Elementary Row Operations

1. Scalar multiplication
2. (Row swap) Exchange two rows
3. (Row swap) Add to one row a multiple of another row.

An augmented matrix A of a liner system → A finite sequence of elementary row operations. → Augmented matrix B represents an equivalent system

Definition (Row-reduced Echelon form)

A matrix is a row-reduced echelon form (reduced echelon form) if it satisfied the following conditions.

1. The first non-zero entry in any rows is 1 (Called a leading-1)
2. All entries in a column containing a leading-1 are zero
3. Each leading-1 of a row in a column to the right of the leading-1 of the row above.
4. All nonzero rows are above any rows of zeroes.

$$\begin{matrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \qquad \therefore \text{the matrix is in reduced echelon form}$$

$$\begin{pmatrix} 1 & -2 & -2 & 0 & 7 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \therefore \text{the matrix is in reduced echelon form}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \therefore \text{the matrix is not in reduced echelon form}$$

From lab session

Theorem:

Every matrix can be transformed by a finite sequence of elementary row operations into one that is in row-reduced echelon form.

Theorem:

The row-reduced echelon form of a matrix is unique.

Method of Gauss-Jordan Elimination

$$x_1 - 2x_2 - 6x_3 = 15$$
$$8x_1 - 15x_2 = 12$$
$$2x_1 + 16x_2 - x_3 = 56$$

$$\begin{Bmatrix} 1 & -2 & -6 & | & 15 \\ 8 & -15 & 0 & | & 12 \\ 2 & 16 & -1 & | & 56 \end{Bmatrix}$$

Put the system in standard form and find the augmented matrix.

$$5x_2 + x_1 = 4 \quad \rightarrow \qquad x_1 + 5x_2 = 4$$
$$x_3 - 2x_2 = x_1 \quad \rightarrow \qquad x_1 - 2x_2 + x_3 = 0$$

Rest is written…