Effective Annual Interest Rate Formula (covered last time?)

$$i_A = \left(1 + \frac{r}{M}\right)^M - 1$$

r = nominal interest rate

i_a = effective annual interest rate

M = number of interest periods / year

Effective Interest Rates per Payment Period

$$i = \left(1 + \frac{r}{M}\right)^c - 1 = \left(1 + \frac{r}{CK}\right)^c - 1$$

where: M = number of compounding periods / year

C = number of compounding periods / payment period

K = number of payment periods / year

$\boxed{\text{Example 3}}$

$K = 4$ (payments / year)

$r = 8\% = 0.08$

M = vary depending on payment period

C = " "

a) <u>Quarterly</u>:

$K = 4$ (payment periods / year)

$r = 0.08$

$C = 1$ (compounding periods / payment period)

$M = CK$ (compounding periods / year)

↪ $M = 4$, $K = 4$ } $C = 1$

$$i_e = \left(1 + \frac{r}{CK}\right)^c - 1$$

$$= \left(1 + \frac{0.08}{(1)(4)}\right)^1 - 1 = 2\%$$

b) Monthly

$r = 8\% = 0.08$

$K = 4$

Since $M = cK$ $\boxed{C = 3}$

$M = \underset{\text{months}}{12}$ (compounding periods / year)

$$i_e = \left(1 + \frac{0.08}{(3)(4)}\right)^3 - 1 = 2.013\%$$

(per quarter)

c) Weekly

$r = 8\% = 0.08$

$K = 4$

$\boxed{C = 13}$ $M = cK$ or $c = M/K$

$M = \underset{\text{weeks}}{52}$ (compounding periods / year)

$$i_e = \left(1 + \frac{0.08}{(13)(4)}\right)^{13} - 1 = 2.019\%$$

(per quarter)

d) Daily

$r = 8\% = 0.08$

$K = 4$

$C = 91.25$ (compounding periods / payment period)

$M = \underset{\text{days}}{365}$ (compounding periods / year)

$$i_e = \left(1 + \frac{0.08}{(91.25)(4)}\right)^{91.25} - 1 = 2.02\%$$

(daily)

$\boxed{\text{Example 4}}$

$r = 6.25\%$ (compounded monthly)

A = per month

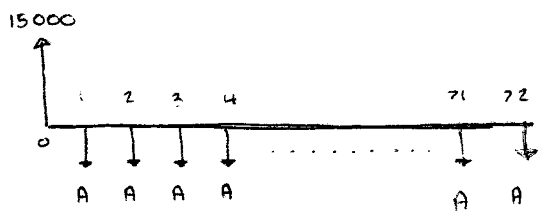
Step 1: $M = 12$ (compound periods / year)

Step 2: $i_e = \dfrac{r}{M} = 0.5208\%$ (per month)

Step 3: $N = M * \text{years} = 72$ months

... Calculating A (payments)

$$A = P(A/P, i, N)$$
$$= 15000\,(A/P,\ 0.5208\%,\ 72)$$
$$= \$250.37$$



Example 5

Step 1 : $M = 12$

$K = 4$ (deposits / year) = / quarter

$C =$ (compounding period / payment period)

$\hookrightarrow M = CK \rightsquigarrow C = 3$

Step 2 : $r = 6\% = 0.06$

$$i_e = \left(1 + \frac{0.06}{(3)(4)}\right)^3 - 1 = 1.5075\%$$

(per quarter)

Step 3 :

$$N = K \times \text{years}$$
$$= 4 \times 2 = 8 \text{ quarters}$$

Step 4 :

balance @ 2years

$$F = A(F/A, i, N)$$
$$= (1500)(F/A,\ 1.5075\%,\ 8)$$
$$= \$12652.60$$

Example 6

$A = 500$ (monthly)

$r = 10\%$

$F =$

$N = 10$ years

Step. 1 : $M = 4$ Compounding periods / Pay

$K = 12$

$C = 1/3 \rightsquigarrow C = M/K$

Step 2 :

$$i_e = \left(1 + \frac{0.10}{(1/3)(12)}\right)^{1/3} - 1 = 0.826\%$$

(per month)

Step 3 :

$N = K \times$ years

$= (12)(10) = 120$ months

Step 4 :

$F = A\,(F/A,\ i,\ N)$

$= 500\,(F/A,\ 0.826\%,\ 120)$

$= \$101{,}907.89$

Example 7

Period 1

$A = 235.37$

$I_n = I_1 = B_{n-1}\,(i)$

$= B_0\,(0.01)$

$= 5000\,(0.01)$

$= 50$

$PP_1 = A - I_1 = 235.37 - 50$

$= 185.37$

$B_1 = B_0 - PP_1 = 5000 - 185.37$

$= 4814.63$

Period 2

$I_2 = 4814.63(0.01) = 48.15$

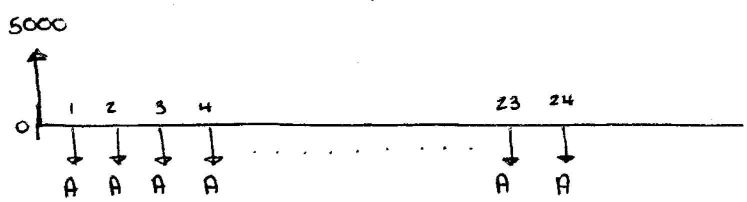
$PP_2 = 235.37 - 48.15$

$= 187.22$

$B_2 = B_1 - PP_2 = 4814.63 - 187.22$

$= 4627.41$

b) $B_6 = A(P/A, i, N-n)$



$= 235.37(P/A, 1\%, 18)$

$= 3869.62$

$I_n = ?$

$I_6 = B_{n-1}(i) = B_5(i)$

$= A(P/A, 1\%, 19)(1\%)$

$= 235.37(P/A, 1\%, 19)(0.01)$

$= 40.54$

$PP_6 = A - I_6$

$= 235.37 - 40.54$

$= 194.83$

Two types of mortgages: - Fixed rate

- Variable mortgage (not covered in course)

$\boxed{\text{Example 9}}$

$P = 100\,000$

$r = 8\%$

$M = 2$ compounding periods / year

amortization = 25 years

term = 3 years

$\hookrightarrow$ $c = M/K$ (per month) $\rightarrow$ $c = 2$ periods / year / 12 months / year

$\hookrightarrow$ $c = 1/6$

$i_e = ?$

$A \rightarrow$ month

So, for i_e:

$$\left.\begin{array}{l} M = 2 \\ K = 12 \\ c = 1/6 \end{array}\right\} \quad i_e = \left(1 + \frac{0.08}{(1/6)(12)}\right)^{1/6} - 1$$

$i_e = 0.6558\%$ <u>(per month)</u>

$N = K * \text{years}$

$= (12)(25) = 300$ months

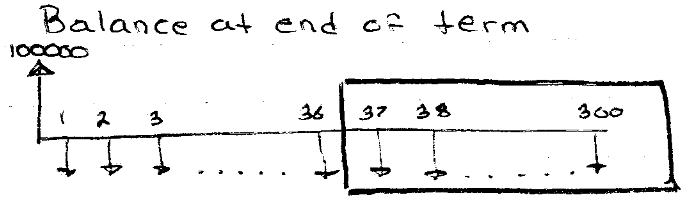
For $A \rightarrow$ payments

$A = P(A/P, i, N)$

$= 100,000\,(A/P,\ 0.6558\%,\ 300)$

$= \$\,763.20$

b) Balance at end of term



$$B_{36} = A(P/A, i, N-n)$$
$$= 763.20(P/A, 0.6558\%, 264)$$
$$= 95655.54$$

c) extra payment monthly

$$B_{36,adj} = 95655.54 - 381.60(F/A, 0.6558\%, 24) - 381.60(F/A, 0.6558\%, 12)$$
$$= \$81023.31$$

d) lump sums

$$B_{36,adj} = 81023.51 - 8000(F/P, 0.6558\%, 24) - 10000(F/P, 0.6558\%, 12)$$
$$= 60848.71$$

NEXT PPT(5)

Independent: costs and benefits of one project do not depend on whether another is chosen

Mutually exclusive: a project is excluded if another is selected

Example 2 Payback period:

$$\text{Payback period} = \frac{\text{Initial cost}}{\text{Uniform annual benefits}}$$
$$= \frac{650{,}000}{162{,}500} \rightarrow 4 \text{ years}$$

Example 4

$$PW(15\%) = -75000 + 24400(P/F, 15\%, 1) + 27340(P/F, 15\%, 2) \cdots$$
$$\cdots + 55760(P/F, 15\%, 3)$$

$$PW(15\%) = 3553$$

$PW(15\%) > 0 \rightarrow$ accept, or recommend other method.

$$FW(15) = -75000\,(F/P,\,15\%,\,3) + 24400\,(F/P,\,15\%,\,2) + 27340\,(F/P,\,15\%,\,1) \cdots$$

$$\cdots + 55760$$

$$FW(15) = 5404$$

$\therefore$ accept or recommend ($FW(15) > 0$)