Example 8

$P = 250000$

$i = 8\%$

$N = 6$

$A = P(A/P, i, N)$

$= (250000)(A/P, 8\%, 6)$

(From table) $\Rightarrow 0.2163$

$A = 54075$

Example 9

P_{adj} or V_1

$A = P(A/P, i, N)$

$= P_{adj}(A/P, 8\%, 6)$

$$= \Big[\underbrace{(250000)(F/P, 8\%, 1)}_{P_{adj} \text{ or } V_1}\Big](A/P, 8\%, 6)$$

$= (250000)(1.08)(0.2163)$

$A = 58401$

Present-worth Factor : $(P/A, i, N)$ — **Uniform Series**

- Find P given A, i, N
- "what would you have to invest now in order to withdraw A dollars after N interest periods."

→ \$17 million lump sum or \$1 million every year for 25 years

$A = 1000000$

$i = 8\%$

$N = 25$

$P = A(P/A, 8\%, 25)$

$= 1000000(10.6748)$

$= 10,674,800$

Present-worth Factor : $(P/G, i, N)$ – Linear Gradient

$\boxed{\text{Example 11}}$

$A_1 = 1000$
$G = 250$
$i = 12\%$
$N = 5$

Find P

$$P = \underbrace{A_1(P/A, 12\%, 5)}_{\text{Uniform Series}} + \underbrace{G(P/G, 12\%, 5)}_{\text{Linear Gradient}}$$

$$P = (1000)(3.6048) + (250)(6.379)$$

$$P = 5204$$

Gradient-to-Equal-Payment series Conversion Factor $(A/G, i, N)$

$\boxed{\text{Example 12}}$

$A_1 = 1000$
$N = 6$
$i = 10\%$
$G = 300$
$N = 6$

$$A_{SAME} = A_1 + G(A/G, 10\%, 6)$$
$$= 1000 + 300(2.22236)$$
$$= 1667.08$$

$\boxed{\text{Example 13}}$ (written as 11 in slides)

$A_1 = 1200$
$i = 10\%$
$N = 5$
$G = 200$

$$F = A_1(F/A, 10\%, 5) - \overbrace{G(P/G, 10\%, 5)}^{P_0}(F/P, 10\%, 5)$$

$$F = (1200)(6.1051) - (200)(6.8615)(1.6105)$$

$$F = 5116$$

Geometric Gradient Series

↳ series of cash flows that increase or decrease by a constant percentage

1. Present-Worth Factor : $(P/A_1, g, i, N)$

Example 14

$A_1 = 54440$

$i = 12\%$

$g = 7\%$

$N = 5$

$$P = A_1 (P/A_1, g, i, N)$$
$$= (54440)(P/A, 7\%, 12\%, 5)$$
$$= 54440 \left[\frac{1 - (1+0.07)^5 (1+0.12)^{-5}}{0.12 - 0.07} \right]$$
$$= 222,283$$

Example 15

OR $\begin{cases} P = A_1 (P/A, g, i, N) \\ P = F(P/F, i, N) \end{cases}$

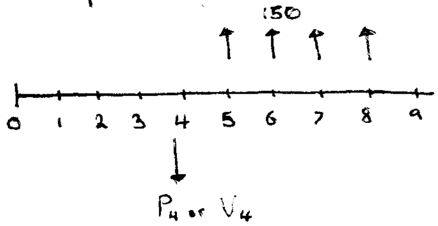
$$= 1,000,000 \, (P/F, 8\%, 20)$$

THEN: $1,000,000 \, (P/F, 8\%, 20) = A_1 (P/A, 6\%, 8\%, 20)$

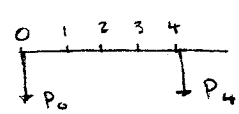
$$A_1 = \frac{1,000,000 \, (P/F, 8\%, 20)}{(P/A, 6\%, 8\%, 20)}$$

$$A_1 = 13757$$

Composite cash Flows



$$P_4 = A(P/A, i, N)$$
$$= (150)(P/A, 15\%, 4)$$



Example 16

Period 2

→ Cash Flow 1

$V_2 = 100(F/A, 12\%, 2) + 300(P/A, 12\%, 3)$

$V_2 = 932.55$

→ Cash flow 2

$V_2 = C(F/A, 12\%, 2) + \overbrace{C(P/A, 12\%, 2)(P/F, 12\%, 1)}^{V_3}$

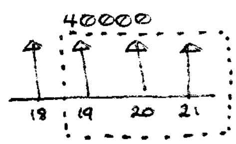
$V_2 = 3.6290\,C$

Now,

$V_2 CF_1 = V_2 CF_2$

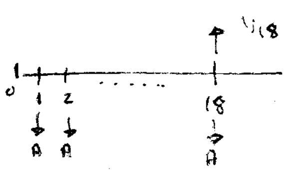
$932.55 = 3.6290C \rightsquigarrow C = 256.97$

Example 17 For withdrawal :



$V_{18} = 40000(P/A, 7\%, 3) + 40000$

$V_{18} = 144{,}972$ → only consider 3 periods



$A = V_{18}(A/F, 7\%, 18)$

$= 144972(A/F, 7\%, 18)$

$= 4264$

<u>Normal interest rate</u> : Stated rate of interest for a given period

<u>Effective interest rate</u> : actual rate of interest, which accounts for the interest amount accumulated over a given period.

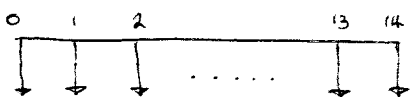
Effective Annual Interest Rate Formula:

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1$$

$$\rightarrow \quad i_a = \left(1 + \frac{0.09}{4}\right)^4 - 1 \quad \rightarrow \quad i_a = 9.3083\%$$

Example 1

Part A:



$A = 1000$

$i = 5\%$

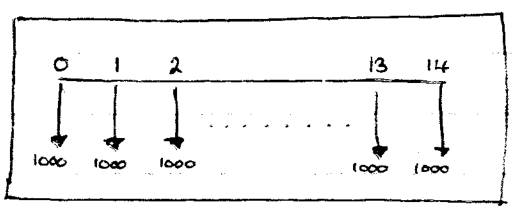
$P = ?$

Approach 1

$P = 1000\,(P/A,\ 5\%,\ 14) + 1000$

$P = 10899$

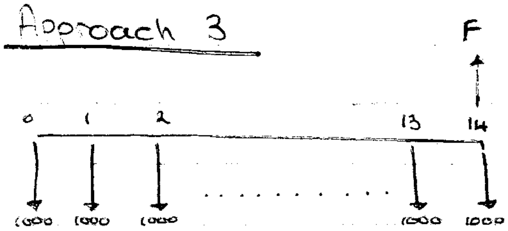
Approach 2



$$P = \underbrace{1000\,(P/A,\ 5\%,\ 15)}_{V_{-1}}(F/P,\ 5\%,\ 1)$$

$P = 10899$

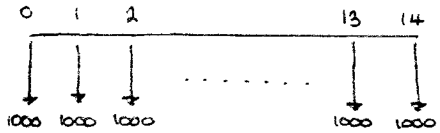
Approach 3



$$P = \overbrace{1000\,(F/A,\ 5\%,\ 15)}^{V_{14}}(P/F,\ 5\%,\ 15)$$

$P = 10899$

Part B



$A = 1000$

$i = 5\%$ comp. monthly

$N =$

Effective interest per year

$i_a = (1 + r/m)^m - 1$

$r = 5\%$, $m = 12$ $\Big\}$ $i_a = 5.12\%$

Now, interest rate and periods match.

Approach 1

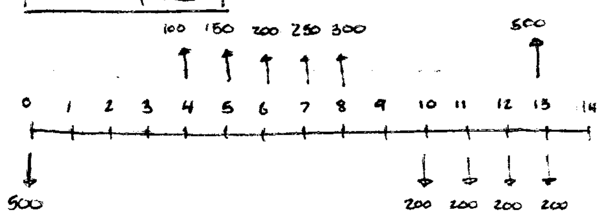
$P = 1000(P/A, 5.12\%, 14) + 1000$

$$P = 1000\left[\frac{(1+0.0512)^{14} - 1}{0.0512(1+0.0512)^{14}}\right] + 1000$$

$\cong 10823$

Review for Exam

Example



$A = ?$

$i = 12\%$

$$P = -500 + \underbrace{100(P/A, 12\%, 5)}_{V_3}(P/F, 12\%, 3) + 50(P/G, 12\%, 5)(P/A, 12\%, 3) \ldots$$

$$\ldots - \underbrace{200(P/A, 12\%, 4)}_{V_9}(P/F, 12\%, 9) + 500(P/F, 12\%, 13)$$

Now A

$A = P(A/P, 12\%, 14)$