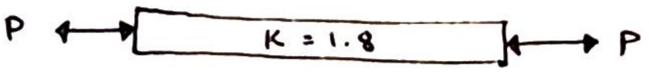
JAN. 30/18
MACHINE DESIGN

Example 2 - Determine the diameter of a circular rod. $\sigma_e = 38{,}000$ psi

$\sigma_{yp} = 50000$ psi, subjected to varying axial load.



$P_{min} = -60000$ lb
$P_{max} = 140000$ lb
$K = 1.8 \quad f_s = 2$

Solution:

$P_m = (P_{max} + P_{min})/2 = [140000 + (-60000)]/2$

$P_m = 40000$ lb

$P_r = (P_{max} - P_m) = (P_{max} - P_{min})/2 = 100000$ ft

$$\sigma_m = \frac{P_m}{A} = 40000/A$$

$$\sigma_r = \frac{P_r}{A} = 100000/A$$

$$K\sigma_r = \frac{\sigma_e}{f_s} - \frac{\sigma_e}{\sigma_{yp}}\sigma_m$$

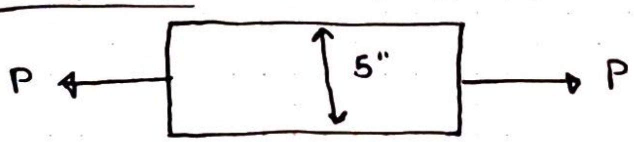
$$1.8 \times \frac{100000}{A} = \frac{38000}{2} - \frac{38000}{50000} \times \frac{40000}{A}$$

$A = 11.1\ in^2$

$A = \pi d^2/4$

$d = \sqrt{4A/\pi} = (4 \times 11.1/\pi)^{1/2} = 3.76$ in

Example 3 - Determine the thickness of the plate.



$20000\ lb \le P \le 50000\ lb$

$\sigma_{yp} = 60000$
$\sigma_e = 45000$
$f_s = 1.6$

Solution:

$A = 5t$

$$\sigma_m = \frac{P_{max} + P_{min}}{2A} = \frac{50000 + 20{,}000}{2 \times 5t} = \frac{70{,}000}{10t}$$

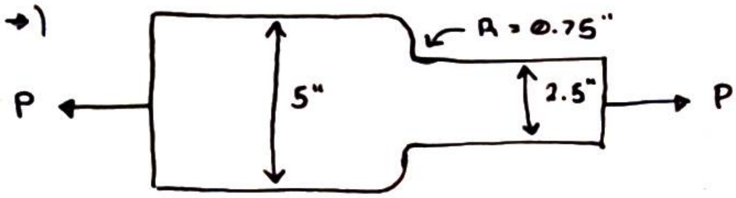
$$\sigma_r = \frac{P_{max} - P_{min}}{2A} = \frac{50{,}000 - 20000}{2 \times 5t} = \frac{30{,}000}{10t}$$

$$\sigma_r = \frac{\sigma_e}{f_s} - \frac{\sigma_e}{\sigma_{yp}}\sigma_m$$

$$\frac{3000}{t} = \frac{45000}{1.5} - \frac{45{,}000}{60{,}000} \times \frac{7{,}000}{t}$$

$$t = 0.275 \text{ in}$$

(b →)



20,000 lb ≤ P ≤ 50,000 lb

$\sigma_{yp} = 60{,}000$

$\sigma_e = 45{,}000$

$f_s = 1.5$

Solution:

$A = 2.5\,t$

(2.16 in book)

$r/d = 0.75 / 2.5 = 0.3$ }
$D/d = 5/2.5 = 2$ } From Fig. X3 4.0. $K = K_t = 1.63$

$$\sigma_m = \frac{50000 + 20000}{2 \times 2.5\,t} = \frac{14000}{t}$$

$$\sigma_r = \frac{50000 - 20000}{2 \times 2.5\,t} = \frac{6000}{t}$$

$$K\sigma_r = \frac{\sigma_e}{f_s} - \frac{\sigma_e}{\sigma_{yp}}\sigma_m$$

$$1.63 \times \frac{6000}{t} = \frac{45000}{1.5} - \frac{45000}{60000} \times \frac{14000}{t}$$

$$t = 0.676 \text{ in}$$

4.4 Loading in the Finite life range

For problems in the finite life range, stresses σ_{av} and $K\sigma_r$ are transformed into an equivalent completely reversing stress σ_R as follows

$$\sigma_R = \frac{K\sigma_r\sigma_{ult}}{\sigma_{ult} - \sigma_{av}}$$

Then σ_R is used in the solution of problems as follows

$A = \sigma_R L^B$ (Basquin's eq'n)

or

$\log A = \log \sigma_R + B \log L$ (equation of straight line)

Experiments show that fatigue curves pass through the following two points:

1 - $0.9\,\sigma_{ult}$; $L = 1000$ reversals

2 - σ_e ; $L = 10^6$ reversals

if σ_{ult} and σ_e are known then A and B in the above equation can be found, and therefore L can be found for any σ_R.

Example 4 - A ground specimen from a steel of 90000 psi tensile strength and 40000 endurance limit is subjected to cyclic stresses, such as:

$\sigma_{av} = 25{,}000$ psi

$\sigma_r = 21.667$ psi

Find the expected number of reversals to failure, if a stress concentration factor of 1.5 applies.

Solution:

$$\sigma_R = \frac{1.5 \times 21.667 \times 90000}{90000 - 25000} = 45000 \text{ psi}$$

$$\log A = \log \sigma_R + B \log L$$

1 - $0.9\,\sigma_{ult} = 0.9 \times 90000 = 81000$ psi

$L = 1000$ cycles

2 - 40.000 psi and $L = 10^6$ cycles

$\log A = \log 81000 + B \log 1000$ (1)

$\log A = \log 40000 + B \log 1{,}000{,}000$ (2)

$\log A = 4.90849 + 3B$

$\log A = 4.60206 + 6B$

Solving for ($\log A$) and (B) to find

$\log A = 5.21492$; $B = 0.10214$

$\therefore\ 5.21492 = \log \sigma_R + 0.10214 \log L$

For $\sigma_R = 45000$ psi

$5.21492 = \log 45000 + 0.10214 \log L$

$\log L = 5.49941$

uR $L = 315{,}800$ cycles

N.B. the results agree with Fig 2.30 (Spotts) the curve thus gives a rough check but logarithms must be used to obtain sufficient accuracy.

4.5 - Miner's Equation (For changing cyclic loading)

A machine part may have one stress for a portion of its life, another stress for another portion, and so on. If we let σ_{Ri} be the applied stress, L_i is the life corresponding to σ_{Ri} applied alone and N_i the applied reversals at this load, then :

$$\sum \frac{N_i}{L_i} = 1 \quad ; \quad \frac{N_1}{L_1} + \frac{N_2}{L_2} + \frac{N_3}{L_3} + \dots = 1$$

If the total number of reversals to failure for the combined loading is N_e, then

$$N_1 = \alpha_1 N_e \; ; \; N_2 = \alpha_2 N_e \; ; \; ---$$

and

$$\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} + \frac{\alpha_3}{L_3} + --- = \frac{1}{N_e}$$

$$\alpha_1 + \alpha_2 + \alpha_3 + --- = 1$$

Example 5 - the ultimate strength of the material is $\sigma_{ult} = 90000$ psi and $\sigma_e = 40000$ psi. For a ground surface. Suppose the loading is as follows, where $K = 1.5$:

$$\left.\begin{aligned} \sigma_{ar} &= 45{,}000 \text{ psi} \\ \sigma_r &= 13641 \text{ psi} \end{aligned}\right\} \text{ for } 80\% \text{ of the time}$$

$$\left.\begin{aligned} \sigma_{ar} &= 35{,}000 \text{ psi} \\ \sigma_r &= 20{,}787 \text{ psi} \end{aligned}\right\} \text{ for } 20\% \text{ of the time}$$

Find the expected number of cycles (N_e) to failure

Solution :

$$\sigma_{R1} = \frac{K\sigma_r \sigma_{ult}}{\sigma_{ult} - \sigma_{av}} = \frac{1.5 \times 13641 \times 90000}{90000 - 45000} = 40923 \text{ psi}$$

$$\sigma_{R2} = \frac{1.5 \times 20787 \times 90000}{90000 - 35000} = 51{,}023 \text{ psi}$$

From example 4, (previous)

$$B = 0.10214 \quad ; \quad \log A = 5.21492$$

For $\sigma_{R1} = 40{,}923$ psi:

$$0.10214 \log L_1 = 5.21492 - 4.61197 = 0.60295$$

$$L_1 = 800{,}150 \text{ cycles}$$

For $\sigma_{R2} = 51{,}023$ psi:

$$0.10214 \log L_2 = 5.21492 - 4.7077 = 0.50715$$

$$L_2 = 92{,}310 \text{ cycles}$$

then $\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} = \frac{1}{N_c}$ / where $\alpha_1 = 0.8$
$\alpha_2 = 0.2$

$\rightarrow \frac{0.8}{800160} + \frac{0.2}{92310} = \frac{1}{N_c}$

$N_c = 315,800$ reversals

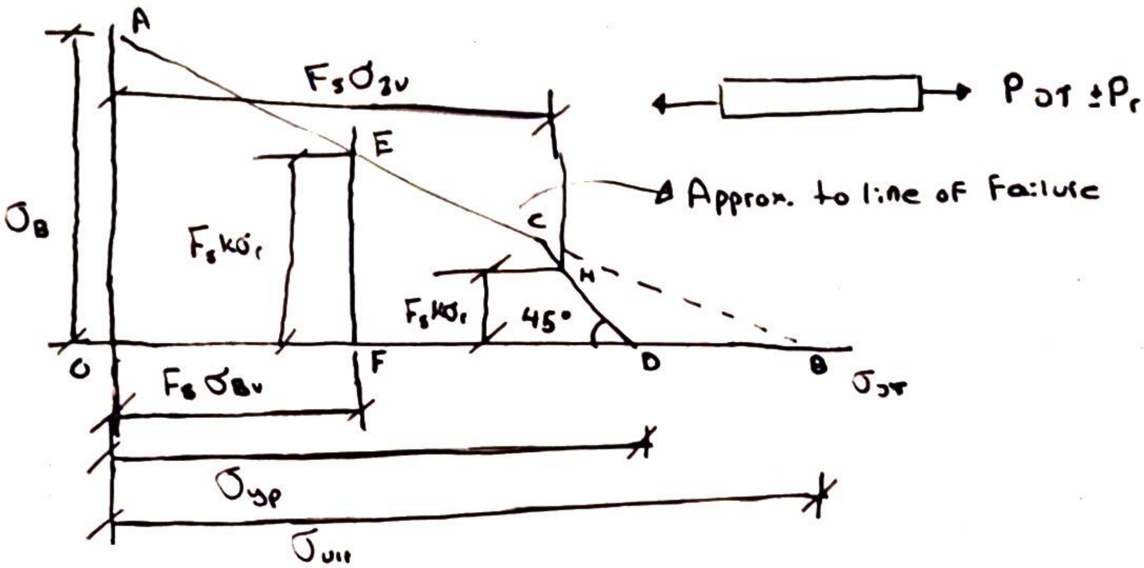
4.6 - Fatigue life determined by short-time testing

The testing time can be reduced if a portion of the test is done at the service stress, and the remainder is done at some exclusively high stress which produces failure in a shorter time. Then:

$$\frac{N_1}{L_1} + \frac{N_2}{L_2} = 1$$

4.7 - Modified Goodman Diagram

Feb. 1/18
MACHINE DESIGN



By similar triangles:

$$\frac{FB}{EF} = \frac{OB}{OA} \quad \text{or} \quad \frac{OB}{OA} \times EF = FB$$

$$FB = \frac{\sigma_{ult}}{\sigma_e} f_s K\sigma_r$$

The equivalent static stress
$\sigma = \sigma_{ult}/f_s$ is valid for segment AC
Then;

$$OB = OF + FB$$

$$\sigma_{ult} = f_s\sigma_{av} + f_s K \frac{\sigma_{ult}}{\sigma_e}\sigma_r$$

or

$$\sigma = \frac{\sigma_{ult}}{f_s} = \sigma_{av} + K\frac{\sigma_{ult}}{\sigma_e}\sigma_r$$

and

$$\boxed{\frac{\sigma_{av}}{\sigma_{ult}} + \frac{K\sigma_r}{\sigma_e} = \frac{1}{f_s}} \quad \underline{\underline{AB}}$$

if the working stress is on segment CD, then
$\sigma = \sigma_{yp}/f_s$ And;

$$\therefore \sigma = \frac{\sigma_{yp}}{f_s} = \sigma_{av} + K\sigma_r$$

or

$$\boxed{\sigma_{yp} = f_s(\sigma_{av} + K\sigma_r)} \quad \underline{\underline{CB}}$$

Example 6 :

A Part with a machined surface has continuously varying tension loads. P_{max} = 45000, P_{min} = 15000 lb. Material tests σ_{ult} = 90000 psi, and σ_{yp} = 70000 psi. A stress concentration factor of 1.42 is present. Area of the part is 2.5 in²

a - Find the factor of safety
b - if P_{max} = 55,400 lb and P_{min} = 44600 lb, Find the Factor of safety

Solution :

a - $$P_{av} = \frac{45000 + 15000}{2} = 30000 \text{ lb}$$

$$\sigma_{av} = \frac{30000}{2.5} = 12000 \text{ psi}$$

(From Fig. 2-26 σ_e = 34000 psi)

$$P_r = \frac{45000 - 15000}{2} = 15000 \text{ lb}$$

$$\sigma_r = \frac{15000}{2.5} = 6000 \text{ psi}$$

Assuming that the working stress point E lies on segment AC of the modified Goodman Curve,

then : $$\frac{\sigma_{av}}{\sigma_{ult}} + K\frac{\sigma_r}{\sigma_e} = \frac{1}{f_s}$$

$$\frac{12000}{90000} + \frac{1.42 \times 6000}{34000} = \frac{1}{f_s} \; ; \; f_s = 2.60$$

A rough sketch indicates assumption O.K.

b - $$P_{av} = \frac{55400 + 44600}{2} = 50000 \text{ lb}$$

$$\sigma_{av} = \frac{50000}{2.5} = 20000 \text{ psi}$$

$$P_r = \frac{55400 - 44600}{2} = 5400 \text{ lb}$$

$$\sigma_r = \frac{5400}{2.5} = 2160 \text{ psi}$$

Assuming that the working stress point E lies on segment CD of the modified Goodman curve,

Then ; $$f_s = \sigma_{yp} / (\sigma_{av} + K\sigma_p)$$

$$f_s = 70000 / (20000 + (1.42)(2.160))$$

$$f_s = 3.03$$

Recheck to find assumption O.K.

Shafting

1- Torsion of Circular Shaft

$$\tau = \frac{Tr}{J}$$

2 - Power Transmitted

Power = Force × Velocity

$1\,hp = 33000\ ft \cdot lb/min$

$hp = F \cdot V / 33000$

where F is in lb; V is in Feet/min

$T = 63030\ hp / RPM$

where T is in Ft.lb

in SI units

$W = NV$ watts

where force N is in newtons and velocity V is in meter/sec

3 - Maximum Static Shearing Stress (Torsion combined with bending)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

For solid circular shaft

$$\sigma_{bending} = \frac{Mc}{I} = \frac{32M}{\pi d^3}$$

$$\tau = Tr/J = \frac{16T}{\pi d^3}$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

4 - ASME Code For Design of Transmission Shafting

To make proper allowance for the harmful effects of the Fluctuation, the ASME code inserts the constants C_m and C_t from Table 3-1 (H.O.) in the above eq'ns

$$\tau_{max} = \sqrt{\left(\frac{C_m\sigma}{2}\right)^2 + (C_t\tau)^2}$$

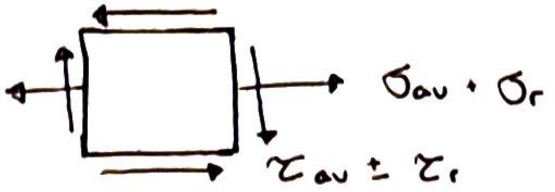
or $$\tau_{max} = \frac{16}{\pi d^3}\sqrt{(C_m M)^2 + (C_t T)^2}$$

where, C_m = Shock and Fatigue Factor to be applied to the computed bending moment.

C_t = to corresponding factor to be applied to the computed torque.

5 - Fluctuating Loads (Max Shear Stress)

If the element is subjected to Fluctuating stresses as shown;



The equivalent static normal stress is

$$\sigma = \sigma_{av} + \frac{K\,\sigma_{ult}}{\sigma_e}\,\sigma_r$$

Where K_t is the stress concentration Factor in torsion
then;

$$\tau_{max} = \sqrt{\frac{1}{4}\left(\sigma_{av} + \frac{K\sigma_{ult}}{\sigma_e}\right)^2 + \left(\tau_{av} + \frac{K_t\sigma_{ult}}{\sigma_e}\right)^2}$$

<u>OR</u>

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{\left(M_{av} + \frac{K\sigma_{ult}}{\sigma_e}M_r\right)^2 + \left(T_{av} + \frac{K_t\sigma_{ult}}{\sigma_e}T_r\right)^2} = \frac{0.5\,\sigma_{yp}}{f_s}$$

Midterm - February 15th