MARCH 20/18
MACHINE DESIGN

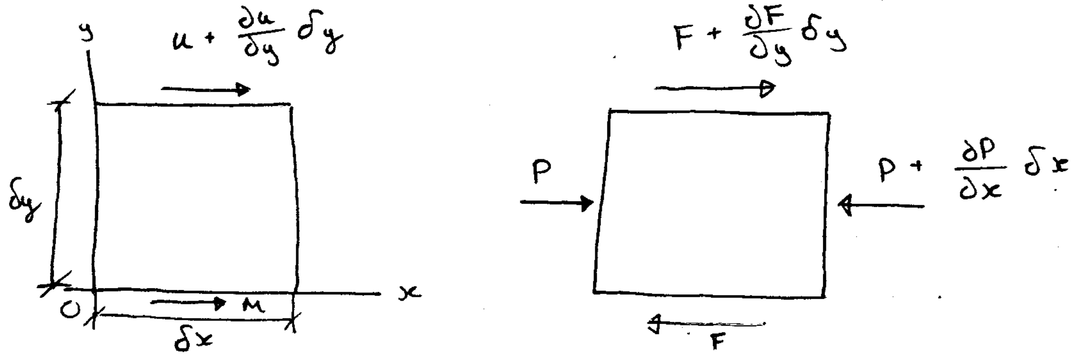
From Newton's Law of Viscous Flow

$$F = \mu \, du/dy \qquad (1.1)$$

F = Shearing Force

μ = viscosity of Fluid

du/dy = rate of shear or velocity gradient



(Element of a Fluid)

$\sum F_x = 0 \quad \therefore$

$$\delta x \, F + \frac{\partial F}{\partial y} dy\,dx + P dy - F\delta x - P dy - \frac{\partial p}{\partial x}\delta x \delta y = 0$$

$$\therefore \frac{\partial F}{\partial y} dy\,dx = \frac{\partial P}{\partial x}\delta x \delta y$$

OR $\dfrac{\partial F}{\partial y} = \dfrac{\partial P}{\partial x}$

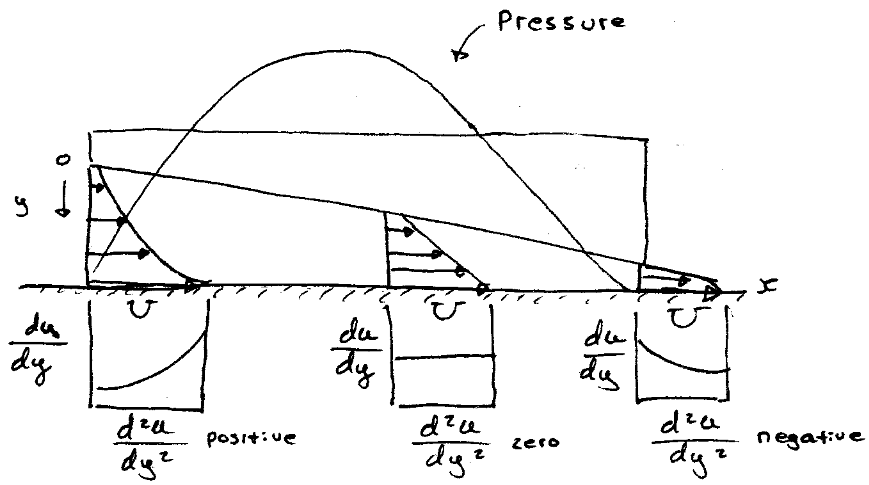
But from equation 1.1

$$F = \mu \frac{\partial u}{\partial y} \qquad \therefore \qquad \frac{\partial F}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2}$$

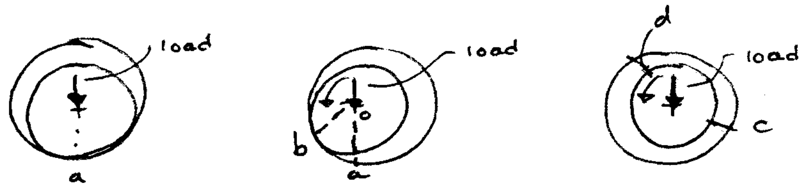
and $\dfrac{\partial P}{\partial x} = \mu \dfrac{\partial^2 u}{\partial y^2}$

For constant h : $\dfrac{\partial^2 u}{\partial y^2} = 0$

and therefore no pressure gradient, i.e. no pressure can be built up in a parallel film.



(Pressure dist. in Converging Film)

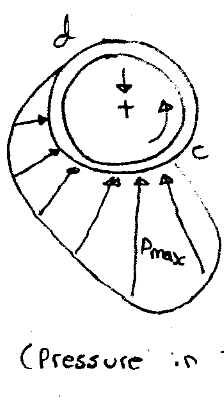


(at rest) (slow speed) (high speed)

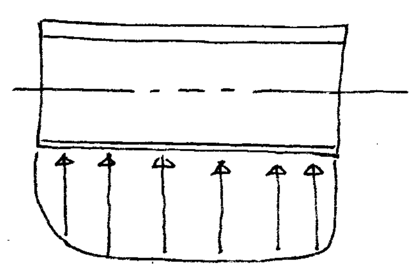
(Formation of continuous oil Film)

- At rest metal contact at a
- At low speed contact point shifts to b where some lubricant may be present but not a continuous Film.
- At high speed a converging Film is created and therefore, a positive pressure is present From d to c to support the load. A negative pressure gradient exists From c to d to draw lubricant From the source if conditions are Favorable. The pressure distribution is as shown in the figure.

Radial:



Axial:



(Pressure in Journal Bearing)

3 - Friction in Journal Bearings

- coefficient of friction

$$f = F/P = T/Pr$$

by dimensional analysis it is found that

$$f = \phi\left(\frac{ZN}{P}, \frac{d}{c}, \frac{L}{d}\right)$$

where : f = coefficient of friction

ϕ = a functional relationship

Z = absolute viscosity of lubricant, centipoises

N = speed of journal, RPM

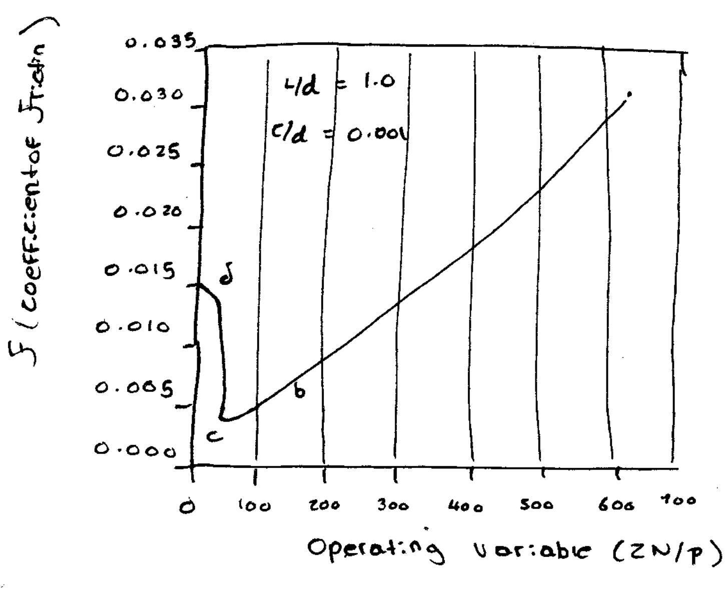
P = bearing pressure on projected bearing area, psi

d = diameter of journal, in

c = clearance of diameters, in

L = length of bearing, in

$\frac{ZN}{P}$ = bearing characteristic number



Cd : boundary or imperfect lubrication

bc : partial metal to metal contact

ab : Fluid Film lubrication

For well-lubricated full journal bearings, the McKee and McKee equation may be used for estimating values of the coefficient of friction, f:

$$f = \frac{473}{10^{10}} \times \frac{ZN}{p} \times \frac{d}{c} + K$$

where $K = 0.002$ for $0.75 \le \frac{L}{d} \le 2.8$ and is the end leakage correction factor. Operating values for ZN/p should be compared with values from Table 2-1 (H.O.) for safe operation.

4. – Impact Lubrication

if ZN/p is too low (cd on curve)

$$f = \frac{C_1 C_2}{250} \sqrt[4]{P/V}$$

where C_1, C_2 = are from table 21.2 (H.O.)

p = bearing pressure, psi

V = rubbing velocity, fpm

5 – Thermal Equilibrium

- heat generated

$H_1 = fPV$

H_1 = heat generated, ft-lb per min

f = coefficient of friction

P = radial load on bearing

V = rubbing velocity, fpm

- heat dissipated

$H_2 = CA(t_b - t_a)$

H_2 = heat dissipated by bearing, ft-lb per min

A = projected bearing area, Ld, in^2

t_b = temperature of bearing surface, °F
t_a = temperature of surrounding air, °F
t_o = temperature of oil film, °F
C = heat dissipation coefficient, ft-lb per min per in² of projected bearing area per °F

Values of $C(t_b - t_a)$ are obtained from Figure 21-19 (H.O.)

Experiments show that for industrial bearing

$$t_b - t_a = \frac{1}{2}(t_o - t_a)$$

6 - Bearing design based on McKee and McKee Eq'n:

1. Choose L/d ratio from table 21-1 (H.O.)
2. Check $p = P/Ld$ from table 21-1
3. Assume a clearance ratio c/d table 21-1
4. Assume a lubricant from Fig. 21-16
   - Assume operating temp $80°F \le t_o \le 140°F$
   (t_o should not exceed 180°F under any conditions)
5. Determine operating value of ZN/p
   Then check table 21-1 for film lubrication
6. Find coefficient of friction
7. Find H_1 and H_2
8. If thermal equilibrium is indicated by comparing H_1 and H_2, design O.K.
9. If approx. equilibrium is not indicated, try again with different temp., oil, ...

MARCH 22/18
MACHINE DESIGN

Example 1 - A Journal bearing is proposed for a steam turbine

$P = 585$ lb
$d = 2\,1/4$ in
$L = 3\,3/8$ in
$N = 1800$ rpm
$c/d = 0.001$
$t_a = 60$ °F
Oil = SAE 10W
$t_o = 140$ °F

Determine H_1 and H_2:

Solution - From Fig 21-16 ; $Z = 14$ cp

$p = P/Ld = 585/(2.25 \times 3.375) = 77$ psi

$\therefore ZN/p = 14 \times 1800/77 = 327$

Checking table 21-1 for item 16

- $p = 77$ psi is well within usual limits
- $ZN/p = 327 > 100$ is high enough to assure hydrodynamic operating conditions

$$f = \frac{473}{10^{10}} \times \frac{ZN}{p} \times \frac{d}{c} + k$$

$$f = \frac{473}{10^{10}} \times 327 \times \left(\frac{1}{0.001}\right) + 0.002 = 0.0175$$

$$V = \frac{\pi D \text{ rpm}}{12} = \frac{\pi \times 2.25 \times 1800}{12} = 1060 \text{ fpm}$$

→ $H_1 = fPV$ (heat generated)

$H_1 = 0.0175 \times 585 \times 1060 = 10{,}852$ ft-lb/min

→ $H_2 = C(t_b - t_a)A$ (heat dissipated)

$t_b - t_a = \frac{1}{2}(t_o - t_a) = (140-60)/2 = 40$ °F

From Fig 21-19 (H.O.) for well-ventilated

$t_b - t_a = 40$ °F $\Rightarrow$ $C(t_b - t_a) = 190$

$H_2 = 190 \times 2.25 \times 3.375 = 1450$ ft-lb/min

$\therefore$ heat generated > heat dissipated. The bearing must be artificially cooled to remove

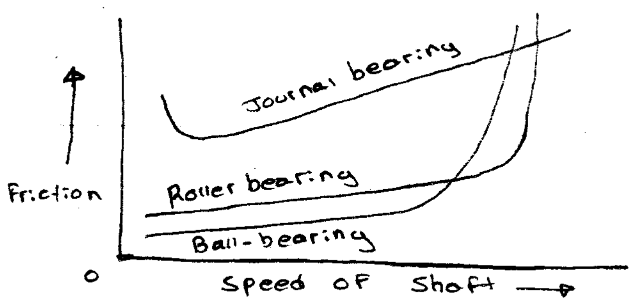
$10{,}852 - 1{,}450 = 9{,}402$ ft-lb/min

or design changed for thermal equilibrium

↳ See assignment #9 questions for practice

Rolling - contact Bearings

- General considerations
- Advantages as compared with journal bearings
  1 - Friction is low except at high speeds (V ≥ 20,000 rpm)
  2 - relatively accurate shaft alignment can be maintained
  3 - Heavy momentary overload can be carried
  4 - lubrication is simple
  5 - both radial and axial loads can be carried by some types
  6 - replacement is easy
  7 - Selection of bearing from manufacturers info is relatively simple



(comparison of friction bearings)

- Disadvantages
  1 - The cost of bearing and mounting is greater
  2 - Failure can occur without warning

2 - Selection of Ball Bearings

If a number of apparently identical bearings are tested under identical conditions, the life at which 10 % of them have failed and 90 % are good is called the rating life or L_{10} life for this bearing

The basic load rating C for a rating life of one million revolutions for radial and angular contact ball-bearings, except filling slot bearings, is given by:

$$C = f_c (i \cos\alpha)^{0.7} Z^{2/3} D^{1.8} \qquad D \le 1 \text{ in.}$$

$$C = f_c (i \cos\alpha)^{0.7} Z^{2/3} D^{1.4} \qquad D > 1 \text{ in.}$$

Where:

f_c = a constant from Table 9-2 as determined by the value of $(D\cos\alpha)/d_m$

i = number of rows of balls in the bearing

α = nominal angle of contact (angle between line of action of ball load and plane perpendicular to bearing axis)

Z = number of balls per row

D = bal diameter

d_m = pitch diameter of ball races

Example 1 – Find the value of C for a 207 radial bearing.

From Table 9.1 $\quad d_m = \tfrac{1}{2}(2.8346 + 1.3780) = 2.1063$ in

↳ $D\cos\alpha / d_m = \dfrac{0.4375}{2.1063} = 0.208 \quad \{ \alpha = 0$ here

With Table 9.2 $\quad f_c = 4550$

From Table 9.1 $\quad D = 7/16 = 0.4375$

$D^{1.8} = 0.2258$

$Z = 9 \; ; \; Z^{2/3} = 4.327$

and $\quad C = 4550 \times 4.327 \times 0.2258 = 4440$ lb

For L_{10} of 1 million revolutions

Rating lives for other loads other than the rating load can be found as follows:

$$10^6 C^3 = N_1 P_1^3 = N_2 P_2^3 = N_3 P_3^3$$

where N may be replaced by

$$N = 60 n L$$

where n = speed, rpm

L = life, hr

Example 2 - For the 207 bearing of Example 1, find the radial load P for a rating life of 500 hr at 1,500 rpm

Solution:

$$10^6 C^3 = N_1 P_1^3 = 60 n L P_1^3$$

$$P_1^3 = 10^6 C^3 / 60 n L$$

$$= \frac{10^6 \times 4440^3}{60 \times 1500 \times 500} = 1,945,000,000$$

$$P_1 = 1250 \text{ lb}$$

3 - Effect of Axial load

If an axial loading is present with the radial loading, the equivalent radial load P is the larger of the values given by

$$P = C_1 V_1 F_r$$

or

$$P = C_1 (X V_1 F_r + Y F_a)$$

where: F_r = radial component of load

F_a = axial component of load

X = radial factor from Table 9-2

Y = axial or thrust factor from table 9-2 as determined from $F_a / i Z D^2$

C_1 = service or shock factor (Table 9-3)

V_1 = race rotation factor $\begin{cases} 1 \text{ for inner race rotation} \\ 1.2 \text{ for outer ring} \end{cases}$

Example 3 -

Suppose the bearing in Example 1 carries a combined load of 400 lb radially and 300 lb axially at 1200 rpm. The outer ring rotates and the bearing is subjected to moderate shock. Find the rating life for this bearing in hours.

Solution:

$$\frac{F_a}{iZD^2} = \frac{300}{9 \times 0.4375^2} = 174$$

From table 9-2 $\quad y = 1.5 \quad ; \quad x = 0.56$

From table 9-3 $\quad C_1 = 2$

$$P = C_1 V_1 F_r = 2 \times 1.2 \times 400 = 960 \text{ lb}$$

or $$P = 2(0.56 \times 1.2 \times 400 + 1.5 \times 300) = 1440 \text{ lb}$$

$$N = \frac{10^6 C^3}{P^3} = 60nL$$

$$L = \frac{10^6 C^3}{60 n P^3} = \frac{10^6 \times 4440^3}{60 \times 1200 \times 1440^3} = 410 \text{ hr}$$

4 - Design for variable loading

If P_i is the bearing load, N_i is the rated bearing life if operated exclusively at the constant load P_i, and N_i' the actual number of application then

$$\sum_i \frac{N_i'}{N_i} = 1$$

OR $$\frac{N_1'}{N_1} + \frac{N_2'}{N_2} + \frac{N_3'}{N_3} = 1$$

And if the life of the bearing under the combined loading is N_c such that

$$N_1' = \alpha_1 N_c \quad ; \quad N_2' = \alpha_2 N_c \quad ; \quad N_3' = \alpha_3 N_c \quad \ldots$$

Then $$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \frac{\alpha_3}{N_3} \cdots = \frac{1}{N_c}$$

and $\alpha_1 + \alpha_2 + \alpha_3 \cdots = 1$

since $N_i = \dfrac{10^6 C^3}{P_i^{\,3}}$

Then $\dfrac{\alpha_1 P_1^{\,3}}{10^6 C^3} + \dfrac{\alpha_2 P_2^{\,3}}{10^6 C^3} + \cdots = \dfrac{1}{N_e}$

or $\dfrac{10^6 C^3}{N_e} = \alpha_1 P_1^{\,3} + \alpha_2 P_2^{\,3} + \cdots$