JAN. 16/18

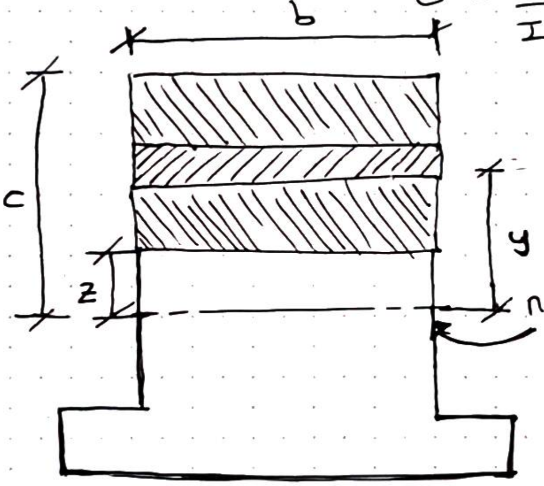
MACHINE DESIGN

5.3 - Transverse Shearing Stress in Beams

In addition to normal stresses induced by bending of a beam, transverse shearing stresses are induced between the elements of Fibres, Provided the bending moment varies along the length of the beam. According to the strength-of-materials method,

$$\tau = \frac{V}{Ib}\int_z^c y\,dA = \frac{VQ}{Ib}$$

first moment of area



$\therefore$ $\tau = 0$ if $z = c$

$\tau = \tau_{max}$ if $z = 0$

Where ;

τ = Shear stress

V = Shearing Force at Section under consideration

b = beam width at section

$Q = \int_z^c y\,dA$ = moment of inertia about N.A.

z = location where shear is desired

For rectangular cross-section:

$$\tau_{max} = \frac{3V}{2A}$$

For solid circular cross-section:

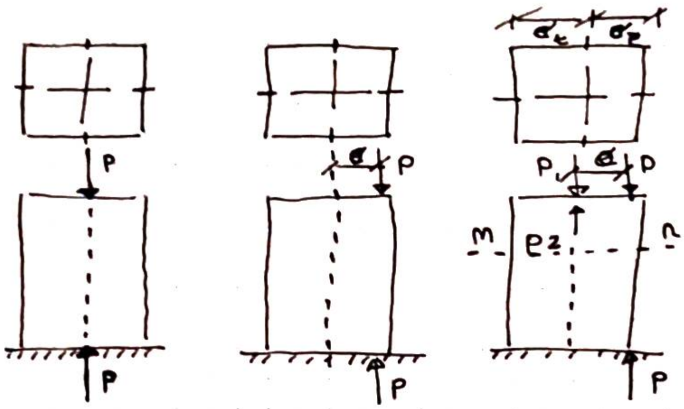
$$\tau_{max} = \frac{4V}{3A}$$

For thin-walled circuar tube:

$$\tau_{max} = 2V/A$$

6 - Combined Stresses

6.1 - Stresses Due to Eccentric Loading

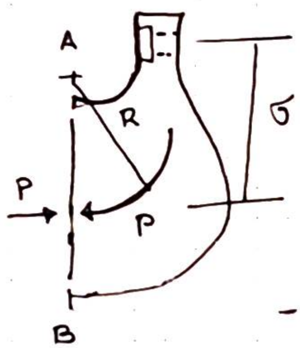


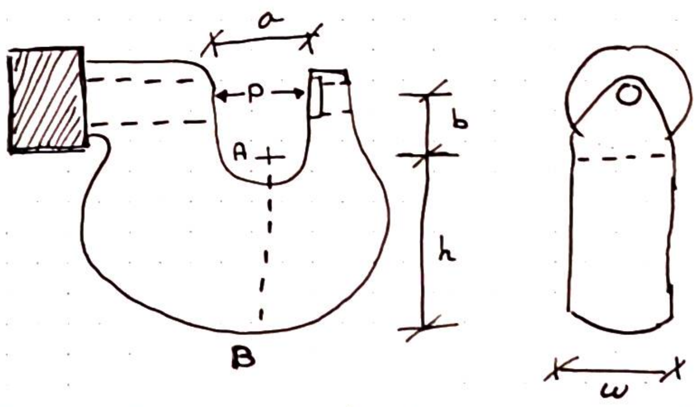
$$\sigma_c = \frac{P e C_c}{I} + P/A \qquad \sigma_t = \frac{P e C_t}{I} - P/A$$

If the loading is Tension:

$$\sigma_c = \frac{P e C_c}{I} - \frac{P}{A} \qquad \sigma_t = \frac{P e C_t}{I} + \frac{P}{A}$$

Example -





- This is a portable hydraulic riveter yoke

where: $P = 10$ tons $= 20.000$ lb

$a = 2$ in ; $b = 2$ in ; $\omega = 1\ ^1/_2$ in

allowable stress $= 75.000$ lb

↳ determine the depth of yoke h.

Solution: $\sigma_t = \dfrac{P}{A} + \dfrac{K P e c}{I}$

$$I = \frac{wh^3}{12} \; ; \quad c = \frac{h}{2} \; ; \quad A = wh$$

$$\sigma_t = \frac{P}{wh} + \frac{6 K P e}{w h^2} = 75000 \text{ psi}$$

$$\frac{75000}{20000} = \frac{1}{wh} + \frac{6 K e}{w h^2} = \frac{1}{1.5 h} + \frac{6 K e}{1.5 h^2}$$

or $h^2 - 0.178 h = 1.07 Ke$

Assume a reasonable value for h ; take $h = 2$ in

then $e = \dfrac{h}{2} + b = 3$ in ; $R = \dfrac{a}{2} + \dfrac{h}{2} = 2$ in

$$R/c = 2 \quad \therefore K \simeq 1.6$$

$$h^2 - 0.178h = 1.07\, Ke = 1.07 \times 1.6 \times 3 = 5.14$$

or $h = 2.358$ in > 2

Repeat with $h = 2.5$ in

then $e = 3.25$; $R = 2.25$; $R/c = 1.8$

$\therefore K = 1.65$

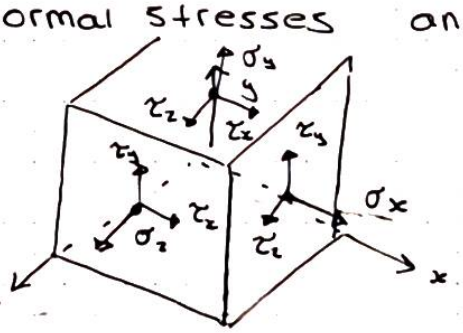
$$h^2 - 0.178h = 1.07 \times 1.65 \times 3.25 = 5.74$$

$$h = 2.48 \text{ in} \simeq 2.5 \text{ in}$$

$h =$ **2.5 in** is a good design value

6.2 - Determination of Principal Stresses

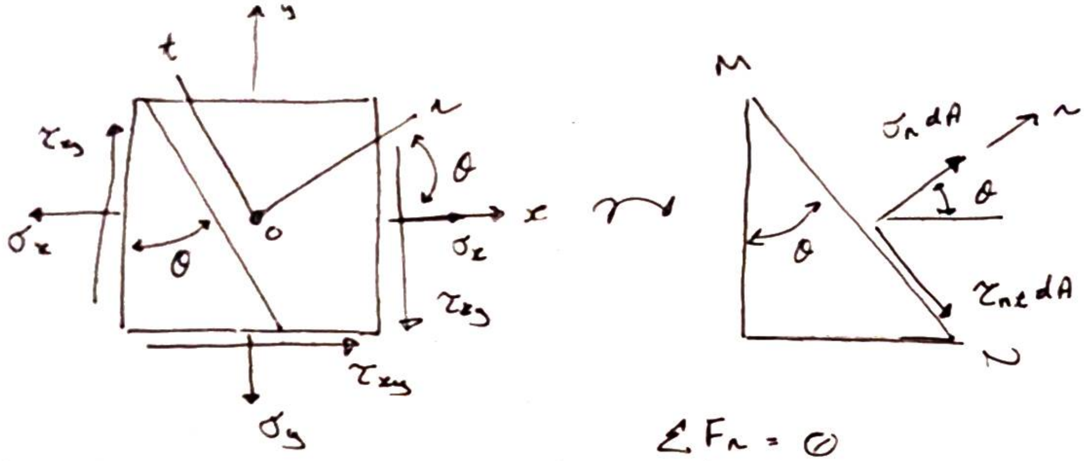
Whatever the aspect of the stress at a joint may be, it can always be expressed in terms of normal stresses and shear stresses



Where : σ_x , σ_y , σ_z are normal stresses

$$\left.\begin{aligned} \tau_{yx} &= \tau_{xy} \\ \tau_{yz} &= \tau_{zy} \\ \tau_{xz} &= \tau_{zx} \end{aligned}\right\} \text{ are shear stresses}$$

Consider a section of this element.



$$\Sigma F_n = 0$$

$$\sigma_n dA = \sigma_x \cos\theta \, dA \cos\theta \; \ldots$$
$$\ldots \; - \sigma_y \sin\theta \, dA \sin\theta + \tau_{xy} \cos\theta \, dA \sin\theta \ldots$$
$$\ldots \; + \tau_{xy} \sin\theta \, dA \cos\theta = 0$$

then; $\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta - \tau_{xy} \sin 2\theta$

$$\{ 2 \sin\theta \cos\theta = \sin 2\theta \}$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \cos^2\theta - \tau_{xy} \sin 2\theta$$

Replacing $\cos^2\theta = \frac{1}{2} (1 + \cos 2\theta)$

$\sin^2\theta = \frac{1}{2} (1 - \cos 2\theta)$

to get: $$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta \ldots$$
$$\ldots - \tau_{xy} \sin 2\theta$$

$\Sigma F_t = 0$ leads to ...

$$\tau_{nt} = (\sigma_x - \sigma_y) \sin\theta \cos\theta + \tau_{xy} (\cos^2\theta - \sin^2\theta)$$

OR

$$\tau_{nt} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

The direction of the principal stresses (maximum and minimum values) is found by ...

.. differentiating σ_n with respect to θ, setting the derivative to zero and solving for θ.

The result is:

$$\tan 2\theta_{1,2} = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

Substituting in the expression of σ_n to find

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

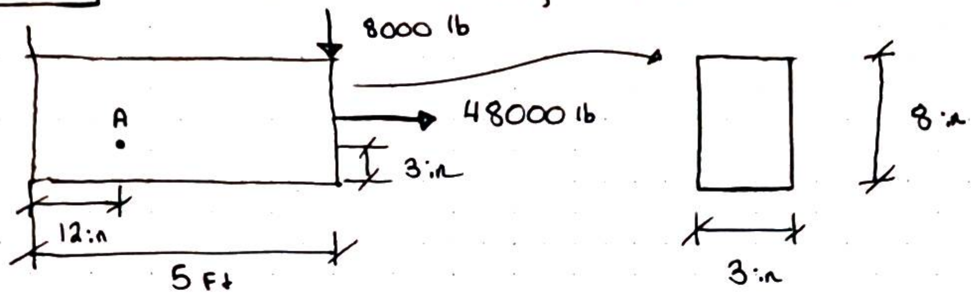
$$\tau_{1,2} = \emptyset$$

In a similar manner, τ_{max} is found to be

$$\tau_{max} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \tfrac{1}{2}(\sigma_1 - \sigma_2)$$

Example: Find physical stresses, max shear stress.



$\sigma_y = 0$

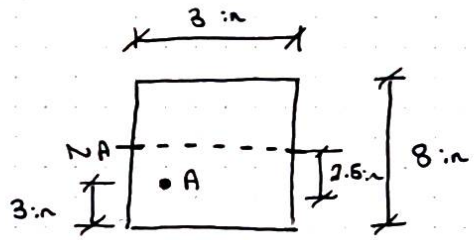
$$\sigma_x = \frac{P}{A} - \frac{My}{I} = \frac{48000}{3 \times 8} - \frac{8000 \times (5 \times 12 - 12) \times (4-3)}{3 \times 8^3/12}$$

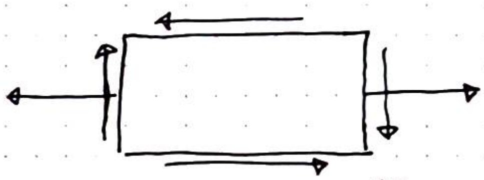
$\sigma_x = -1000$ psi

$$\tau_{xy} = \frac{VQ}{It} = \frac{(8000) \times 3 \times 3 \times 2.5}{\left(\frac{3 \times 8^3}{12}\right) \times 3} = 469 \text{ psi}$$

where $I = bh^3/12 = 3 \times 8^3/12$

$Q = 3 \times 3 \times 2.5$





$\sigma_x = -1000$ psi

$\tau_{xy} = 469$ psi

$$2\theta = \tan^{-1} - \frac{469}{-1000/2} = \tan^{-1}(0.938)$$

$2\theta = 43.2°$ or $2\theta = -136.8° = 136.8$

$2\theta = 223.2°$

and $\theta = 68.4°$

$$\sigma_1 = \frac{-1000}{2} + \sqrt{\left(\frac{-1000}{2}\right)^2 + 469^2} = 186 \text{ psi}$$

$$\sigma_2 = \frac{-1000}{2} - \sqrt{\left(\frac{-1000}{2}\right)^2 + 469^2} = 1186 \text{ psi}$$

$$\tau_{max} = \pm \sqrt{\left(\frac{-1000}{2}\right)^2 + 469^2} = \pm 686 \text{ psi}$$

7- instability Considerations

The Euler Formula :

$$F_{crit} = n\pi^2 EI / L^2$$

The J.B. Johnson Formula :

$$F_{crit} = A\sigma_{yp}\left(1 - \frac{\sigma_{yp} L^2}{4 n \pi^2 E \rho^2}\right)$$

where:

F_{crit} = Critical load causing Failure

A = Cross-sectional area

I = moment of inertia of area

L = length of Column

ρ = least radius of gyration of cross-section

= $\sqrt{I/A}$; for circular section $\rho = d/4$

n = end-fixity coefficiend

E = modulus of elasticity

σ_{yp} = yield point of material

Let $B = \dfrac{\sigma_{yp} L^2}{(n \pi^2 E)}$

Then the Euler Formula (For $B/\rho^2 > 2$)
(long and small cross-section)

$$F_{crit} = \frac{n\pi^2 AE}{(L/\rho)^2} = \frac{\sigma_{yp} A\rho^2}{B}$$

Then J.B. Johnson Formula (For $B/\rho^2 < 2$)

$$F_{crit} = A\sigma_{yp}\left(1 - \frac{B}{4\rho^2}\right)$$

Most struts used in machinery are of proportions in the J.B. Johnson range

Start with Johnson, Find B/ρ^2 if < 2 o.k.
if not, use Euler.

For column with initial crookedness See spotts
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8 - Factor of Safety (f_s)

In order to provide a margin against failure, it is common practise in machine design to determine the allowable stress by dividing the failure stress for the member by a factor of safety (f_s)

$$\sigma_{allow} = \frac{\sigma_{crit}}{f_s}$$

In Buckling:

$$F_{allow} = \frac{F_{crit}}{f_s}$$

$\boxed{\text{Example}}$ Circular cross-section
SAE 1030 ; $\sigma_{yp} = 42000$ psi ; $E = 30 \times 10^6$ psi

$L = 6$ in
$P = 2000$
diameter of loading pin = 0.5 in
$f_s = 1.5$
Allowable bearing pressure at the pin = 10000 psi
↳ determine the dimensions for the strut.

Solution :

$n = 1$; $B = \dfrac{\sigma_{yp} L^2}{n \pi^2 E} \Rightarrow \dfrac{42.000 \times 6^2}{1 \times \pi^2 \times 30 \times 10^6} = 0.0051 \text{ in}^2$

$F_{crit} = f_s \times F_{allow} = 1.5 \times 2000 = 3000$ lb

Starting with Johnson's eq'n :

$$F_{crit} = A \sigma_{gyp} \left(1 - B/4\rho^2\right)$$

$$A = \pi d^2/4 \quad ; \quad \rho = d/4$$

Substituting and solving for d^2

$$d^2 = \frac{4 \cdot F_{crit}}{\pi \sigma_{yp}} + 4B \Rightarrow \frac{4 \times 3000}{\pi \times 42000} + 4 \times 0.0051$$

$d^2 = 0.111 \text{ in}^2$; $d = 0.333$ in

Using a standard 3/8 in we check for B/ρ^2

$$B/\rho^2 = \frac{16B}{d^2} = \frac{16 \times 0.0061}{(3/8)^2} = 0.680$$

$\therefore B/\rho^2 < 2$
↳ Johnson justified

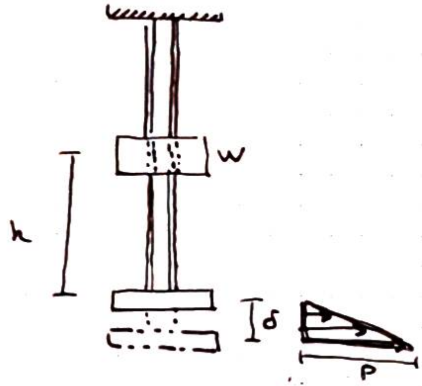
For the eye: $\sigma = P/td$

$$10000 = \frac{2000}{t \times 0.5}$$

$$t = 0.2/0.5 = 0.4 \text{ in}$$

↳ use 1/2 in to allow for machining of the faces of the eye

9- Stresses due to shock and impact loading



W = falling weight, lb
h = height of free fall, in
δ = deflection, in
P = impact load, lb
$c = P/\delta$ = lb/in of deflection

Energy balance:

$$(1/2)P\delta = W(h + \delta)$$

$$P = 2\frac{W}{\delta}(h + \delta)$$

$$\frac{P}{W} = 2\left(\frac{h}{\delta} + 1\right)$$

But $\delta = P/c$

$$\therefore \frac{P}{W} = 2\left(\frac{hc}{P} + 1\right) \rightsquigarrow P^2 = 2W(hc + P)$$

$$P^2 - 2WP - 2Whc = 0$$

$$P = \frac{2W \pm \sqrt{4W^2 + 8Whc}}{2}$$

$$P = W\left(1 + \sqrt{1 + \frac{2hc}{W}}\right)$$

$$P/W = 1 + \sqrt{1 + \frac{2hc}{W}}$$

For a bar in tension

$$\delta = \frac{PL}{AE} \quad \therefore C = \frac{P}{\delta} = \frac{\cancel{P}}{\cancel{P}L/AE} \Rightarrow \frac{AE}{L}$$

Special case: if the load is applied instantaneously without velocity of approach then $P = 2W$