Heat Transfer, Chapter 3: Steady Heat Conduction

Oct. 31/17
Thermal sci.

Obj:

1) Understand concept of thermal resistance, develop resistance network for practical heat cond. prob.
2) Multilayer wall, cylinder, sphere
3) Identify when insulation increases heat transfer
4) Conduction shape factor

Steady heat conduction in plane wall:

$$\dot{Q} = -kA\frac{dt}{dx}$$

$$\dot{Q}_{cond,wall} = \frac{kA(T_1 - T_2)}{L} \quad (\text{Watt}) \quad \text{or} \quad \left[\frac{(T_1 - T_2)}{R_{wall}}\right]$$

Thermal Resistance Concept:

$$I = \frac{V_1 - V_2}{R_e}$$

$V_1 - V_2$ → potential diff.

R_e → electrical resistance

ohm's law

$$\dot{Q}_{cond,wall} = \frac{kA(T_1 - T_2)}{L} = \frac{(T_1 - T_2)}{(L/kA)} = \frac{(T_1 - T_2)}{(R_{wall})}$$

where

$$\left[R_{wall} = L/kA\right]$$

Newton's Law of Cooling (convection):

A_s, h, T_s $\quad T_\infty$

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = \frac{T_s - T_\infty}{(1/hA_s)} = \frac{T_s - T_\infty}{R_{conv.}}$$

$R_{conv} = K/W$ or $°C/W$

$$\dot{Q}_{rad} = \epsilon\sigma A_s(T_s^4 - T_{surr}^4)$$

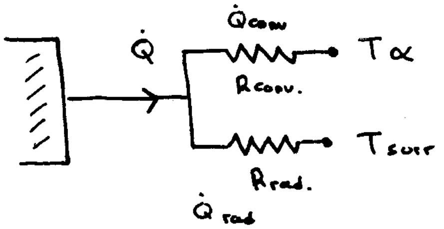
$$= \epsilon\sigma A_s\left[(T_s^2 - T_{surr}^2)(T_s^2 + T_{surr}^2)\right]$$

$$= \epsilon\sigma A_s\left[(T_s - T_{surr})(T_s + T_{surr})(T_s^2 + T_{surr}^2)\right]$$

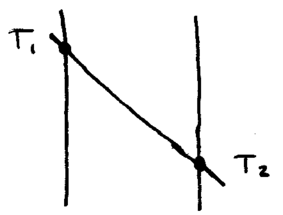
$$\left[\left(\frac{T_s - T_{surr}}{R_{rad}}\right)\right]$$

$$R_{rad} = \frac{1}{h_{rad} A_s}$$

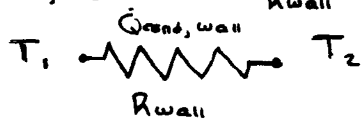
$$h_{rad} = \epsilon\sigma\left(T_2^{2} + T_{surr}^{2}\right)\left(T_s + T_{surr}\right) \quad (\text{in } W/m^2 \cdot K)$$



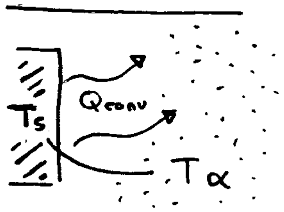
$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

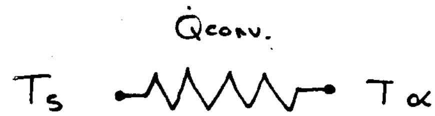


$$\dot{Q}_{cond,\,wall} = \frac{T_1 - T_2}{R_{wall}}$$



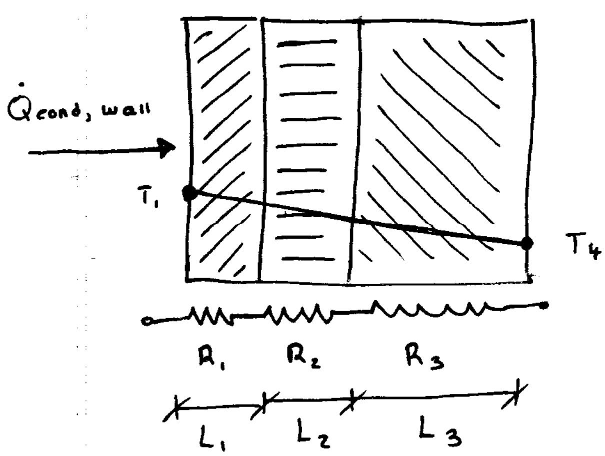
Convection





$$R_{conv.} = \frac{1}{hA_s}$$

$$h_{comb} = h_{conv.} + h_{rad}$$

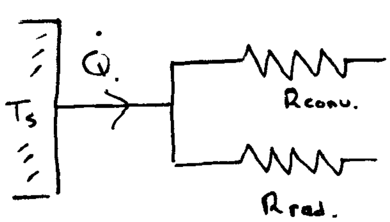


$$\dot{Q}_{cond} = \frac{T_1 - T_4}{R_{TOTAL}}$$

$$R_{TOTAL} = R_{wall_1} + R_{wall_2} + R_{wall_3}$$

$$R_{wall_1} = \frac{L}{kA} \; ; \quad R_{wall_2} = \frac{L}{k_2 A}$$

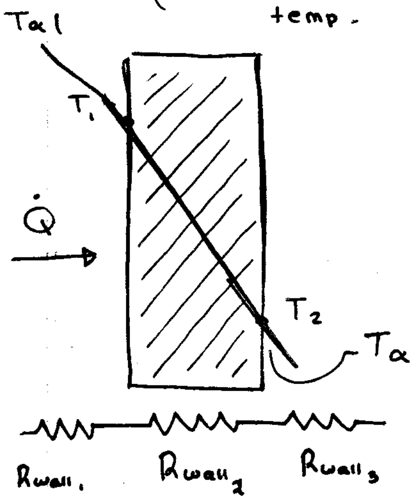
$$R_{wall_3} = \frac{L}{k_3 A}$$



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_n}$$

$$\frac{1}{R_{TOTAL}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$$

(Tα1 is the highest temp.)

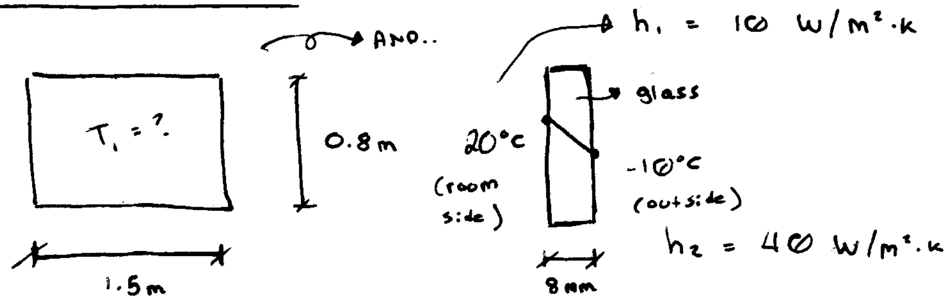


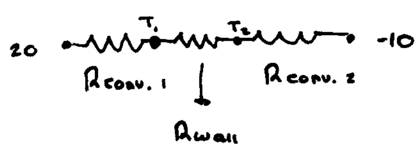
$$\dot{Q} = \frac{T_{\alpha 1} - T_{\alpha 2}}{R_{TOTAL}}$$

$$R_{TOTAL} = \underbrace{R_{conv1}}_{1/hA} + \underbrace{R_{wall}}_{1/kA} + R_{conv2}$$

$$\dot{Q}_1 = hA(T_\alpha - T_1) = \frac{kA(T_1 - T_2)}{L} = hA(T_2 - T_{\alpha 2})$$

Example 3-2 :





$$\dot{Q} = \frac{T_{\alpha 1} - T_{\alpha 2}}{R_{TOTAL}}$$

$$R_{conv1} = \frac{1}{h_1 A} = \frac{1}{10 \times (0.8 \times 1.5)}$$

$$= 0.08333 \ ^\circ C/W$$

$$R_{wall} = \frac{L}{kA} = \frac{(0.008)}{(0.78)(0.8 \times 1.5)}$$

$$= 0.00855 \ ^\circ C/W$$

$$R_{conv2} = \frac{1}{h_2 A} = \frac{1}{40 \times (0.8 \times 1.5)}$$

$$= 0.02083 \ ^\circ C/W$$

$$\therefore R_{TOTAL} = 0.1127 \ ^\circ C/W$$

$$\dot{Q} = \frac{20 - (-10)}{(0.1127)}$$

$$\boxed{\dot{Q} = 266 \text{ W}}$$

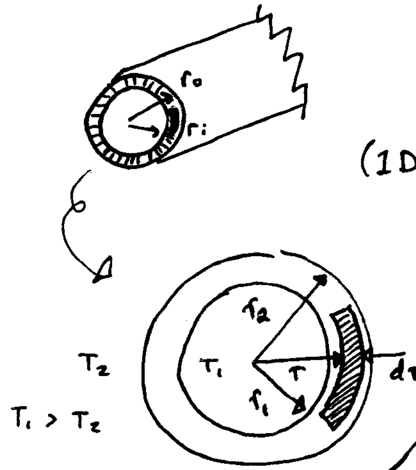
$$Q = \frac{T_{\alpha 1} - T_1}{R_{conv.1}}$$

$$\therefore \boxed{T_1 = -2.2^\circ C}$$

Nov. 2/17

Heat conduction in cylinders:

Thermal Sc:.



$$\dot{Q}_{cond} = -KA\frac{dT}{dx}$$

$$(1D): \dot{Q}_{cond,\,cyl} = -KA\frac{dT}{dx}$$

$$\int_{r_1}^{r_2} \frac{\dot{Q}_{cond,\,cyl}}{A} = -\int_{T_1}^{T_2} K\,dT$$

$$A = 2\pi r L$$

$$\int_{r_1}^{r_2} \dot{Q}_{cond,\,cyl}\frac{dr}{r} = -2\pi K L\int_{T_1}^{T_2} dT$$

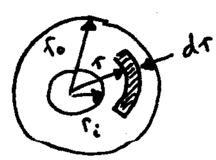
$$\Rightarrow \dot{Q}_{cond,\,cyl}\ \ln r\Big|_{r_1}^{r_2} = -2\pi K L\ T\Big|_{T_1}^{T_2}$$

$$\dot{Q}_{cond,\,cyl}(\ln r_2 - \ln r_1) = -2\pi K L(T_2 - T_1)$$

$$\boxed{\dot{Q}_{cond,\,cyl}\left(\ln\left(\frac{r_2}{r_1}\right)\right) = 2\pi K L(T_1 - T_2)}$$

$$\dot{Q}_{cond,\,cyl} = \frac{T_1 - T_2}{R_{cyl}} \quad \therefore \quad R_{cyl} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K L}$$

For sphere: $\dot{Q}_{cond,\,sphere} = -KA\frac{dT}{dr}$



$$\dot{Q}_{cond,\,sphere} = \frac{T_1 - T_2}{R_{sphere}}$$

$$R_{sphere} = \frac{r_2 - r_1}{4\pi r_1 r_2 K}$$

$$R_{wall} = \frac{L}{KA}$$

$$R_{cylinder} = \frac{\ln(r_2/r_1)}{2\pi K L} \qquad R_{sphere} = \frac{r_2 - r_1}{4\pi r_1 r_2 K}$$

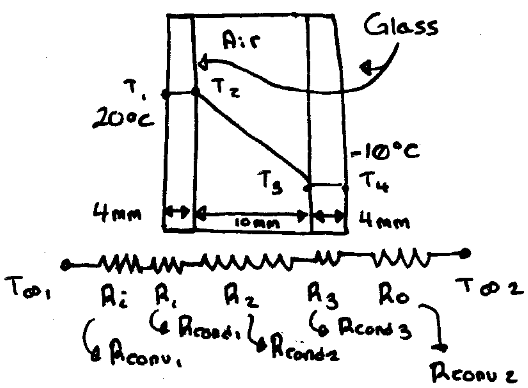
Cartesian coordinate	Plane wall	$\dot{Q}_{conduction} = \frac{T_1 - T_2}{R_{wall}}$
Cylindrical	cylinder	$\dot{Q}_{conduction,\,cyl} = \frac{T_1 - T_2}{R_{cyl}}$
Spherical	Sphere	$\dot{Q}_{conduction,\,sphere} = \frac{T_1 - T_2}{R_{sph}}$

Example 3.3 - (Textbook)

$A = 0.8m \times 1.5m$

$t = 4mm = 0.004m$ (two panes), with $0.01m$ air gap.

$K = 0.78 \; W/m.K$



$R_i = R_{conv-1} = \frac{1}{hA} \Rightarrow \frac{1}{10 \times 0.8 \times 1.5}$

$R_i = 0.08333 \; °C/W$

$R_1 = R_{cond-1} = \frac{L}{KA} = \frac{(0.004)}{(0.78)(0.8 \times 1.5)}$

$R_1 = 0.00427 \; °C/W = R_3$

$R_2 = R_{cond-2} = \frac{L}{KA} = \frac{(0.010)}{(0.026)(0.8 \times 1.5)}$

$R_2 = 0.3205 \; °C/W$

$R_o = R_{conv-2} = \frac{1}{hA} = \frac{1}{(40)(0.8 \times 1.5)}$

$R_o = 0.02083 \; °C/W$

$R_{TOTAL} = R_{conv-1} + R_{conv-2} \cdots$

$\cdots + R_{cond-1} + R_{cond-2} + R_{cond-3}$

$R_{TOTAL} = 0.4332 \; °C/W$

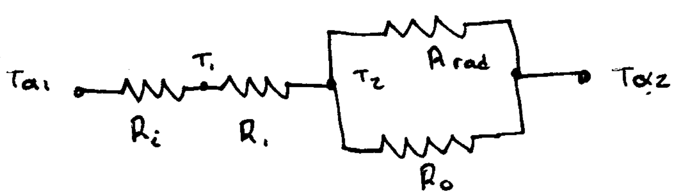
$\dot{Q} = \frac{T_{\alpha 1} + T_{\alpha 2}}{R_{TOTAL}} \Rightarrow \frac{20 - (-10)}{0.4332} = 69.2 \; W$

$\hookrightarrow T_1 = T_{\alpha 1} - \dot{Q} R_{conv-1}$

$= 20 - 69.2 \times (0.08333)$

$\therefore T_1 = 14.2 °C$

Example 3.7 - (Textbook)



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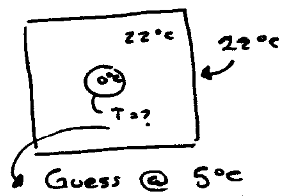


$$A_1 = \pi D_1^2 = \pi(3^2) = 28.3\ m^2$$

$$A_2 = \pi D_2^2 = \pi(3.04^2) = 29\ m^2$$

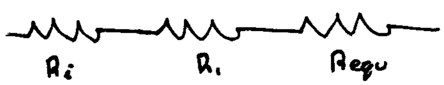
$$h_{rad} = \overset{(1)}{\epsilon}\sigma(\overset{5^\circ C}{T_s^2} + \overset{22^\circ C}{T_{surr}^2})$$

$$\therefore h_{rad} = 5.34\ W/m^2\cdot K$$

$$R_i = R_{conv\text{-}1} = \frac{1}{h_1 A_1} = \frac{1}{80 \times 28.3} = 0.000442\ ^\circ C/W$$

$$R_1 = R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 K} = (----) = 0.0000047\ ^\circ C/W$$

$$R_{equ}. \quad \frac{1}{R_{equ}} = \frac{1}{R_o} + \frac{1}{R_{rad}} \quad \therefore R_{equ} = 0.00225\ ^\circ C/W$$



$$\therefore R_{total} = 0.00274\ ^\circ C/W$$

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{0.00274} = 8029\ W$$

$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{equ}} \Rightarrow T_2 = 4^\circ C$$

In 24 hours: $\dot{Q} = \frac{Q}{\Delta t} \Rightarrow Q = \dot{Q}\cdot\Delta t$

$$\Rightarrow (8029)(24)(60)(60)$$

$$= 693700\ kJ$$

Ice's latent heat of fusion = 333.7 kJ

Mass of ice $\quad \therefore m_{ice} = \frac{Q}{h_{if}} = \frac{693700\ kJ}{338.7\ kJ/kg}$

$$= 2079\ kg$$

Example 3-8