \dots, c_k$ not equal to zero

lin combination x_0 $\quad x_0 = c_1 x_1 + c_2 x_2 + \dots + c_k x_k$

PARTICULAR VECTOR

Span x $\quad x = c_1 x_1 + c_2 x_2 + \dots + c_k x_k$

ARBITRARY VECTOR

Lindependent

$\emptyset$ $\quad \emptyset = c_1 x_1 + c_2 x_2 + \dots + c_k x_k$

ZERO VECTOR $\quad c_1 = \dots = c_k = \emptyset$

$S = \{(x_1, \dots, x_k)\}$ is linearly independent

Determine whether $x_0 = (2, -6, 3)$ is a linear comb. of $x_1 = (1, -2, -1)$, $x_2 = (3, -5, 4)$

$(2, -6, 3) = c_1(1, -2, -1) + c_2(3, -5, 4)$

$$\begin{aligned} 2 &= c_1 + 3c_2 \\ -6 &= -2c_1 - 5c_2 \\ 3 &= -c_1 + 4c_2 \end{aligned} \quad \leadsto \quad A = \begin{array}{ccc} x_1 & x_2 & x_0 \\ \end{array} \left(\begin{array}{cc|c} 1 & 3 & 2 \\ -2 & -5 & -6 \\ -1 & 4 & 3 \end{array}\right)$$

$\downarrow$

Row reduced

$$\begin{array}{cc} x_1 & x_2 \end{array} \left(\begin{array}{cc|c} 1 & \emptyset & 8 \\ \emptyset & 1 & -2 \\ \emptyset & \emptyset & 19 \end{array}\right)$$

no solution $\rightarrow$

$\emptyset x_1 + \emptyset x_2 \neq 19$

x_0 is not a L.C. of x_1, x_2

Determine whether $x_0 = (-7, 7, 11)$ is a L.C. of $x_1 = (1, 2, 1)$, $x_2 = (-4, -1, 2)$, $x_3 = (-3, 1, 3)$

$$A = \left(\begin{array}{ccc|c} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \end{array}\right) \xrightarrow{\text{Row reduced}}$$

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \\ \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \end{array}$$

c_3 - Free $\to c_3 = t$

$c_2 + c_3 = 3 \to c_2 = 3 - c_3$

$c_1 + c_3 = 5 \to c_1 = 5 - c_3$

$x_0 = (5-t)x_1 + (3-t)x_2 + t x_3$

@ $t = 1$ $x_0 =$

A subspace W generated by vectors $x_1, \dots, x_k$ means S spans W

$x = c_1 x_1, \dots, c_k x_k$ $\qquad S = \{x_1, \dots, x_k\}$ Spanning set

any vector (arbitrary vector) $\qquad W =$ Spans S

$S_1 = \{e_1, e_2\}$ Spans $\mathbb{R}^2$

$S_2 = \{(1,1), (1,0), (0,1)\}$ Spans $\mathbb{R}^2$ $\qquad e_1 = (1,0)$, $e_2 = (0,1)$

$W = \{(x, y, z) : x, y \text{ real}\}$

$x_1 = (-2, -2, -2)$

$x_2 = (5, 1, 1)$

is

Is the set $S = \{(1, 2, -1), (1, 0, 1)\}$ Spanning set For R^3

$$A = \left(\begin{array}{cc|c} 1 & 1 & x_1 \\ 2 & 0 & x_2 \\ -1 & 1 & x_3 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & x_1 \\ 0 & -2 & x_2 + 2x_1 \\ 0 & 2 & x_3 + x_1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cc|c} 1 & 1 & x_1 \\ 0 & -2 & x_2 + 2x_1 \\ 0 & 0 & x_3 + x_2 - x_1 \end{array}\right) \qquad x_1 = x_3 + x_2$$

Impossible to Follow, he's writing Sideways.

Next Q : Find a spanning set for the solution set of the system.

$AX = 0$

$$A = \begin{pmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \\ 4 & 4 & -1 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{c} \quad x_1 \; x_2 \; x_3 \; x_4 \\ A = \left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

$x_1 = -x_2 - 2x_4$

$x_3 = x_4$

x_2, x_4 Free

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -x_2 - 2x_4 \\ x_2 \\ x_4 \\ x_4 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$= x_2 V_1 + x_4 V_2$

$S \{v_1, V_2\}$

Determine whether the vector $x = (2, -1, -1)$ belongs to the subspace generated by $S = \{(1, 0, 1), (0, 1, 1)\}$

$x \in W$

$x = c_1(1, 0, 1) + c_2(0, 1, 1)$

$(2, -1, -1) = c_1(1, 0, 1) + c_2(0, 1, 1)$

$$A = \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 1 & -1 \end{array}\right) \longrightarrow \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -2 \end{array}\right) \end{array} \quad 0x_1 + 0x_2 \neq -2$$

system has no solution

x does not belong to the subspace

R^n $\quad x \in R^n$ $\quad x = (x_1, \ldots, x_n)$

$k > n$ set $S = \{x_1, \ldots, x_k\}$ linearly dependent

$k = n$ ~~set~~ can use det or row-reducing procedure

$k < n$ can use row ~~r~~educing procedure

Determine whether $x_1 = (-2, -2, -2)$, $x_2 = (5, 1, 1)$ are lin. indep.

$(0, 0, 0) = c_1(-2, -2, -2) + c_2(5, 1, 1)$

$$\left(\begin{array}{cc|c} -2 & 5 & 0 \\ -2 & 1 & 0 \\ -2 & 1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} c_1 = 0 \\ c_2 = 0 \end{array}$$

x_1 and x_2 are lin. indep.

$v_1 = (3, 0, 1, 2)$

$v_2 = (1, -1, 0, 1)$

$v_3 = (1, 2, 1, 0)$

$0 = c_1(3, 0, 1, 2) + c_2(1, -1, 0, 1) + c_3(1, 2, 1, 0)$

$$\left(\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$c_1 = -c_3$ linearly dependent

$c_2 = 2c_3$

c_3 is free

The set $S = \{x_1, \ldots, x_k\}$ is a basis for W if

1. $x_1, \ldots,$ are linearly independent
2. $x_1, \ldots,$ span W

A basis for W is a linearly indep. spanning set for W.

$k < n$ — lin indep. vector but not spanning set

$k > n$ — vectors span R^n but lin indep. (can be)

$k = n$ — vectors span and linearly independent.