ECON

1

JAN. 7 / 20

- Attendance will be done through the Top Hat app.
- Most classes are 1hr. / 5 min. break / 1 hr.

Engineering Economics: The science that deals with techniques of quantitative analysis, used for selecting a preferred alternative from technically viable ones.

- Engineering economic analysis are decisions based upon established facts.

Proprietorship: a business owned by 1 individual.
Partnership: a business with 1 or more owner.
Corporation: a legal entity created under provincial or federal law, entity separate from owners & managers
Equipment + process selection: Selecting best alternative
Equipment Replacement: Consider replacement expenditure
New Product + Product Expansion: Decisions for increasing revenue
Cost Reduction: lower firms operating costs
Improvement of Quality Design: continuously improve quality of product

Engineers must estimate:

1. Required investment in a project
2. Product demand
3. Selling price
4. Manufacturing cost
5. Product life

Principles of Engineering Economics

Principle 1: nearby penny is worth a distant dollar
Principle 2: all that counts are the differences among alt's
Principle 3: marginal revenue must exceed marginal cost
Principle 4: additional risk is not taken without the expected additional return.

- in this course we're only considering compound interest

<u>Market Interest Rate</u> : interest rate quoted by financial institutions (the cost of money to the borrowers).

<u>Earning power</u>: money earns more over time.

<u>Purchasing Power</u> : loss of value due to inflation.

<u>Time value</u> : A dollar today is worth more than a dollar in the future.

<u>Principal</u> : Initial money.

<u>Interest Rate</u>: cost, expressed as percent per unit time.

<u>Interest period</u> : length of time, often a year (how frequently interest is calculated).

<u>Number of Interest Periods</u> : length of time of transaction.

<u>Plan for Receipts (or payments)</u> : particular cash flow over specified time.

<u>Future amount of money</u> : cumulative effects of the interest rate over a number of interest periods.

→ Exam on March 2nd (?)

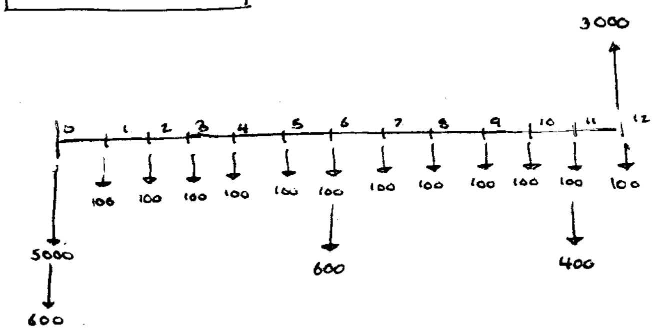
Cash flow diagram : graphical summary of the timing and magnitude of a set of cash flows.

↳ Upward arrows represent positive flow

Downward arrows represent negative flow

end of period convention : placing all cash-flow transactions at the end of an interest period.

Example 1



Simple interest : interest rate charged to initial sum

Compound interest : interest rate charged to initial sum + uncollected interest

(*) General compound interest equation

$$\boxed{F = P(1+i)^N}$$

where N is number of periods
i is interest rate
P is principal amount

Economic equivalence : exists between individual cash flows and/or patterns of cash flows that have the same economic effect (in the end).

Principle 1 : Equivalence calculations made to compare alternatives need the same timescale

Principle 2 : Equivalence depends on interest rate

Principle 3 : Equivalence calculations ...

Principle 4 : ...

Five types of cash flows :

Single cash flow

Equal series

Linear gradient series

Geometric gradient series

Irregular series

Compound amount factor : $F = P(1+i)^N = P(F/P, i, N)$

Example

$P = 20000$

$i = 12\%$

$N = 15$ years

$F = ?$

$F = P(F/P, i, N)$

$F = P(F/P, 12\%, 15)$

By equation :

$F = 20000(1 + 0.12)^{15}$

$F = 109472$

By tables :

$F = P(F/P, 12\%, 15)$

$= 5.4736$ (compound amount factor)

$= 20000(5.4736)$

$= 109472$

Present worth factor : $P = F/(1+i)^N = F(P/F, i, N)$

Example 2

$F = 1000$

$i = 12\%$

$N = 5$

$P = ?$

$P = F(P/F, 12\%, 5)$

$= 0.5674$ (

$= 1000(0.5674)$

$= 567.40$

Example 3

$P = 10$
$F = 20$
$N = 5$
$i = ?$

$F = P(F/P, i, N)$
$20 = 10(1+i)^5$
$\hookrightarrow i = 14.87\%$

Example 4

$P = 6000$
$F = 12000$
$i = 20\%$
$N = ?$

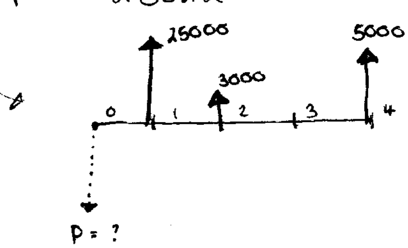
$F = P(F/P, i, N)$
$12000 = 6000(1+0.2)^N$
$\hookrightarrow N = 3.8$ years

Example 5

$P = 25000(P/F, 10\%, 1) + 3000(P/F, 10\%, 2) + 5000(P/F, 10\%, 4)$

$P = 25000(0.9091) + 3000(0.8264) + 5000(0.6830)$
$P = 28622$

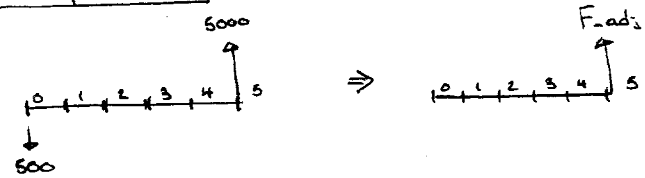
here's the question...



The sinking Fund Factor (A)

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$$

Example 6



Step 1: $F_{adj} = 5000 - 500(F/P,\ 7\%,\ 5)$
$\cong 4299$

Step 2:
$$A = F_{adj}(A/F,\ 7\%,\ 5)$$
$$= 4299\left[\frac{0.07}{(1+0.07)^5 - 1}\right]$$
$$= 4299\,[0.1739]$$
$$= 747.55$$

Example 7

$A = 3000$
$i = 7\%$
$N = 10$
$F = ?$

$$F = A(F/A,\ 7\%,\ 10)$$
$$= 3000\left[\frac{(1+0.07)^{10} - 1}{0.07}\right]$$
$$\cong 41449$$

<u>Capital recovery Factor</u> $A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P,\ i,\ N)$

$\left\{ \begin{array}{l} \text{P is always 1 period before first period.} \\ \text{F is always last period.} \end{array} \right\}$

Example 8

$P = 250000$

$i = 8\%$

$N = 6$

$A = P(A/P, i, N)$

$= (250000)(A/P, 8\%, 6)$

(From table) $\Rightarrow 0.2163$

$A = 54075$

Example 9

P_{adj} or V_1

$A = P(A/P, i, N)$

$= P_{adj}(A/P, 8\%, 6)$

$= \Big[\underbrace{(250000)(F/P, 8\%, 1)}_{P_{adj} \text{ or } V_1}\Big](A/P, 8\%, 6)$

$= (250000)(1.08)(0.2163)$

$A = 58401$

Present-worth Factor : $(P/A, i, N)$ – **Uniform Series**

- Find P given A, i, N
- "what would you have to invest now in order to withdraw A dollars after N interest periods."

→ \$17 million lump sum or \$1 million every year for 25 years

$A = 1000000$

$i = 8\%$

$N = 25$

$P = A(P/A, 8\%, 25)$

$= 1000000(10.6748)$

$= 10,674,800$

Present-worth Factor : $(P/G, i, N)$ – Linear Gradient

$\boxed{\text{Example 11}}$

$$\left.\begin{array}{l} A_1 = 1000 \\ G = 250 \\ i = 12\% \\ N = 5 \end{array}\right\} \text{Find } P$$

$$P = \underbrace{A_1(P/A, 12\%, 5)}_{\text{Uniform Series}} + \underbrace{G(P/G, 12\%, 5)}_{\text{Linear Gradient}}$$

$$P = (1000)(3.6048) + (250)(6.379)$$

$$P = 5204$$

Gradient-to-Equal-Payment series
Conversion Factor $(A/G, i, N)$

$\boxed{\text{Example 12}}$

$A_1 = 1000$

$N = 6$

$i = 10\%$

$G = 300$

$N = 6$

$$A_{SAME} = A_1 + G(A/G, 10\%, 6)$$
$$= 1000 + 300(2.22236)$$
$$= 1667.08$$

$\boxed{\text{Example 13}}$ (written as 11 in slides)

$A_1 = 1200$

$i = 10\%$

$N = 5$

$G = 200$

$$F = A_1(F/A, 10\%, 5) - \overbrace{G(P/G, 10\%, 5)}^{P_0}(F/P, 10\%, 5)$$

$$F = (1200)(6.1051) - (200)(6.8615)(1.6105)$$

$$F = 5116$$

Geometric Gradient Series

↳ series of cash flows that increase or decrease by a constant percentage

1. Present-Worth Factor : $(P/A_1, g, i, N)$

Example 14

$A_1 = 54440$

$i = 12\%$

$g = 7\%$

$N = 5$

$$P = A_1 (P/A_1, g, i, N)$$
$$= (54440)(P/A, 7\%, 12\%, 5)$$
$$= 54440 \left[\frac{1 - (1+0.07)^5 (1+0.12)^{-5}}{0.12 - 0.07} \right]$$
$$= 222,283$$

Example 15

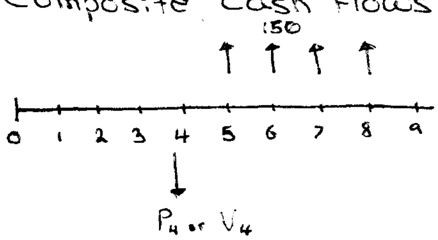
$$\text{OR} \begin{cases} P = A_1 (P/A, g, i, N) \\ P = F(P/F, i, N) \end{cases}$$
$$= 1,000,000 (P/F, 8\%, 20)$$

THEN: $1,000,000 (P/F, 8\%, 20) = A_1 (P/A, 6\%, 8\%, 20)$

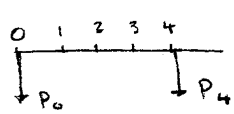
$$A_1 = \frac{1,000,000 (P/F, 8\%, 20)}{(P/A, 6\%, 8\%, 20)}$$

$A_1 = 13757$

Composite Cash Flows



$$P_4 = A(P/A, i, N)$$
$$= (150)(P/A, 15\%, 4)$$



$\boxed{\text{Example 16}}$

Period 2

→ Cash Flow 1

$V_2 = 100(F/A, 12\%, 2) + 300(P/A, 12\%, 3)$

$V_2 = 932.55$

→ Cash flow 2

$V_2 = C(F/A, 12\%, 2) + \overbrace{C(P/A, 12\%, 2)(P/F, 12\%, 1)}^{V_3}$

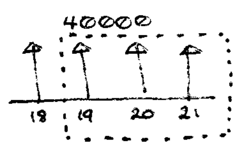
$V_2 = 3.6290\,C$

Now,

$V_2 CF_1 = V_2 CF_2$

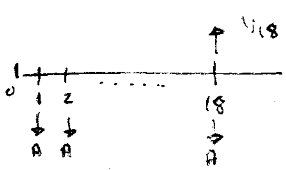
$932.55 = 3.6290C \rightarrow C = 256.97$

$\boxed{\text{Example 17}}$ For withdrawal :



$V_{18} = 40000(P/A, 7\%, 3) + 40000$

$V_{18} = 144{,}972$ → only consider 3 periods



$A = V_{18}(A/F, 7\%, 18)$

$= 144972(A/F, 7\%, 18)$

$= 4264$

<u>Normal interest rate</u> : Stated rate of interest for a given period

<u>Effective interest rate</u> : actual rate of interest, which accounts for the interest amount accumulated over a given period.

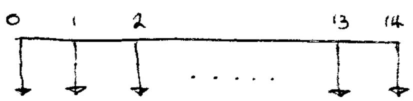
Effective Annual Interest Rate Formula:

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1$$

$$\rightarrow \quad i_a = \left(1 + \frac{0.09}{4}\right)^4 - 1 \quad \rightarrow \quad i_a = 9.3083\,\%$$

Example 1

Part A:



$A = 1000$

$i = 5\%$

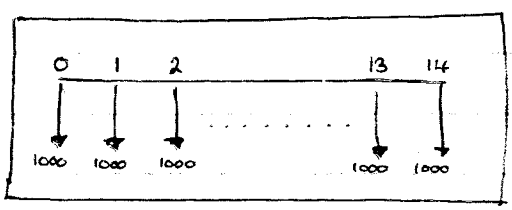
$P = ?$

Approach 1

$P = 1000\,(P/A,\ 5\%,\ 14) + 1000$

$P = 10899$

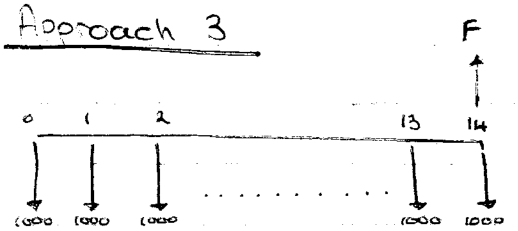
Approach 2



$$P = \underbrace{1000\,(P/A,\ 5\%,\ 15)}_{V_{-1}}\,(F/P,\ 5\%,\ 1)$$

$P = 10899$

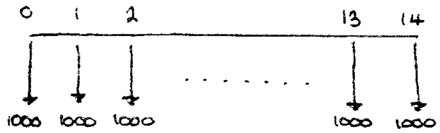
Approach 3



$$P = \overbrace{1000\,(F/A,\ 5\%,\ 15)}^{V_{14}}\,(P/F,\ 5\%,\ 15)$$

$P = 10899$

Part B



$A = 1000$

$i = 5\%$ comp. monthly

$N =$

Effective interest per year

$i_a = (1 + r/m)^m - 1$

$r = 5\%$, $m = 12$ } $i_a = 5.12\%$

Now, interest rate and periods match.

Approach 1

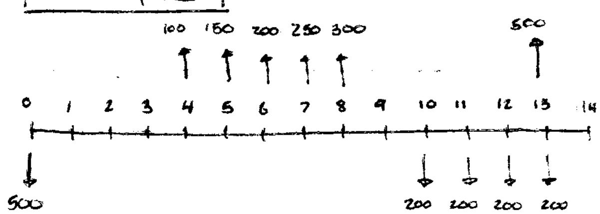
$P = 1000(P/A, 5.12\%, 14) + 1000$

$$P = 1000\left[\frac{(1+0.0512)^{14} - 1}{0.0512(1+0.0512)^{14}}\right] + 1000$$

$\cong 10823$

Review for Exam

Example



$A = ?$

$i = 12\%$

$$P = -500 + \underbrace{100(P/A, 12\%, 5)}_{V_3}(P/F, 12\%, 3) + 50(P/G, 12\%, 5)(P/A, 12\%, 3) \cdots$$

$$\cdots - \underbrace{200(P/A, 12\%, 4)}_{V_9}(P/F, 12\%, 9) + 500(P/F, 12\%, 13)$$

Now A

$A = P(A/P, 12\%, 14)$

Effective Annual Interest Rate Formula (covered last time?)

$$i_A = \left(1 + \frac{r}{M}\right)^M - 1$$

r = nominal interest rate

i_a = effective annual interest rate

M = number of interest periods / year

Effective Interest Rates per Payment Period

$$i = \left(1 + \frac{r}{M}\right)^c - 1 = \left(1 + \frac{r}{CK}\right)^c - 1$$

where: M = number of compounding periods / year

C = number of compounding periods / payment period

K = number of payment periods / year

Example 3

$K = 4$ (payments / year)

$r = 8\% = 0.08$

M = vary depending on payment period

C = "

a) Quarterly:

$K = 4$ (payment periods / year)

$r = 0.08$

$C = 1$ (compounding periods / payment period)

$M = CK$ (compounding periods / year)

$\hookrightarrow$ $M = 4$, $K = 4$ } $C = 1$

$$i_e = \left(1 + \frac{r}{CK}\right)^c - 1$$

$$= \left(1 + \frac{0.08}{(1)(4)}\right)^1 - 1 = 2\%$$

b) Monthly

$r = 8\% = 0.08$

$K = 4$

Since $M = cK$ $\boxed{C = 3}$

$M = \underset{\text{months}}{12}$ (compounding periods / year)

$$i_e = \left(1 + \frac{0.08}{(3)(4)}\right)^3 - 1 = 2.013\%$$

(per quarter)

c) Weekly

$r = 8\% = 0.08$

$K = 4$

$\boxed{C = 13}$ → $M = cK$ or $c = M/K$

$M = \underset{\text{weeks}}{52}$ (compounding periods / year)

$$i_e = \left(1 + \frac{0.08}{(13)(4)}\right)^{13} - 1 = 2.019\%$$

(per quarter)

d) Daily

$r = 8\% = 0.08$

$K = 4$

$C = 91.25$ (compounding periods / payment period)

$M = \underset{\text{days}}{365}$ (compounding periods / year)

$$i_e = \left(1 + \frac{0.08}{(91.25)(4)}\right)^{91.25} - 1 = 2.02\%$$

(daily)

$\boxed{\text{Example 4}}$

$r = 6.25\%$ (compounded monthly)

A = per month

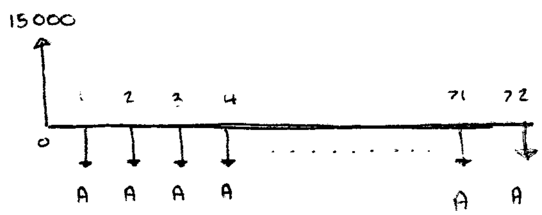
Step 1: $M = 12$ (compound periods / year)

Step 2: $i_e = \dfrac{r}{M} = 0.5208\%$ (per month)

Step 3: $N = M * \text{years} = 72$ months

... Calculating A (payments)

$$A = P(A/P, i, N)$$
$$= 15000\,(A/P,\ 0.5208\%,\ 72)$$
$$= \$250.37$$



Example 5

Step 1 :

$M = 12$

$K = 4$ (deposits / year) = / quarter

$C =$ (compounding period / payment period)

$\hookrightarrow M = CK \rightsquigarrow C = 3$

Step 2 :

$r = 6\% = 0.06$

$$i_e = \left(1 + \frac{0.06}{(3)(4)}\right)^3 - 1 = 1.5075\%$$

(per quarter)

Step 3 :

$N =$ K × years

$= 4 \times 2 = 8$ quarters

Step 4 :

balance @ 2 years

$$F = A(F/A, i, N)$$
$$= (1500)(F/A,\ 1.5075\%,\ 8)$$
$$= \$12652.60$$

Example 6

$A = 500$ (monthly)

$r = 10\%$

$F =$

$N = 10$ years

Step. 1 : $M = 4$ Compounding periods / Pay

$K = 12$

$C = 1/3 \rightsquigarrow C = M/K$

Step 2 :

$$i_e = \left(1 + \frac{0.10}{(1/3)(12)}\right)^{1/3} - 1 = 0.826\%$$

(per month)

Step 3 :

$N = K \times \text{years}$

$= (12)(10) = 120$ months

Step 4 :

$F = A\ (F/A,\ i,\ N)$

$= 500\,(F/A,\ 0.826\%,\ 120)$

$= \$101{,}907.89$

Example 7

Period 1

$A = 235.37$

$I_n = I_1 = B_{n-1}\,(i)$

$= B_0\ (0.01)$

$= 5000\ (0.01)$

$= 50$

$PP_1 = A - I_1 = 235.37 - 50$

$= 185.37$

$B_1 = B_0 - PP_1 = 5000 - 185.37$

$= 4814.63$

Period 2

$I_2 = 4814.63(0.01) = 48.15$

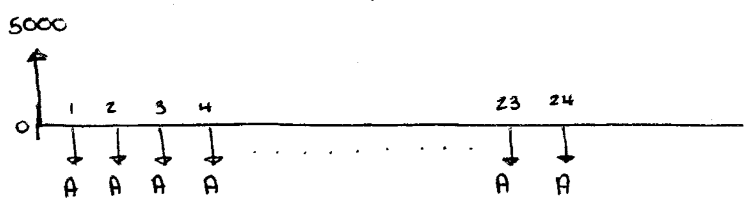
$PP_2 = 235.37 - 48.15$

$= 187.22$

$B_2 = B_1 - PP_2 = 4814.63 - 187.22$

$= 4627.41$

b) $B_6 = A(P/A, i, N-n)$



$= 235.37(P/A, 1\%, 18)$

$= 3869.62$

$I_n = ?$

$I_6 = B_{n-1}(i) = B_5(i)$

$= A(P/A, 1\%, 19)(1\%)$

$= 235.37(P/A, 1\%, 19)(0.01)$

$= 40.54$

$PP_6 = A - I_6$

$= 235.37 - 40.54$

$= 194.83$

Two types of mortgages: - Fixed rate
- Variable mortgage (not covered in course)

Example 9

$P = 100\,000$

$r = 8\%$

$M = 2$ compounding periods / year

amortization = 25 years

term = 3 years

$\hookrightarrow c = M/K \rightarrow c = 2$ periods / year / 12 months / year
(per month)
$\hookrightarrow c = 1/6$

$i_e = ?$

$A \rightarrow$ month

So, for i_e:

$$\left.\begin{array}{l} M = 2 \\ K = 12 \\ c = 1/6 \end{array}\right\} \quad i_e = \left(1 + \frac{0.08}{(1/6)(12)}\right)^{1/6} - 1$$

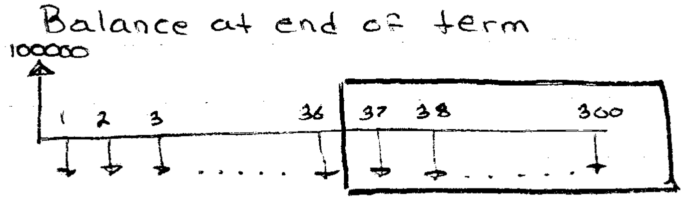
$i_e = 0.6558\%$ (per month)

$N = K * \text{years}$
$= (12)(25) = 300$ months

For $A \rightarrow$ payments

$A = P(A/P, i, N)$
$= 100,000\,(A/P, 0.6558\%, 300)$
$= \$\,763.20$

b) Balance at end of term



$$B_{36} = A(P/A, i, N-n)$$
$$= 763.20(P/A, 0.6558\%, 264)$$
$$= 95655.54$$

c) extra payment monthly

$$B_{36,adj} = 95655.54 - 381.60(F/A, 0.6558\%, 24) - 381.60(F/A, 0.6558\%, 12)$$
$$= \$\,81023.31$$

d) lump sums

$$B_{36,adj} = 81023.51 - 8000(F/P, 0.6558\%, 24) - 10000(F/P, 0.6558\%, 12)$$
$$= 60848.71$$

NEXT PPT(5)

<u>Independent</u>: costs and benefits of one project do not depend on whether another is chosen

<u>Mutually exclusive</u>: a project is excluded if another is selected

$\boxed{\text{Example 2}}$ Payback period:

$$\text{Payback period} = \frac{\text{Initial cost}}{\text{Uniform annual benefits}}$$
$$= \frac{650{,}000}{162{,}500} \longrightarrow 4 \text{ years}$$

$\boxed{\text{Example 4}}$

$$PW(15\%) = -75000 + 24400(P/F, 15\%, 1) + 27340(P/F, 15\%, 2) \cdots$$
$$\cdots + 55760(P/F, 15\%, 3)$$

$$PW(15\%) = 3553$$

$PW(15\%) > 0 \longrightarrow$ <u>accept, or recommend</u>

→ other method.

$$FW(15) = -75000\,(F/P, 15\%, 3) + 24400\,(F/P, 15\%, 2) + 27340\,(F/P, 15\%, 1) \cdots$$

$$\cdots + 55760$$

$$FW(15) = 5404$$

$\therefore$ <u>accept or recommend</u> $(FW(15) > 0)$

ECON

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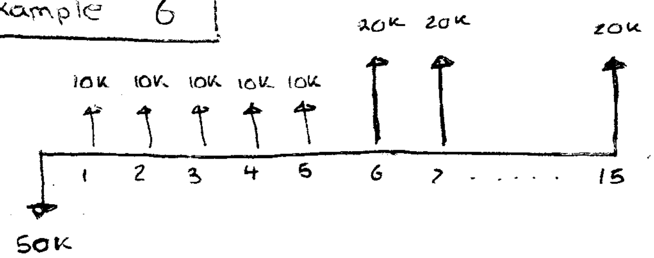
JAN. 28/20

Example 6



MARR = 9%

AE = ?

PW(9%) = -5000 + 10000(P/A, 9%, 5) + 20000(P/A, 9%, 10)(P/F, 9%, 5)

= 73318

AE(9%) = PW(A/P, i, N)

= 73318(A/P, 9%, 15)

= 9096

AE > 0 (accept or recommend)

Example 7

First cycle — MARR = 12% (not given in question)

PW(12%) = -1000000 + 800000 (P/A, 12%, 4) - 100 000 (P/G, 12%, 4)

= 1017150

AE(12%) = PW(A/P, 12%, 4)

= 1017150 (A/P, 12%, 4)

= 334880

Two cycles

PW(12%) = -1000000 - 1000000 (P/F, 12%, 4) . . .

. . . + 800000 (P/A, 12%, 8) - 100000 (P/G, 12%, 4) . . .

. . . - 100000 (P/G, 12%, 4) (P/F, 12%, 4)

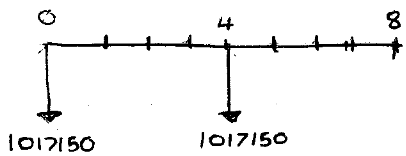
PW(12%) = 1663 560

AE(12%) = 1663560 (A/P, 12%, 8)

= 334 880 (same as one cycle)

Simplify:

→ 2 cycles



Example 8

Capital cost

$$CR(10\%) = (P-S)(A/P, 10\%, 5) + Si$$

$$= (20000 - 4000)(A/P, 10\%, 5) + (4000)(0.10)$$

$$= 4620.76$$

Total Annual Cost = Cap. cost + Oper. cost

$$= 4620.76 + 500$$

$$= 5120.76$$

→ Compare to 5000 per year

Example 9

$$PW(15\%) = 3553$$

$$AW(15\%) = 3553(A/P, 15\%, 3)$$

$$= 1556$$

Savings per machine hour

$$\Rightarrow \frac{1556}{2000} = 0.78 /hr$$

START CLASS NOTES 6

Break-even interest rate : i^*

Simple investments change sign once

Example 3

$PW(i^*) = -1250000 + 731500(P/A, i^*, 15) + 80000(P/F, i^*, 15) = 0$

$i^* = 58.71\%$ (from software)

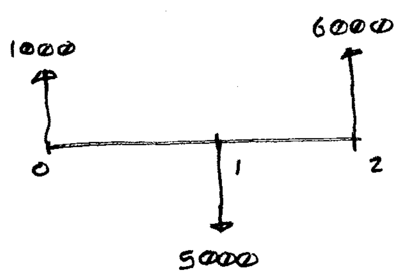
MARR = 18%

IRR > MARR (accept or recommend)

$PW(18\%) = -1250000 + 731500(P/A, 18\%, 15) + 80000(P/F, 18\%, 15)$

$\longrightarrow > 0$; so IRR > MARR

Example 5



MARR = 25%

Non-simple inv.

ERR

ECON

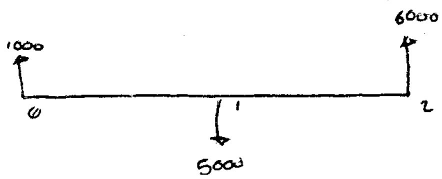
1

JAN. 30/20

Example 5

→ Exact ERR

End of year 1 :



→ non-simple, more than one sign change, apply ERR (otherwise use IRR)

$$FW = 1000(F/P, 25\%, 1) - 5000 = -3750$$

End of year 2

$$FW = -3750(F/P, ERR, 1) + 6000 = 0$$

(Somehow) → Trial and error, $ERR = 60\%$

$ERR > MARR$ (accept or recommend)

→ Approximate ERR

$$FW(rec) = 1000(F/P, 25\%, 2) + 6000$$

$$FW(dist) = 5000(F/P, ERR, 1)$$

$$FW(rec) = FW(dist)$$

$$1000(F/P, 25\%, 2) + 6000 = 5000(F/P, ERR, 1)$$

app $ERR = 61.25\%$

app $ERR > MARR$ (accept or recommend)

→ **BEGIN CLASS NOTES 7**

Example 1

m. $PW(12\%) = -209000 + 55000(P/A, 12\%, 5) + 80000(P/F, 12\%, 5)$

$PW(12\%) = 34657$

M_2

$PW(12\%) = -294600 + 74000(P/A, 12\%, 5) + 120\,000(P/F, 12\%, 5)$

$PW(12\%) = 40245$

M_3

$PW(12\%) = -294600 + 58000(P/A, 13\%, 12\%, 5) \ldots$

$\ldots + 120000(P/F, 12\%, 5)$

$PW(12\%) = 37085$

$\therefore$ M_2 is the recommended machine

Example 2

→ $B_2 - B_1$

0	-9000
1	2850
2	4425
3	4830

Simple

IRR

$PW(IRR) = -9000 + (2850)(P/F, IRR, 1) + (4425)(P/F, IRR, 2) \ldots$

$\ldots + (4830)(P/F, IRR, 3) = 0$

→ IRR = 15%

IRR > MARR → The inves is good

15 10 B2 is best

Example 4

Analysis period = 2 years

MARR = 15%

→ model A

$PW(15\%) = -300000 - 80000(P/A, 15\%, 2) + 90000(P/F, 15\%, 2)$
$= -362000$

→ model B

$PW(15\%) = -480000 - 45000(P/A, 15\%, 2) + 250000(P/F, 15\%, 2)$
$= -364000$

↳ model A > model B , recommend model A

Example 5

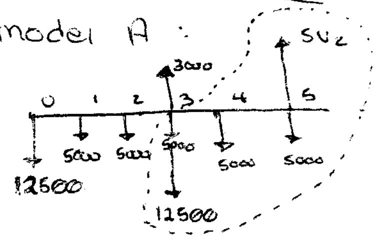
model A → $PW(15\%) = -12500 - (5000)(P/A, 15\%, 5) - (11000)(P/A, 15\%, 2)(P/F, 15\%, 3) \cdots$
$\cdots + (2000)(P/F, 15\%, 3) = -34359$

model B → $PW(15\%) = -15000 - 4000(P/A, 15\%, 4) - (5000 + 11000)(P/F, 15\%, 5) \cdots$
$\cdots + (1500)(P/F, 15\%, 4) = -31031$

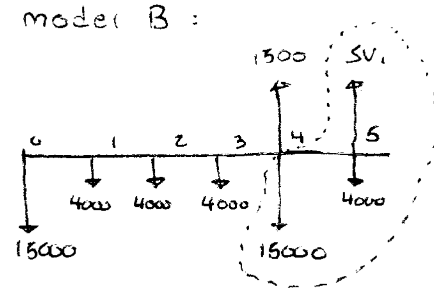
↳ model B > model A , recommend model B

Second approach

model A :



model B :



** exam will likely test each type of question

LCM = least common multiple

↳ LCM(3,4) = 12 years

Example 7

Model A: MARR = 15% → should have been given

First cycle

$PW(15\%) = -12500 - 5000(P/A, 15\%, 3) + 2000(P/F, 15\%, 3)$

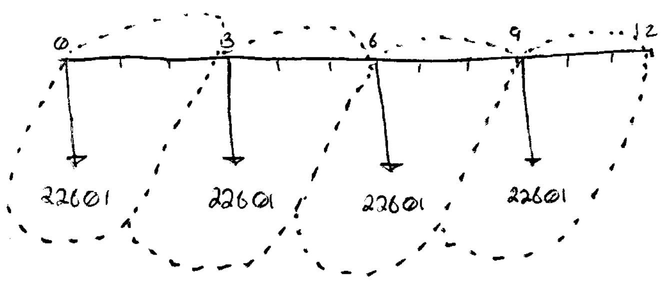
$PW(15\%) = -22601$

$AW(15\%) = PW(A/P, 15\%, 3)$

$= -22601(A/P, 15\%, 3)$

$= -9899$

Now For LCM:



$PW(15\%) = (-22601) - (22601)(P/F, 15\%, 3) - (22601)(P/F, 15\%, 6) \cdots$

$\cdots - (22601)(P/F, 15\%, 3)$

$PW(15\%) = -53657$

$AW(15\%) = -53657(A/P, 15\%, 12)$

$AW(15\%) = -9899$

Model B

First cycle

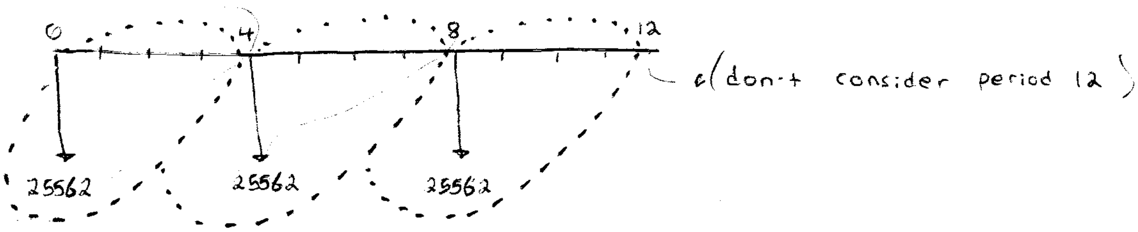
$PW(15\%) = -15000 - 4000(P/A, 15\%, 4) + (1500)(P/F, 15\%, 4)$

$PW(15\%) = -25562$

$AW(15\%) = -25562(A/P, 15\%, 4)$

$= -8954$

- can't compare PW for different payment period lengths



$PW(15\%) = -25562 - (25562)(P/F, 15\%, 4) - (25562)(P/F, 15\%, 8)$

$PW(15\%) = -48534$

$AW(15\%) = -48534(A/P, 15\%, 12)$

$AW(15\%) = -8954$

$\hookrightarrow PW(15\%)_{,\text{CASE B}} > PW(15\%)_{,\text{CASE A}}$

END OF PPT

→ Start of Chapter 8

$\boxed{\text{Example 3}}$

$P = 10000$

$N = 5$ years

$S = 2000$

$$D_n = \frac{P-S}{N} = \frac{10000-2000}{5} = 1600$$

Book value at end of period 4

$BV_4 = P - n\,(P-S/N)$

$BV_4 = 10000 - 4(1600)$

$BV_4 = 3600$

Period (n)	BV_{n-1}	D_n	BV_n
1	10000	1600	8400
2	8400	1600	6800
3	6800	1600	5200
4	5200	1600	3600
5	3600	1600	2000

Example 4

DB

P = 10000

N = 5 years

S = 3277

Period	BV_{n-1}	D_n	BV_n
1	10000	2000	8000
2	8000	1600	6400
3	6400	1280	5120
4	5120	1024	4096
5	4096	819	3277

$d = (1/N)$ multiplier $= (1/5)(1)$

$= 20\%$. (decrease D_n by 20% every period)

" Summary Version of Schedule 8 : Capital Cost Allowance Form "

- review all columns
  ↳ heading is given, but not process

Column: 1, 2, 3, 5, 6, 7, 8, 9, 12, 13

Example 5

Year	UCC Begin	Acq	Disp	UCC	50%	UCC Red.	UCC Rate	CCA	UCC End
2006	0	50000	0	50000	25000	25000	25%	6250	43750
2007	43750	0	0	43750	0	43750	25%	10 937.5	32 812.5
2008	32812.5	0	0	32 812.5	0	32812.5	25%	8 203	24609
2009	24609	0	0	24609	0	24609	25%	6152	18962

col 3 = Acq. → $2 \times 25000 = 50000$

col 6 = $2 + 3 - 5 = 0 + 50000 - 0 = 50000$

col 7 = $\frac{3 - 5}{2} = \frac{50000 - 0}{2} = 25000$

col 8 = $6 - 7 = 50000 - 25000 = 25000$

col 9 = 25%

col 12 = $8 \times 9 = (25000)(0.25) = 6250$

col 13 = $6 - 12 = 50000 - 6250 = 43750$

→ CLASS-NOTES-9 :

Example 2

Net income : (First year)

Revenues : 52000

Expenses :

Cost of goods sold	20000
oper. cost.	5000
CCA	6000

diff = 52000 − 20000 − 5000 − 6000 = 21000

Taxable income : 21000

Taxes (40%) : 8400 → 40% × 21000

Net income : 12600

ECON

1

Feb. 11/20

Example 4

a) $S = 150000$

$G = t(U_{disp} - S)$

where $t = 40\%$

$G = (0.4)(104125 - 150000)$

$G = -18350$

Then...

Net Salvage value $= S + G$

$150000 - 18350 = 131650$

b) $S = 104125$

$S = U_{disp}$

$\rightarrow G = 0$

c) $S = 90000$

$G = (0.4)(104125 - 90000)$

$= 5650$

d) $S = 280000$

$S >$ cost basis

$\rightarrow$ cap. gain

$G = t(U_{disp} - P) - t_{CG}(S - P)$

$= (0.4)(104125 - 250000) \cdots$

$\cdots - (0.4/2)(280000 - 250000)$

$G = -64350$

START Chapter 10

Example 1

Step 1 :

Income statement : Year 1

Revenues :	100 000
Expenses :	
Labour :	20 000
Material :	12 000
Overhead :	8 000
CCA :	18750
Taxable Inc.	41250
Taxes (40%) :	16500
Net income :	24750

half for first period!

CCA system

Period	CCA	UCC
0		125000
1	18750	106250
2	31875	74375
3	22313	52062
4	15619	36444
5	10933	24511

(*) CCA Rate = 30%

$$CCA_p = \frac{125000}{2}(0.30) = 18750$$

Step 2 : Cash Flow Statement

	0	1
Operating Act.		
Net income		24750
CCA		18750
Investing Act		
Initial cost	125000	
Salvage		
Disp. Tax Effect		
Financing Act.		
Net Cash Flow	125 000	43500

↳ Disposal tax effect

$G = t(U_{disp} - S)$

$t = 40\%$

$U_{disp} = 24511$

$S = 50000$

→ $G = 0.4(24511 - 50000)$

$= -9796$

Example 1

Salary in 2008 = 310,800

Salary in 1968 = 25000

$\bar{f}$ = 4.611%

years = 40 years

$$A_{2008} = 25000(1+0.04611)^{40}$$
$$= 151729$$

Example 2

Actual → Constant dollars

$$A_n' = A_n((P/A, \bar{f}, n)$$

Period 0

$$A_0' = 20000(1+0.05)^0 = -20000$$
$$A_1' = 20000(1+0.05)^{-1} = -19048$$

⋮

$$A_4' = 20000(1+0.05)^{-4} = -16454$$

Example 3

$i' = 12\%$

Constant dollars

$$PW(12\%) = -250000 + 100000(P/A, 12\%, 4) + 10000(P/G, 12\%, 4) \ldots$$
$$\ldots + (120000)(P/F, 12\%, 4)$$

Example 4

Year 1

Act. → const. dollar

$A_1' = A_1(P/F, f, n)$

$= 32000(1+0.05)^{-1}$

$= 30476$

[actual ↓ constant (w/ inflation)]

b) $i' = 10\%$

$PW(10\%) = -75000 + 30476(P/F, 10\%, 1) + 32381$

Example 5

$i = i' + f + i'f$

$= 0.10 + 0.05 + (0.10)(0.05)$

$= 15.5\%$

$PW(15.5\%) = -75000 + 32000(P/F, 15.5\%, 1) + (35700)(P/F, 15.5\%, 2) + \ldots$

$= \$45268$

Final:
- 2nd March
- 25 questions
- problems may be 2 parts
- problems may have 5 parts
  - → review fundamentals
  - → review practice midterm / Final
- ROR won't ask for exact value, (just above/below MARR)