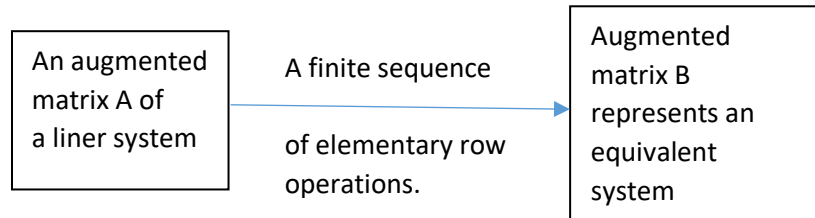


Elementary Row Operations

1. Scalar multiplication
2. (Row swap) Exchange two rows
3. (Row swap) Add to one row a multiple of another row.



Definition (Row-reduced Echelon form)

A matrix is a row-reduced echelon form (reduced echelon form) if it satisfied the following conditions.

1. The first non-zero entry in any rows is 1 (Called a leading-1)
2. All entries in a column containing a leading-1 are zero
3. Each leading-1 of a row in a column to the right of the leading-1 of the row above.
4. All nonzero rows are above any rows of zeroes.

$$\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \therefore \text{the matrix is in reduced echelon form}$$

$$\begin{pmatrix} 1 & -2 & -2 & 0 & 7 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \therefore \text{the matrix is in reduced echelon form}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \therefore \text{the matrix is not in reduced echelon form}$$

From lab session

Theorem:

Every matrix can be transformed by a finite sequence of elementary row operations into one that is in row-reduced echelon form.

Theorem:

The row-reduced echelon form of a matrix is unique.

Method of Gauss-Jordan Elimination

$$x_1 - 2x_2 - 6x_3 = 15$$

$$8x_1 - 15x_2 = 12$$

$$2x_1 + 16x_2 - x_3 = 56$$

$$\left\{ \begin{array}{ccc|c} 1 & -2 & -6 & 15 \\ 8 & -15 & 0 & 12 \\ 2 & 16 & -1 & 56 \end{array} \right\}$$

Put the system in standard form and find the augmented matrix.

$$\begin{array}{lcl} 5x_2 + x_1 = 4 & \rightarrow & x_1 + 5x_2 = 4 \\ x_3 - 2x_2 = x_1 & \rightarrow & x_1 - 2x_2 + x_3 = 0 \end{array}$$

Rest is written...