K = the highest value of actual stress on fillet, notch, hole, etc.

nominal stress, given by elementary equation for minimum cross-section.

Values of stress-concentration factors can be found experimentally. For a number of simple cases, solutions have been obtained by mathematical analysis.

Some of these values can be found in Fig. 2.8, through Fig 2.21 in Spotts book.

1.4 - Seriousness of Stress Concentration

Material	Static Load	Cyclic Load
Brittle	Serious	very serious
Ductile	Not serious	Serious

2 - Design Criteria

2-1 → modes of Failure

I - yielding

- maximum induced stress exceeding the yield strength of the material, causing it to deform plastically.
- creep deformation, whereby the member deforms under a constant load, usually at an elevated temperature

II - Fracture

- Due to static loads
- Due to Fatigue loads
- Due to impact loads

III - Excessive elastic deflection

IV - Wear

V - Buckling

VI - Corrosion Fatigue

2.2 - Factors affecting the mode of Failure

- Type and duration of the load
- Shape of the part (stress concentration)
- Nature of the material (ductile or brittle)

Normally ductile materials will act like brittle materials under any of the following conditions:

- repeated or fatigue loading
- impact, particularly at low temperatures
- creep
- triaxial state of stress
- severe quenching without tempering
- strain hardening accomonying yielding

3 - Theories of Failure by yielding and by Fracture under State of Stress

3.1 - Maximum-normal stress theory
(generally applies to failure of brittle mat's.)

According to this theory, failure (yielding or fracture) occurs at a point in a body when one of the principal stresses at that point equals a critical stress for the material (yield stress or ultimate strength)

$$\text{if } |\sigma_1| > |\sigma_2| > |\sigma_3|$$

Failure for combined stresses occurs when

$$\sigma_1 = \pm \sigma_{crit}$$

The allowable stress would be (doesn't matter if it is comp. or tensile)

$$\sigma_{1(all)} = \pm \frac{\sigma_{crit}}{f_s}$$

This theory generally applies to the failure of brittle materials. However, the maximum-normal-stress theory for yielding does not agree with test results.

3.2 - Max. shear stress Theory
(widely used for predicting failure of ductile material by yielding)

This theory assumes that failure (yielding or fracture) occurs for a combined stress condition when the maximum shear stress equals the value of a critical shear stress (yield shear stress or ultimate shear stress) produced by an element subjected to simple tension, which would be:

$$(\sigma_s)_{yp} = \frac{\sigma_{yp}}{2}$$

For 3.D. the maximum shear stresses are given by one of the following

$$\frac{\sigma_1 - \sigma_2}{2} \quad ; \quad \frac{\sigma_2 - \sigma_3}{2} \quad ; \quad \frac{\sigma_3 - \sigma_1}{2}$$

OR

$$\frac{\sigma_{yp}}{2} = \begin{cases} (\sigma_1 - \sigma_2)/2 \\ (\sigma_2 - \sigma_3)/2 \\ (\sigma_3 - \sigma_1)/2 \end{cases} \quad \text{OR} \quad \sigma_{yp} = \begin{cases} \sigma_1 - \sigma_2 \\ \sigma_2 - \sigma_3 \\ \sigma_3 - \sigma_1 \end{cases}$$

For 2.D. $\sigma_3 = 0$ then,

- if σ_1 and σ_2 are of opposite sign
  $\sigma_1 - \sigma_2 = \pm \sigma_{yp}$ or $f_s = \sigma_{yp} / \sigma_1 - \sigma_2$
- if σ_1 and σ_2 are of the same sign
  $\sigma_1 = \pm \sigma_{yp}$ if $|\sigma_1| > |\sigma_2|$
  or $f_s = \sigma_{yp} / \sigma_1$ (the larger stress)

  $\sigma_2 = \pm \sigma_{yp}$ if $|\sigma_1| < |\sigma_2|$
  or $f_s = \sigma_{yp} / \sigma_2$

This theory is in good agreement with experimental results and is on the safe side.

The above equations can be used to predict failure by fracture σ_{yp} is replaced by σ_{ult}. However, most brittle materials lost higher ultimate strength in comp. than in tension. Let: σ_{uc} = ultimate strength in comp.
σ_{ut} = ultimate strength in tension

Then for simple compression

$$\sigma_c = \sigma_{uc} \quad \text{or} \quad f_s = \frac{\sigma_{uc}}{\sigma_c}$$

Then for simple Tension

$$\sigma_t = \sigma_{ut} \quad \text{or} \quad f_s = \frac{\sigma_{ut}}{\sigma_t}$$

- 2D stresses

if σ_1 and σ_2 have opposite signs and $\sigma_1 > 0$

$$\frac{\sigma_1}{\sigma_{ut}} + \frac{\sigma_2}{\sigma_{uc}} = \frac{1}{f_s}$$

If σ_1 and σ_2 have the same sign, failure is assumed to be due only to the principal stress of larger magnitude.

$$f_s = \frac{\sigma_{ut}}{\sigma_1} \qquad |\sigma_1| > |\sigma_2| \text{ and } \sigma_1 > 0$$

$$f_s = \frac{\sigma_{uk}}{\sigma_1} \qquad |\sigma_1| > |\sigma_2| \text{ and } \sigma_1 < 0$$

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3.3 - Mises-hencky or Distortion - Energy Theorem (concerned mainly with predicting yielding)

Yielding will occur when the strain energy of distortion per unit volume equals the strain energy of distortion per unit volume for a specimen in uniaxial tension or compression (strained to the yield stress). This energy is found to be :

→ For a body under 3D stresses :

$$U_s = \frac{1+\mu}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

→ For a specimen :

$$U_s = \frac{1+\mu}{6E}\left[2\sigma_{yp}^2\right]$$

$$\therefore \left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] = \left[2\sigma_{yp}^2\right]$$

→ For 2D stresses, $\sigma_3 = 0$, so

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{yp}^2$$

→ using a factor of safety (f_s)

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_{yp}}{f_s}\right)^2$$

In the case of pure shear, $\sigma_1 = -\sigma_2 = \tau$, then

$$3\sigma_1^2 = \sigma_{yp}^2$$

and

$$\tau = \sigma_1 = 0.577\,\sigma_{yp} \quad \text{or} \quad (\sigma_s)_{yp} = 0.577\,\sigma_{yp}$$

From maximum shear stress theory $(\sigma_s)_{yp} = 0.56\,\sigma_{yp}$

experiment shows distortion energy theory closer to experimental results.

See " graphic for the three failure theories "

Example The stresses at a point in a body are:

$\sigma_x = 13000$ psi

$\sigma_y = 3000$ psi

$\tau_{xy} = 12000$ psi

The material tests $\sigma_{yp} = 40{,}000$ psi

a - Find the Factor of safety by the maximum shear stress theory

b - Find the Factor of safety by the Mises - Hencky theory.

Solution: a → $\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$\sigma_{1,2} = \frac{13000 + 3000}{2} \pm \sqrt{\left(\frac{13000 - 3000}{2}\right)^2 + (12000)^2}$$

$\sigma_1 = 21000$; $\sigma_2 = -5000$ psi

a - maximum shear stress theory

σ_1 and σ_2 are of opposite sign

$$\therefore \sigma_1 - \sigma_2 = \pm \sigma_{yp} / f_s$$

$$21000 - (-5000) = 40.000 / f_s$$

$$f_s = 40.000 / 26.000 \simeq \boxed{1.54}$$

b → $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = (\sigma_{yp} / f_s)^2$

$$(21000)^2 - (21000)(-5000) + (-5000)^2 = \left(\frac{40000}{f_s}\right)^2$$

$$f_s = 40{,}000 / 23{,}900 \simeq \boxed{1.67}$$

4 - Fatigue Stress

4.1 - Cyclic or Fatigue stress

- see graph for "Fatigue - stress variations"

$\sigma_{av} = \sigma_m = S_m = (\sigma_{max} + \sigma_{min})/2$ = mean stress

$\sigma_r = S_r = (\sigma_{max} - \sigma_{min})/2$ = alternating stress

Range of stress $= R = 2\sigma_r$

$\sigma_{max} = S_{max}$ = maximum stress

$\sigma_{min} = S_{min}$ = minimum stress

4.2 - Key Factors in Fatigue Failures

1 - a maximum stress of sufficient magnitude

2 - an applied stress fluctuation of large enough magnitude

3 - a sufficient number of cycles of the applied stress

4.3 - Fatigue Design procedures

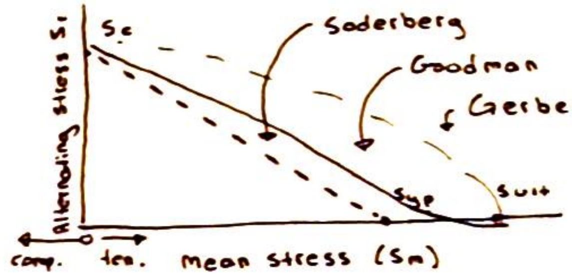
One of the most common methods of presenting engineering fatigue data is by the means of the "S-N curve"

→ see "S-N curve"

In this particular graph, if SAE No. or Ultimate stress is known, and N is known,

then the fatigue or endurance limit of the material can be found. However, the above data is for zero mean stress $\sigma_m = S_m = 0$.

To solve for cases where $\sigma_m \neq 0$, first $\sigma_r - N$ curves are plotted as shown, then for a given $N = N_1$, the $\sigma_r - \sigma_m$ is plotted.



From the $\sigma_r - \sigma_m$ curve, the following empirical solution was found;

$$\sigma_r = \sigma_e \left[1 - \left(\frac{\sigma_m}{\sigma_{ult}} \right)^p \right]$$

Gerber curve : $p = 2$

Goodman line : $p = 1$

When design is based on yield strength, then the Soderberg Law is followed;

$$\sigma_r = \sigma_e \left(1 - \frac{\sigma_m}{\sigma_{yp}} \right)$$

When a factor of safety is required

$$\sigma_r = \frac{\sigma_e}{f_s} - \frac{\sigma_e}{\sigma_{yp}} \sigma_m$$

where, σ_e = endurance strength for the case $\sigma_m = 0$

σ_{yp} = yield strength under static load

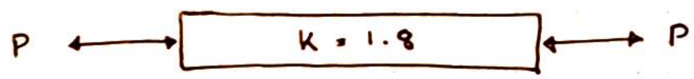
σ_{ut} = ultimate strength under static load

$$K\sigma_r = \frac{\sigma_e}{f_s} - \frac{\sigma_e}{\sigma_{yp}}\sigma_m$$

N.B. endurance limit in shear may be taken as 0.55 times the endurance limit

$$\sigma_{es} = 0.55\,\sigma_e$$

Example - Determine the diameter of a circular rod, σ_e = 38,000 psi ; σ_{yp} = 50,000 psi , subjected to varying axial load.



P_{min} = - 60,000 lb

P_{max} = 140,000 lb

K = 1.8

f_s = 2