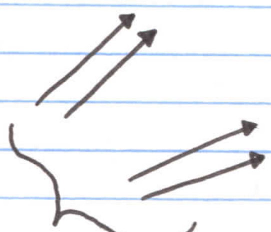
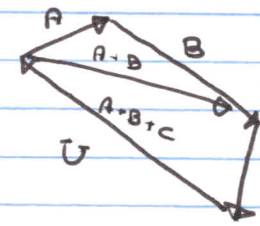
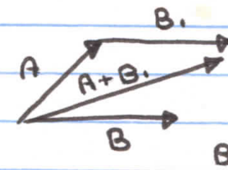
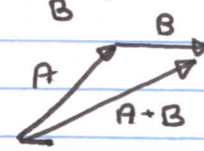


2 vectors

Express the vector U in terms of vectors A, B, C

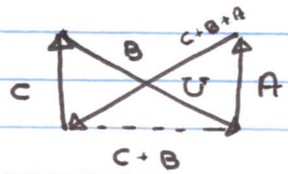






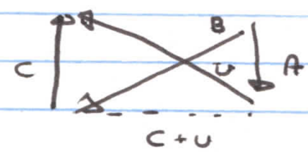
$A + B + C - U = \emptyset$

$U = A + B + C$



$C + B + A - U = \emptyset$

$U = C + B + A$



$B + C + U - A = \emptyset$

$U = A - B - C$

Find non-zero scalars a, b such that for all vectors x and y

$a(x + 2y) - bx + 4y - x = \emptyset$

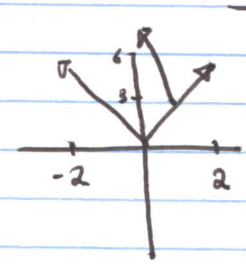
$ax + 2ay - bx + 4y - x = \emptyset$

$(a - b - 1)x + (4 + 2a)y = \emptyset$

$a - b - 1 = \emptyset \rightarrow b = -3$

$4 + 2a = \emptyset \quad a = 2$

Find $x + y$, $2x$ if $x = (2, 3)$, $y = (-2, 5)$



$x + y = (2, 3) + (-2, 5) = (\emptyset, 8)$

$2x = 2(2, 3) = (4, 6)$

Find the norm of the vector $x = 2i + 3j + 4k$

$\cancel{\|x\| = \sqrt{x^2}}$

$$\|x\| = \sqrt{2^2+3^2+4^2} = \sqrt{29}$$

Find the distance between the two points
$P(2,3)$, $Q(3,4)$

$$\begin{aligned} \text{dist } PQ &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(2-3)^2+(3-4)^2} \\ &= \sqrt{2} \end{aligned}$$

Find $x \cdot y$ if $x = (2,3,1)$, $y = (3,-2,0)$

$$x = (x_1, x_2, x_3) \qquad y = (y_1, y_2, y_3)$$

$$x \cdot y = x_1 y_1 + x_2 y_2 + x_3 y_3$$

$$x \cdot y = 2(3) + (3)(-2) + 1(0) = 0$$
$$x \cdot y = 0$$

~~Find the angle between~~ $\cancel{x = (-3,-4)}$ ~~and~~ $\cancel{y = (4,-3)}$

Find the unit vector parallel to, and in the direction of the vector $x = (-5, 12)$

$$U = \frac{1}{\|x\|} x = \frac{1}{13}(-5,12) = \left(\frac{-5}{13}, \frac{12}{13}\right)$$

$$\|U\| = \sqrt{\left(\frac{-5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} \qquad \|x\| = \sqrt{(-5)^2+12^2} = 13$$

Given $x = 3i + 2j - k$, $y = i - 3j + 2k$.
Find the unit vector in the direction $x - 2y$

$$x - 2y = (3i + 2j - k) - 2(i - 3j + 2k) = i + 8j - 5k$$

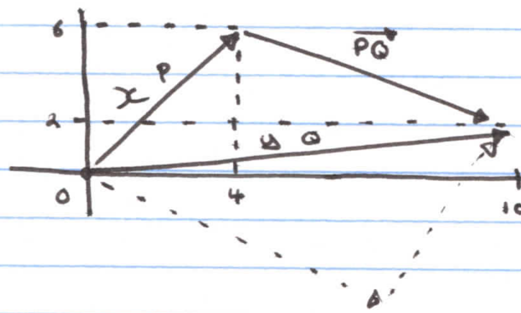
$$U = \frac{1}{\|x - 2y\|}(x - 2y) = \frac{1}{3\sqrt{10}}(i + 8j - 5)$$

$$\Rightarrow \left(\frac{1}{3\sqrt{10}}, \frac{8}{3\sqrt{10}}, -\frac{5}{3\sqrt{10}}\right) = \frac{1}{3\sqrt{10}} i + \frac{8}{3\sqrt{10}} j - \frac{5}{3\sqrt{10}} k$$

$$\|x - 2y\| = \sqrt{(1)^2 + 8^2 + (-5)^2}$$

$$= \sqrt{90} \Rightarrow \sqrt{9}\sqrt{10} = 3\sqrt{10}$$

Find the vector in standard position that is equivalent to the vector x from $P(4, 6, 1)$ to $Q(10, 2, 3)$



$$x + PQ = y$$
$$PQ = y - x$$

$$y - x =$$
$$(10, 2, 3) - (4, 6, 1)$$
$$= (6, -4, 2)$$

$$\text{Proj}_y x = \frac{x \cdot y}{y \cdot y} y = \frac{x \cdot y}{\|y\|^2}$$

$$\|y\|^2 = y \cdot y = a_1(a_1) + c_2(a_2) = a_1^2 + a_2^2$$
$$y = (a_1, c_2)$$

Projection of x on to y

$$\text{Proj}_y x = \frac{x \cdot y}{\|y\|^2} y = \quad x = (4, 5)$$
$$y = (8, 10)$$

$$= \frac{4(8) + 5(10)}{8^2 + 10^2} \Rightarrow \frac{82}{164}$$

The cross product of two vectors is defined in $\mathbb{R}^3$ only.

$x = (2, 0, 2)$ $\quad y = (-1, 7, 6)$ $\quad$ Find $x \times y = ?$

$$x + y = \begin{vmatrix} i & j & k \\ 2 & 0 & 2 \\ -1 & 7 & 6 \end{vmatrix} = i \begin{vmatrix} 0 & 2 \\ 7 & 6 \end{vmatrix} - j \begin{vmatrix} 2 & 2 \\ -1 & 6 \end{vmatrix} + k \begin{vmatrix} 2 & 0 \\ -1 & 7 \end{vmatrix}$$

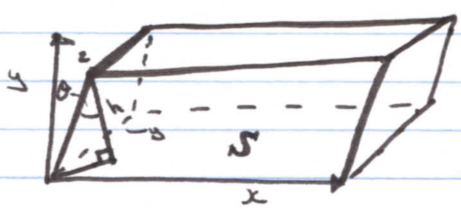
$$\Rightarrow 14i - 14j + 14k$$

Remember $x \times y \neq y \times x$

Find the area of the parallelogram, defined by $x = 3i - 6j + 4k$

$$x \times y = \begin{vmatrix} i & j & k \\ 3 & -6 & 4 \\ -5 & 2 & 0 \end{vmatrix} = -8i - 20j - 24k$$

$$\|y \times y\| = \sqrt{(-8)^2 + (-20)^2 + (-24)^2}$$
$$= 4\sqrt{65}$$



$V = |z \cdot (x \times y)|$

This formula can be used to ~~define~~ determine whether three vectors x, y, z are coplanar

Determine whether the vector $x = (3, 0, -3)$ is orthogonal to the solution space of $Ax = 0$ with $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

x_3 = free