MARCH 6/18
MACHINE DESIGN

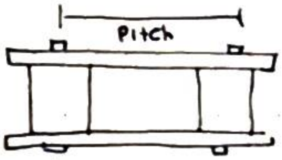
Chain numbering system

Roller chains . RC: # : one digit #

RC : the number to the left of the right hand digit is the number of 1/8 in in the pitch
Rollerchain :

: 6 indicated rollerless bushing chain
: 1 indicates a lightweight chain
: 0 indicate a chain of usual proportions with roller

Chain no. :	25	35	41	40	50	60	80	100	120	140	160	180	200	240
Pitch :	1/4	3/8	1/2	1/2	5/8	3/4	1	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2	3



Inverted-tooth or Silent chains SC: #

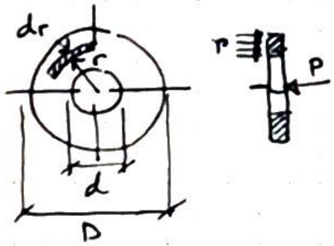
SC : The numeral following is the number of 1/8 in. in the pitch
Silent chain :

Chain no. :	SC3	SC4	SC5	SC6	SC8	SC10	SC12	SC16
Pitch :	3/8	1/2	5/8	3/4	1	1 1/4	1 1/2	2

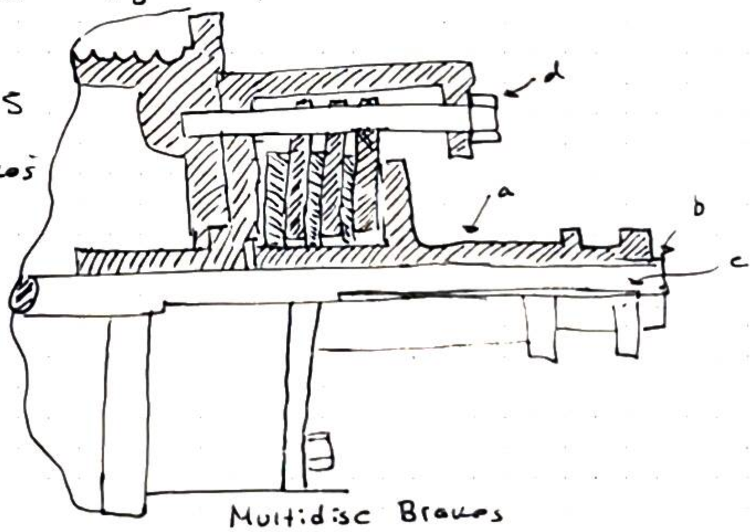
Clutches and Brakes

1 - Plate clutches and Brakes

1.1 - Force Analysis



Friction Disc



Multidisc Brakes

$dA = 2\pi r\,dr$

$dF = p\,2\pi r\,dr$

$dF_{friction} = f\,p\,2\pi r\,dr$

Where, p = surface pressure
f = coefficient of friction

$$P = 2\pi \int_r^R p r\,dr \qquad (1)$$

$$T = 2\pi f \int_r^R p r^2\,dr \qquad (2)$$

1-2 - Uniform Pressure (new clutch)

$$P = 2\pi p \int_r^R r\,dr = \frac{\pi p}{4}(D^2 - d^2) \qquad (3)$$

$$T = 2\pi f p \int_r^R r^2\,dr = \frac{2\pi f p}{24}(D^3 - d^3) \qquad (4)$$

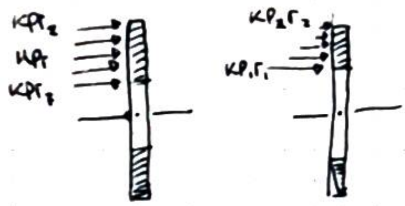
Eliminating p from (3) and (4)

$$T = \frac{fP}{3} \cdot \frac{D^3 - d^3}{D^2 - d^2} \qquad \text{(For new clutch)}$$

where, T = torque for one pair of friction surfaces in contact

↳ if you have n pair, multiply by n (*)

1.3 - Uniform axial wear (old clutches)



$$p_1 r_1 = p_2 r_2 = \text{const.}$$

$$pr = C = \text{const}$$

$$p = \frac{C}{r}$$

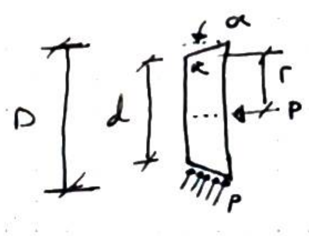
and $$P = 2\pi \int_r^R p r\,dr = 2\pi C \int_{r_1}^{r_2} dr = 2\pi C (r_2 - r_1)$$

$$T = 2\pi f \int_r^R p r^2\,dr = 2\pi f C \int_{r_1}^{r_2} r\,dr = \pi f C (r_2^2 - r_1^2)$$

eliminating C

$$T = \frac{Pf}{2}(r_2 + r_1) = \frac{fP}{4}(D + d) \qquad \text{(old clutches)}$$

2 - Cone clutches



$$dA = 2\pi r\,dr / \sin\alpha$$

$$dF = p\,2\pi r\,dr / \sin\alpha$$

Frictional Force on dA = $f p\,2\pi r\,dr / \sin\alpha$

$$\therefore \quad P = \frac{2\pi}{\sin\alpha} \int_r^R p r\,dr$$

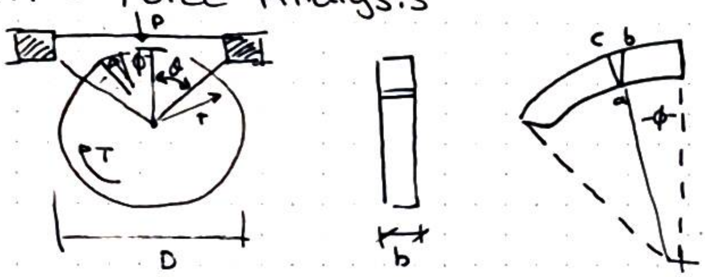
and $$T = \frac{2\pi f}{\sin\alpha} \int_r^R p r^2\,dr$$

replacing $p = C/r$ and solving

$$T = f\frac{P(D+d)}{4\sin\alpha}$$

3 - Block Brakes and Clutches

3.1 - Force Analysis



$$dA = \frac{D}{2} b\, d\phi$$

$$dF = p\frac{D}{2} b\, d\phi \quad = \text{normal Force on } dA$$

The component parallel to P is:

$$dP = p\frac{D}{2} b\cos\phi\, d\phi$$

$$P = \int_{-\theta}^{\theta} p\frac{D}{2} b\cos\phi\, d\phi$$

$$= \frac{Db}{2}\int_{-\theta}^{\theta} p\cos\phi\, d\phi$$

Friction Force on $dA = fp\frac{D}{2} bd\phi$

$$T = \int_{-\theta}^{\theta} fp\frac{D^2}{4} bd\phi$$

if f is constant, then

$$T = f\frac{D^2 b}{4}\int_{-\theta}^{\theta} p\, d\phi$$

For normal wear $p = C\cos\phi$

$$\therefore \quad P = \frac{CDb}{2}\int_{-\theta}^{\theta} \cos^2\phi\, d\phi$$

$$= \frac{CDb}{4}(2\theta + \sin 2\theta)$$

and

$$T = \frac{CfD^2 b}{4}\int_{-\theta}^{\theta}\cos\phi\, d\phi$$

$$= 2cfb\left(\frac{D}{2}\right)^2\sin\theta$$

eliminating C

$$T = f P \frac{D}{2} \frac{4 \sin \theta}{2\theta + \sin 2\theta}$$

The tangential Force is

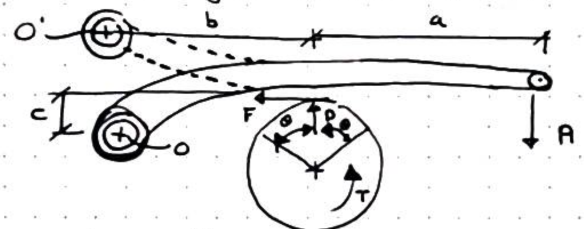
$$F = 2\frac{T}{D} = fP \underbrace{\frac{4 \sin \theta}{2\theta + \sin 2\theta}}_{} = f' P$$

always in radians

Where, f' = equivalent coefficient of friction

$$f' = f \frac{4 \sin \theta}{2\theta + \sin 2\theta}$$

3.2 Single Block Brake



Focus on Brake Lever

$$\Sigma M_o = 0$$

$$A = (a + b) - Pb - Fc = 0$$

but $F = f'P$

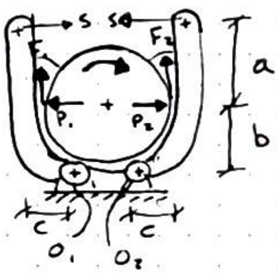
$$\therefore A(a+b) - Pb - f'Pc = 0$$

or $P = \dfrac{A(a+b)}{b + f'c}$

$$T = F\frac{D}{2} = f' \frac{PD}{2}$$

$$\therefore T = \frac{f' AD(a+b)}{2(b + f'c)}$$

3.3 Double Block Brake



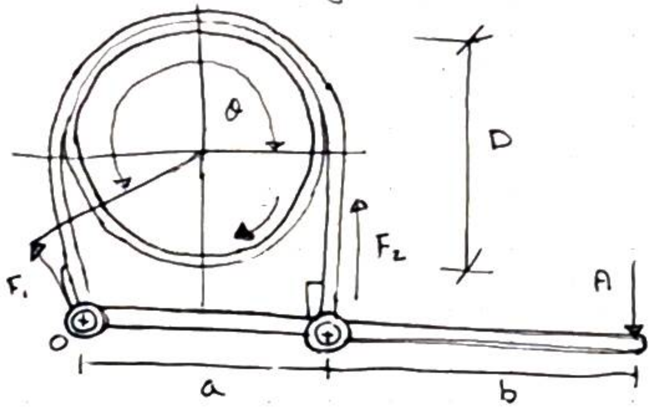
(Double-block brake)

$$\Sigma M_{o1} = S(a+b) + F_1 C - P_1 b = 0$$
$$\Sigma M_{o2} = S(a+b) - F_2 C - P_2 b = 0$$

$$F = f' p$$
$$F_1/P_1 = F_2/P_2 = f'$$
$$T = (F_1 + F_2)\frac{D}{2}$$

4 - Band brakes and Clutches

4.1 - Force Analysis



(Band brake)

For clockwise rotation $F_1 > F_2$

$\Sigma M_o = F_1 0 + F_2 a - A(a+b) = 0$ (1)

it can be shown that ; $F_1/F_2 = e^{f\theta}$ (2)

The torque on the brake wheel is ; $T = (F_1 - F_2)\frac{D}{2}$ (3)

From (1) $A = \dfrac{F_2 a}{a+b}$ (4) From (2) $F_1 = F_2 e^{f\theta}$

Substituting in (3)

$$T = (F_2 e^{f\theta} - F_2)\frac{D}{2} = F_2(e^{f\theta} - 1)\frac{D}{2}$$

and $F_2 = \dfrac{2T}{D(e^{f\theta} - 1)}$

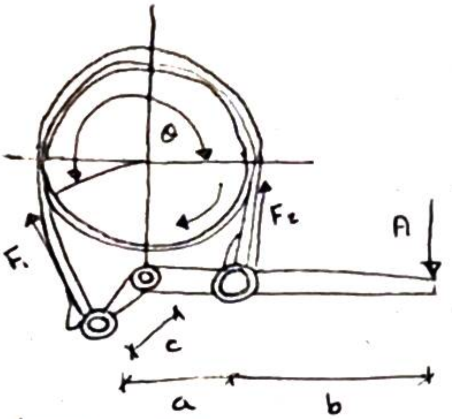
Substituting in (4)

$$A = \frac{2Ta}{D(e^{f\theta} - 1)(a+b)}$$

(if the value is -ve, it is self locking ...)

4.2 Differential band brake

In this type of band brake, the tension in the band assists in applying the brake.



(Differential band brake)

$$\Sigma M_o = 0 = A(a+b) - aF_2 + cF_1$$

$$\therefore \; A = \frac{aF_2 - cF_1}{(a+b)}$$

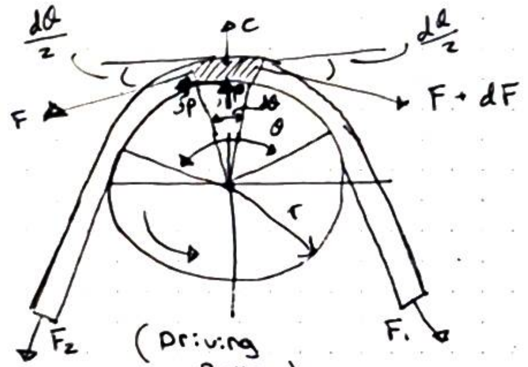
but $\frac{F_1}{F_2} = e^{f\theta}$; $F_2 = \frac{F_1}{e^{f\theta}}$

Substituting the expression of A;

$$A = \frac{aF_1}{e^{f\theta}(a+b)} - \frac{cF_1}{(a+b)} = \frac{F_1}{a+b}\left(\frac{a}{e^{f\theta}} - c\right)$$

If $\frac{a}{e^{f\theta}} \leq c$ the brake will be self-locking for clockwise rotation of the wheel.

4.3 - Maximum normal pressure



(Driving Pulley)

It can be shown that;

$dP = F d\theta$

but $dP = pbr\,d\theta$

$\therefore \; pbr\,d\theta = F d\theta$

and $p = F/br$

The maximum normal pressure P_{max} occurs at $\theta = 0$ where $\theta = 0$ where $F = F_1$

$\therefore \; P_{max} = F_1/br$

Where, P_{max} = maximum pressure between band and wheel
F_1 = max band tension
b = band width
r = wheel radius

5 - Design coefficient

- Braking transforms mechanical energy into heat energy which will raise brake temperature
- Maximum temperature not to exceed ;

Leather, Fibre, and wood Facing - 150 - 160 °F
Asbestos - 200 - 220 °F
Automotive asbestos brake lining - 400 - 500 °F

- Temperature rise in brakes is difficult to predict, therefore, a design coefficient PV is used instead. This is a measure of Foot-pounds of energy absorbed per square inch of surface per minute.

Where, p = pressure, psi
v = rubbing velocity, fpm

- In block brakes $30{,}000 \le PV \le 80{,}000$

Where, $pv = 30.000$ for continuous operation close surrounding.

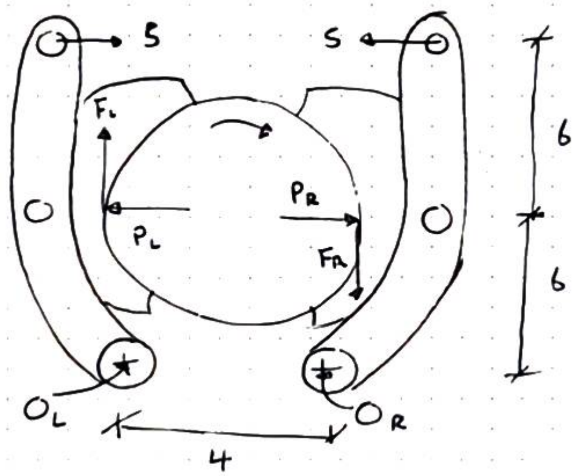
And $PV = 80{,}000$ for intermittent operation in well-ventilated locations.

For good shoe-alignment, the ratio of the shoe width to the wheel diameter must be such that;

$$1/4 \le b/D \le 1/2$$

if $b/D < 1/4 \Rightarrow$ alignment problems
if $b/D > 1/2 \Rightarrow$ non-uniform pressure

Example 1 - (Double-block brake)



- Rated torque = 175 Ft.lb @ 600 rpm
- D = 8 in
- angle of contact of each shoe is = 120°
- $f = 0.3$
- PV = 50.000 ft.lb/min.in² of projected area

a - determine the Force S required to set the brake

b - determine the width of shoe b

a) $2\theta = 120°$; $\theta = 60°$

$$f' = f\,\frac{4\sin\theta}{2\theta + \sin 2\theta} = 0.3 \times 1,17 = 0.351$$

$F/P = f' = 0.351$

$P = F/0.351 = 2.85F$

$$\sum M_{O_L} = 12S + 2F_L - 6P_L = 0$$
$$= 12S + 2F_L - 6 \times 2.85 F_L = 0$$

$F_L = 0.795\,S$

$\Sigma M_{OR} = 12S - 2F_R - 6P_R = 0$

$12S - 2F_R - 6 \times 2.85 F_R = 0$

likely on final / tutorial

$F_R = 0.628\,S$

$F = F_L + F_R = 2\frac{T}{D} = \frac{2 \times 12 \times 175}{8} = 525 \text{ lb}$

$\therefore 0.795S + 0.628S = 525$; thus $S = 369$ lb

b) $(P_L + P_R)/2 = 2.85\,(F_L + F_R)/2 = 2.85 \times 525/2 = 745P$

The projected bearing area per shoe is:

$A = b \times 8\sin 60 = 6.93b$

$P = 745/A = 745/6.93b$

$V = \frac{\pi D\,(\text{rpm})}{12} = \frac{\pi \times 8 \times 600}{12} = 1260$ fpm

$PV = 50.000 = 745 \times 1260/6.93b$

$b = 745 \times 1260 / (50000 \times 6.93) = 2.76$

Use $b = 2\,3/4$ in

$b/D = 2.75/8 = 0.344$

$\therefore 1/4 < 0.344 < 1/2$ Design is O.K.