Feb. 6/18
MACHINE DESIGN

- Review tutorial #3 for exam, ASME code for design
  - ↳ don't calculate wrong I
- three questions (Feb. 15)

*** 6 - Mises-Hencky theory for shafting**

According to this theory, the equivalent static normal stress S for the element considered earlier is:

$$S^2 = \sigma^2 + 3\tau^2$$

$$\therefore S = \sqrt{\left(\sigma_{av} + \frac{K\sigma_{ult}}{\sigma_e}\sigma_r\right)^2 + 3\left(\tau_{av} + \frac{K_t\sigma_{ult}}{\sigma_e}\tau_r\right)^2}$$

distortion energy theory

OR

$$S = \frac{32}{\pi d^3}\sqrt{\left(M_{av} + \frac{K\sigma_{ult}}{\sigma_e}M_r\right)^2 + 0.75\left(T_{av} + \frac{K_t\sigma_{ult}}{\sigma_e}T_r\right)^2}$$

mises-hencky eq'n

7 - Critical speed of rotating shafts

The critical speed of rotating shaft is equal to its natural frequency which can be shown to be

$$f = \frac{1}{2\pi}\sqrt{\frac{g(W_1y_1 + W_2y_2 + \dots)}{W_1y_1^2 + W_2y_2^2 + \dots}} \quad \text{cycles/sec}$$

where; W_1, W_2, etc. represent the weight of the rotating bodies.

y_1, y_2, etc. represent the respective static deflections of the weights

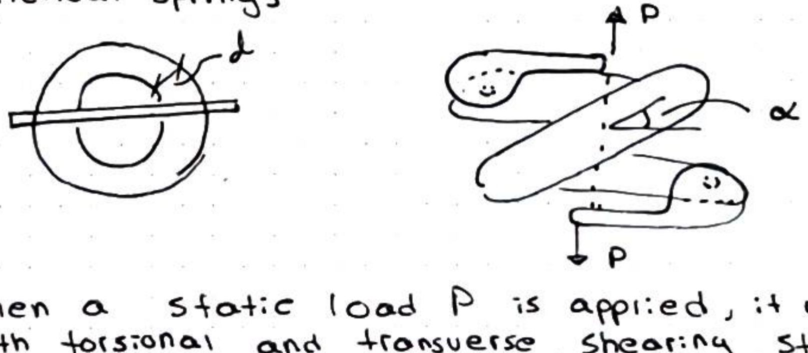
$g = 386\ \text{in/sec}^2$ = gravitational constant

(end of midterm material)

Springs

Symbols are as listed on pp. 261-262 Spotts

1 - Helical Springs



When a static load P is applied, it will introduce both torsional and transverse shearing stresses in the spring wire. The total shearing stress τ on the inside of the coil at the mid-height by static load P is given by:

$$\tau = \frac{16PR}{\pi d^3}\left(1 + \frac{0.615}{c}\right) = K_s\frac{16PR}{\pi d^3}$$

Where $K_s = 1 + \frac{0.615}{C}$; by definition, $C = \frac{2R}{d}$

then $\tau = K_s \frac{8PC}{\pi d^2} = K_s \frac{2PC^3}{\pi R^2}$

and the spring index may be written as

$$C = \frac{\pi d^2}{8P}\tau - 0.615$$

and $$R = \frac{d}{2}\left(\frac{\pi d^2 \tau}{8P} - 0.615\right)$$

1.1 - Deflection of Helical Spring

The work done to compress the spring is

$$W = \int_0^\delta P\,dx = \int_0^\delta Kx\,dx$$

where $P = Kx$

K = spring rate

x = displacement

and $W = \frac{Kx^2}{2}\Big|_0^\delta = K\frac{\delta^2}{2}$

but $K\delta = P$

and $W = \frac{P}{2}\delta$

The strain energy in the spring is

$W = T\theta/2$

Where the total angle of twist is

$\theta = TL/GJ$

and $L = \pi D N_c/\cos\alpha \simeq \pi D N_c$ — number of coils / $\alpha \not> 5°$, $\cos\alpha \simeq 1$

$$\therefore \quad W = \frac{1}{2}\Big(\underbrace{\frac{PD}{2}}_{T/2}\cdot\underbrace{\frac{PD}{2}}_{T}\underbrace{\pi D N_c}_{L}\Big)\Big/\big(\underbrace{G\pi d^4/32}_{J}\big)$$

$$= 8P^2D^3N_c/2Gd^4$$

but $W = P\delta/2$

$$\therefore \quad \frac{P\delta}{2} = 8P^2D^3N_c/2Gd^4$$

or

$$\delta = 8PD^3N_c/Gd^4 = 8PC^3N_c/Gd$$

and $K = \frac{P}{\delta} = \frac{Gd}{8C^3N_c}$ where $D = 2R$

$G = 11{,}500{,}000$ psi

$= 78{,}300$ MPa

Example 1 - A steel helical compression spring is made from wire 4mm in diameter and is to carry a load of 450 N at a deflection of 25mm. Maximum shearing stress to be 550 MPa and number of inactive end coils Q equals 2. Find required values for mean radius R, number of active coils N_c, and volume of spring material.

Solution : $R = \frac{d}{2}\left(\frac{\pi d^2 \tau}{8P} - 0.615\right) = \frac{4}{2}\left(\frac{\pi 4^2 \times 550}{8 \times 450} - 0.615\right)$

$R = 14.14\ mm$

$C = \frac{2R}{d} = \frac{2 \times 14.14}{4} = 7.07$

$N_c = \frac{\delta R G}{4PC^4} = \frac{25 \times 14.14 \times 79300}{4 \times 450 \times 7.07^4} = 6.23$ active coils

$V = \frac{1}{2}\pi^2 d^2 R (N_c + Q)$

$= \frac{1}{2}\pi^2 4^2 \times 14.14\ (6.23 + 2) = 9,188\ mm^3$

1.2 - Helical spring of minimum volume of material (static load)

It can be shown that for minimum volume of spring material, the following condition must be satisfied (for static loading only)

$$B = \frac{\delta G}{Q\sqrt{8P\pi\tau}} = \frac{C^3(5C + 1.23)}{2.46\sqrt{C + 0.615}}$$

Table 4.8 provides values for C for a range of numerical values of B from 94 to 1,830.

→ Table 10-2

1.3 - Design of Helical Spring for Fluctuating loads

The basic equation for the design of springs with continuously fluctuating loading is :

$$\frac{K_c \tau_r}{\frac{\tau_{yp}}{f_s} - \tau_{av}} = \frac{\frac{1}{2}\tau_e'}{\tau_{yp} - \frac{1}{2}\tau_e'}$$

where K_c = Wall Factor for curvature

$K_c = \frac{4C-1}{4C-4}$

τ_{av} = average stress

τ_r = range stress

$\left.\begin{array}{l}\tau_{yp} \\ \tau_e'\end{array}\right\}$ table 4-6 (spotts)

σ_{ult} } table 4-2 to 4-5 (spotts)

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Example 2 - A helical compression spring, made of No.4 music wire, carries a fluctuating load. The spring index is 6, and the factor of safety is 1.5. Find the permissible values for the maximum and minimum loads.

Solution: $R = \dfrac{cd}{2} = \dfrac{6 \times 0.2253}{2} = 0.676$

$$K_c = \frac{4 \times 6 - 1}{4 \times 6 - 4} = 1.15$$

$$\tau = \frac{16PR}{\pi d^3}\left(1 + \frac{0.615}{c}\right)$$

$$\tau_{av} = \frac{16 \times 120 \times 0.676}{\pi \times 0.2253^3}\left(1 + \frac{0.615}{6}\right) = 39,820 \text{ psi}$$

From Table 4-3

$\sigma_{ult} = 235,000$ psi

From Table 4-6

$\tau_{yp} = 0.4$; $\sigma_{ult} = 94,000$ psi

$\tau_e' = 0.23$; $\sigma_{ult} = 54,000$ psi

Then:
$$\frac{K_c \tau_r}{\frac{\tau_{yp}}{f_s} - \tau_{av}} = \frac{(1/2)\tau_e'}{\tau_{yp} - (1/2)\tau_e'}$$

$$\frac{1.15\,\tau_r}{\frac{94000}{1.6} - 39,820} = \frac{27025}{94000 - 27025}$$

$$\tau_r = 8020 \text{ psi}$$

$$P_r = \frac{\tau_r}{\tau_{av}} P_{av} = \frac{8020}{39820}(120) = 24.2 \text{ lb}$$

$P_2 = 120 + 24.2 = 144.2$ lb

$P_1 = 120 - 24.2 = 95.8$ lb

1-4 - Surging of helical springs

A sudden compression of the end of a helical spring will form a compression wave of frequency f such as:

$$f = \frac{d}{2\pi R^2 N_c}\sqrt{\frac{Gg}{32\gamma}} \quad \text{cycles/sec}$$

The spring may exhibit higher modes such that

$$f_n = Nf \qquad N = 2, 3, 4, \ldots$$

$g = 386$ in/sec^2

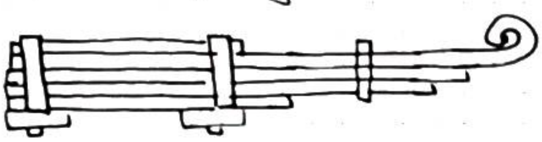
and for steel spring ; $G = 11{,}500{,}000$ psi

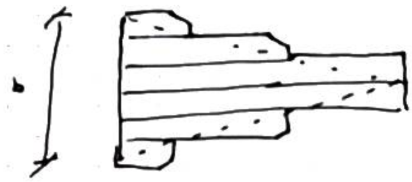
$\gamma = 0.285$ lb/in^3

$$\therefore \; f = 3510\, d / R^2 N_c \quad \text{cycles/sec}$$

Surge will be introduced if the operating spring frequency coincide with one of its natural frequencies given by $f_n = Nf$. $N = 1, 2, 3, 4, \ldots$

2 - Leaf Spring

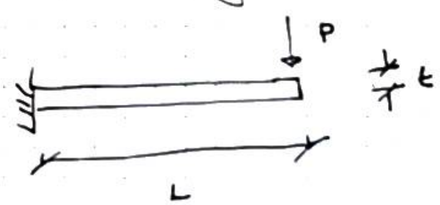




For this type of springs the maximum bending stress is : $\sigma = 6PL/bt^2$

and the maximum deflection is

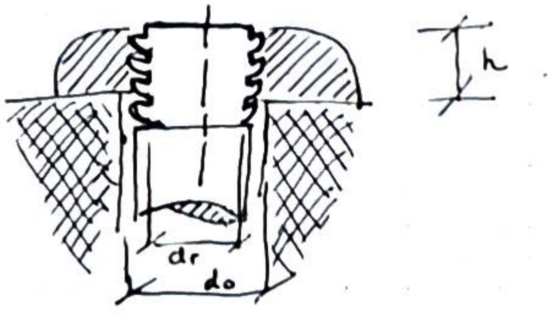
$$\delta = 6PL^3/Ebt^3$$



Detachable Fastenings

Screws

Unless otherwise stated, specifications of screw fastenings refer to the major (outside) diameter, i.e. a ½ in bolt has $d_o = ½$ in



1 - Height of nut

1.1 - strength of the bolt in Tension

$$F_t = \frac{\pi d_r^2}{4} \sigma_t$$

1.2 - Strength of the threads in shear

$$F_s = \pi d_r h S_s$$

1.3 - Determination of h for equal strength

Let $F_s = F_t$

$$\pi d_r h S_s = \frac{\pi d_r^2}{4} \sigma_t$$

based on maximum shear stress theory ;

$$S_s = \sigma_t / 2$$

$$\therefore \; d_r/2 = h$$

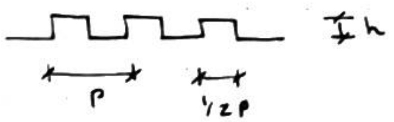
For standard coarse threads $d_r = 0.8 d_o$

$$\therefore \; h = \frac{0.8}{2} d_o = 0.4 \, d_o$$

American standard nuts are 7/8 d_o in height so that the thread would not shear.

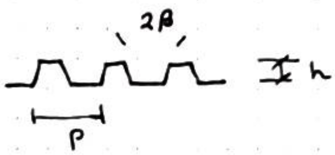
Translation Screws

1 - Form of threads



$h = 7/16 \, p$

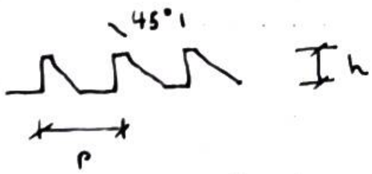
(square thread)



$h = 0.5p + 0.01$

$2\beta = 29°$

(acme thread)



$h = 0.75 \, p$

(buttress thread)

(all forms of transition thread screws)

2 - Multiple threads

Translation screws with multiple threads, such as double, triple, etc, are used when it is desired to secure a large load with Fine threads or high-efficiency.

3 - Efficiency of Screws

Let Q = axial load, lb

d = diameter of mean helix, in.

α = lead angle

ϕ = Friction angle

2β = included angle thread

f = coefficient of thread Friction = $\tan\phi$

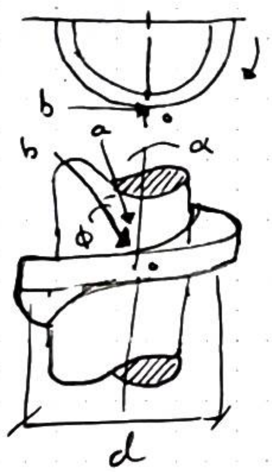
L = lead of threads, in

T = torque required to overcome thread Friction and to move load, lb-in

T_o = torque required to move load, neglecting Friction, lb-in.

e = efficiency of screw

3.1 - Square threads



(Forces on square-threaded screws)

When the screw rotates so that the nut is moved against its external load Q, the line of action of aO will be rotated through the angle of Friction ϕ to bO, as shown above.

For equilibrium of Forces,

$$\sum F_y = 0 \quad \therefore \quad Q = bO\cos(\alpha + \phi) \quad (1)$$

$$\sum F_x = 0 \quad \therefore \quad F = bO\sin(\alpha + \phi) \quad (2)$$

dividing (2) by (1)

$$\therefore \; F = Q\tan(\alpha + \phi)$$

$$T = F\,\frac{d}{2} = Q\,d/2\,\tan(\alpha + \phi)$$

but, $\tan(\alpha + \phi) = \dfrac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}$

$\tan\phi = f \rightarrow$ (coeff. of friction)

and $\tan\alpha = L/\pi d$

$$\therefore \tan(\alpha + \phi) = \frac{(L/\pi d) + f}{1 - fL/\pi d} = \frac{L + \pi d f}{\pi d - fL}$$

and $T = Q\,\dfrac{d}{2}\,\dfrac{L + \pi d f}{\pi d - fL}$

if $f = 0$ then: $T_o = QL/2\pi$

and, $e = \dfrac{T_o}{T} = \dfrac{QL}{2\pi T}$

(From earlier

$$N_c = \frac{G\delta R}{4Pc^4}$$)