JAN. 9TH/1[illegible]

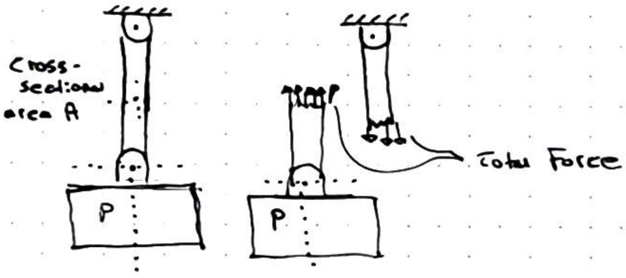
MACHINE DESIG[illegible]

Midterm - week 6 (open book)
↳ either on the tuesday or thursday

First priority for review is from class-notes

Fundamental principles

1 - Tension and Compression stress



Direct axial stress → $\boxed{\sigma = P/A}$ (1) ← axial load

$\varepsilon = \delta / l$ (2)

Hooke's Law: $\sigma = \epsilon E$ (3)

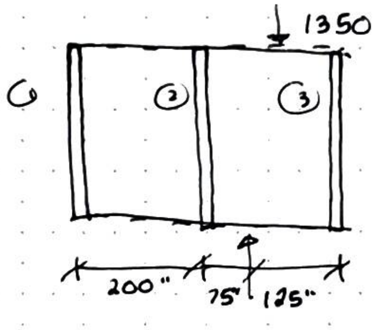
or $\epsilon = \sigma / E$

Substituting (1) and (3) into (2): $\boxed{\delta = \dfrac{Pl}{AE}}$ (4)

2 - Statically Indeterminate Problems in Tension and Compression

Machine parts are sometimes arranged in a manner where the axial forces cannot be determined by the equations of statics alone. For such situations, the deformations of the parts must be taken into consideration

Example 1: The struts in the figure are of the same material and have equal cross-sectional areas. Members on-top and bottom can be considered rigid. Find the forces in each bar.



Solution $F_1 + F_2 + F_3 = 1350 \text{lb}$ (a)

Also, $\delta_2 - \delta_1 = \delta_3 - \delta_2$

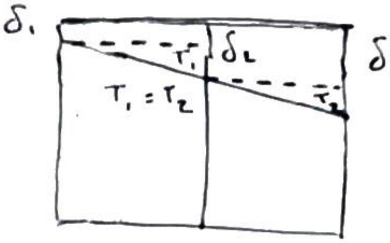
$\delta_1 - 2\delta_2 + \delta_3 = 0$

But the struts are of the same material and have equal cross-sectional areas.

$\therefore F_1 - 2F_2 + F_3 = 0$ (b)

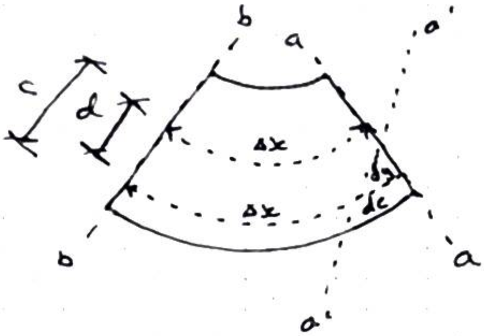
Moment @ load

$275 F_1 + 75 F_2 - 125 F_3 = 0$ (c)



Solving $F_1 = 200$ lb;
$F_2 = 450$ lb;
$F_3 = 700$ lb

3 - Normal Stresses Due to Bending



Similar triangles :

$$\delta y / y = \delta c / c$$

$$\delta y = y \, \delta c / c$$

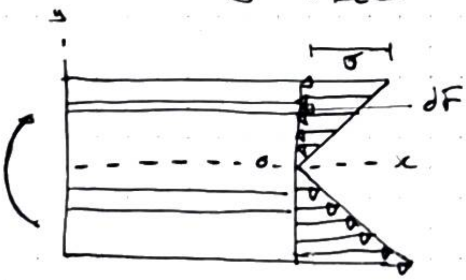
By definition $\epsilon = \Delta L / L$

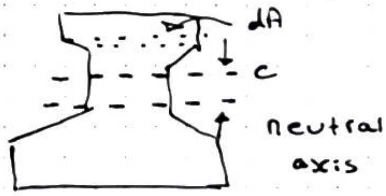
$$\therefore \delta y / \Delta x = y = \frac{\delta c / \Delta x}{c}$$

$$\epsilon = y \frac{\epsilon_c}{c}$$

From Hooke's Law : $\epsilon = \sigma / E$

$$\sigma / E = y \, \sigma_c / E_c c$$





But; $E_c = E$ and

$$\sigma / y = \sigma_c / c \quad (5)$$

For equilibrium $\Sigma F_x = 0$

$$\therefore \int^A \sigma \, dA = 0$$

$$\sigma_c / c \int^A y \, dA = \sigma_c / c \, A \bar{y} = 0$$

Since $\sigma_c / c \, A \neq 0$ Then $\bar{y} = 0$

$\boxed{\therefore \text{ Neutral axis and centroidal axis are the same}}$

Moment of inertia about O is

$dF = y$; $M = \int^A y \, \sigma \, dA$

But $\sigma = y \, \sigma_c / c$ And $M = \sigma_c / c \int^A y^2 \, dA$

$$= \sigma / y \int^A y^2 \, dA$$

but $\int^{A} y^2 \, dA = I$ = moment of inertia

$\therefore \; M = \sigma/y \cdot I$

or $\boxed{\sigma = \dfrac{My}{I}}$ where: σ = stress at distance y from neutral axis

I = moment of inertia of cross-section about neutral axis

M = applied bending moment

For symmetrical beams:

$$\sigma_{t\text{-}max} = \sigma_{c\text{-}max} = \frac{Mc}{I} = \frac{M}{I/c}$$

* I/c is known as "section modulus"

For curved beams:

$$\left[\; \sigma = \frac{M}{AR}\left(1 + \frac{1}{Z}\,\frac{y}{R+y}\right) \quad \text{where } Z = -\frac{1}{A}\int \frac{y}{R+y}\, dA \;\right]$$

For reference

Determination of which may be complicated.
In practise the stress is found using a series of tables.

* see the (K vs. R/c graph)

$$\sigma = \frac{KMc}{I}$$

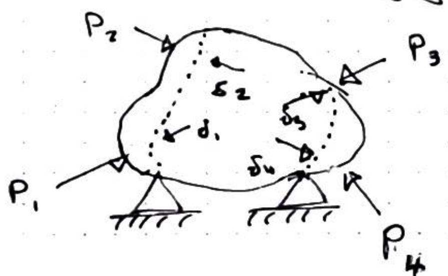
(For $R/c \geq 10$ use $K = 1$) section modulus

4 - Deflection of beams

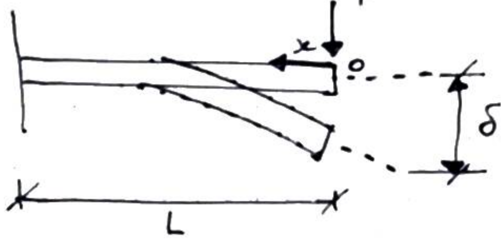
- Castigliano's Theorem

The displacement corresponding to any force of a system of forces acting on an elastic body can be determined by taking the partial derivative of the elastic strain energy with respect to that force

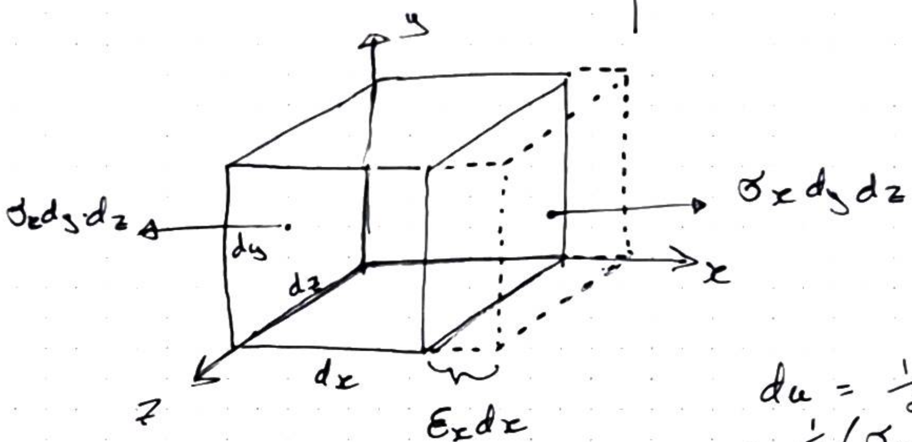
$$\frac{\delta u}{\delta P_r} = \delta_r$$



Example 2 :



Alternative solution:



$$du = \frac{1}{2} P\delta$$
$$= \frac{1}{2}(\sigma_x dy dz)(\varepsilon_x dx)$$
$$= \frac{1}{2}\, \sigma_x \varepsilon_x \, dx dy dz$$

du per unit volume $= \frac{1}{2}\sigma_x \epsilon_x$

but $\epsilon_x = \sigma_x / E$

$$\therefore dU = \sigma_x^2 / 2E$$

$$U = \frac{1}{2}\int^L \int^A \frac{\sigma_x^2}{E}\, dA\, dx = \frac{1}{2}\int^L\int^A \frac{1}{E}\left(\frac{My}{I}\right)^2 dA\, dx$$

$$U = \frac{1}{2}\int^L \frac{1}{E}\frac{M^2}{I^2}\, dx \int^A y^2\, dA$$

$$= \frac{1}{2}\int_0^L \frac{M^2}{EI}\, dx$$

$$\delta = \frac{\partial U}{\partial P} = \frac{1}{2}\int_0^L \frac{\partial}{\partial P}\left(\frac{M^2}{EI}\right) dx$$

$$= \int_0^L M \frac{\partial M/\partial P}{EI}\, dx$$

$$\delta = \int_0^L \frac{(-Px)(-x)}{EI}\, dx = \frac{PL^3}{3EI} \downarrow$$

* The deflection is always in the direction of the force

Cont'd:

$d^2y/dx^2 = -\frac{1}{EI}[P(L-x)]$

$dy/dx = -P/EI\int(L-x)dx = -P/EI\,(Lx - x^2/2 + C_1)$

$y = -P/EI\int(Lx - x^2/2 + C_1)\,dx$

$y = -P/EI\left(\frac{Lx^2}{2} - \frac{x^3}{6} + C_1x + C_2\right)$

B.C.:

at $x = 0$ $\quad y' = 0$ --- $C_1 = 0$

$\quad\quad\quad\quad y = 0$ --- $C_2 = 0$

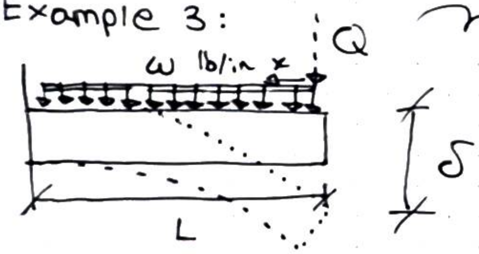
$y = -P/EI\left(\frac{Lx^2}{2} - x^3/6\right)$

at $x = L$ $\quad y = \delta$

$\therefore\ \delta = -P/EI\,(L^3/2 - L^3/6) = -\frac{PL^3}{3EI}$

$$\delta = \boxed{\frac{PL^3}{3EI}\downarrow}$$

Example 3:



Assume a fictitious force Q

$M = -Qx - \frac{wx^2}{2}$

$\frac{\partial M}{\partial Q} = -x$

$$\delta = \frac{\partial U}{\partial Q} = \int_0^L \frac{M(\partial M/\partial Q)}{EI}\,dx$$

$$\Rightarrow \int_0^L \left(-Qx - \frac{wx^2}{2}\right)\frac{-x}{EI}\,dx$$

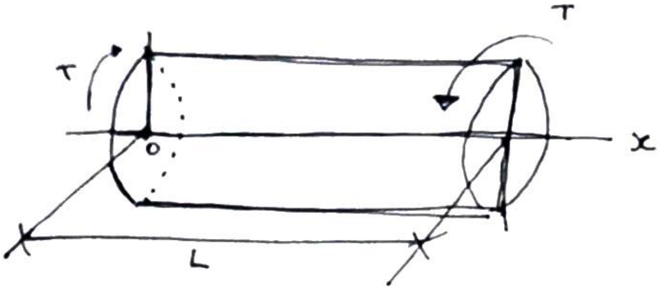
$$\delta = \int_0^L \frac{wx^3}{2EI}\,dx = \boxed{\frac{wL^4}{8EI}\downarrow}$$

The method of superposition: This method uses the fact that for linear problems, the deflection at any point is equal to the sum of the deflections caused by each load acting separately.

→ Figure 1.13 (Pg. 41 in)

- Angular deformations

Castiliano's Theorem may also be employed to Calculate the angle of twist in members s.t. torsion.



(Bar in torsion)

It can be shown that the strain energy per unit volume is : $u = \frac{1}{2}\tau\gamma = \frac{\tau^2}{2G}$

The total strain energy for a cylindrical bar in Torsion is :

$$U = \int^L \int^A \frac{\tau^2}{2G} \, dx\,dA = \int^L \int^A \frac{T^2 r^2}{2GJ^2} \, dx\,dA$$

$$U = \int_0^L \frac{T^2}{2JG} \, dx$$

Where T = torque

J = Polar moment of inertia

G = Shear modulus (or torsional modulus) of elasticity

$$\theta = \frac{\partial U}{\partial T} = \int_0^L \frac{2T}{2JG} dx = \int_0^L \frac{T\,dx}{JG}$$

If the torque is uniform along the length of the shaft

$\boxed{\theta = \frac{TL}{JG}}$ = total angle of twist

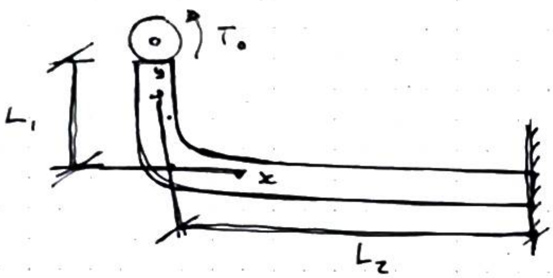
The rotation of a section of a beam at a particular location is found to be :

$$\theta = \frac{\partial U}{\partial C} = \int_0^L \frac{M(\partial M / \partial C)}{EI} \, dx \qquad (C = \text{couple})$$

Where C is the couple at the section of interest in the case of just bending where $M = C$ throughout the length of the beam

$$\boxed{\theta = \frac{ML}{EI}}$$

Example 5 - Determine the rotation θ of the free end of a tube in the plane of a torque T_0; Both portions of the tube lie in the same plane. Neglect the effect of deflection of the radius of the quarter bend.



Length L_1 of pipe is subjected to torque T_0

Length L_2 of pipe is subjected to bending moment T_0 about y-axis

$$U_1 = \int_0^{L_1} \frac{T^2}{2JG}\,dy \quad ; \quad U_2 = \int_0^{L_2} \frac{M^2}{2EI}\,dx$$

where $M = T = T_0$

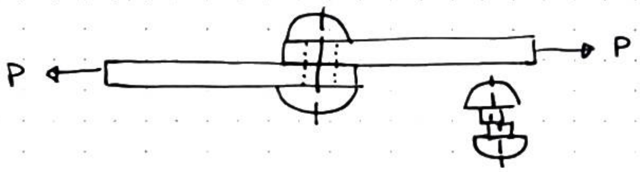
$$U = U_1 + U_2 = \int_0^{L_1} \frac{T_0^2}{2JG}\,dy + \int_0^{L_2} \frac{T_0^2}{2EI}\,dx$$

and $$\theta = \frac{\partial U}{\partial T_0} = \int_0^{L_1} \frac{T_0}{JG}\,dy + \int_0^{L_2} \frac{T_0}{EI} = \frac{T_0 L_1}{JG} + \frac{T_0 L_2}{EI}$$

Assignment #1 : 1.1, 1.4, 1.20, 1.23, 1.28, 1.32, 1.84

5 - Shear stresses

5.1 - direct shear



(Rivet connection, Shear deformation)

The shear stress is the component of the stress on a plane section that is parallel to the section.

$$\tau = P/A$$

P = shearing load

A = area in shear

τ = shear stress

(For the above rivet: $A = \frac{\pi d^2}{4}$)

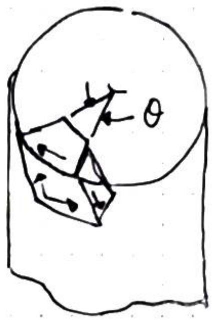
then, $\tau = 4P/\pi d^2$

5.2 - Torsional shear stress

Torsional moments induce shear stresses on cross-section's normal to the axis of bars + shafts

For circular shafts

$\tau = Tr/J$



Shear stress due to torsion

Where:

τ = shear stress

r = distance from the centre of shaft to point of stress

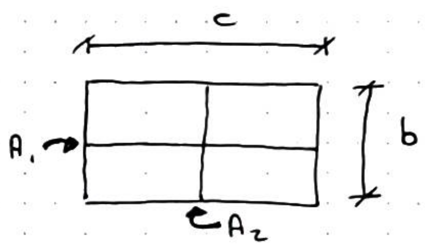
J = Polar moment of inertia

For solid circular shaft

$$\tau_{max} = \frac{Td}{2J} = \frac{Td}{2\pi d^4/32} = \boxed{\frac{16T}{\pi d^3}}$$

For rectangular bars

$$\tau = \frac{T}{\alpha_i b c^2}$$



For wide rectangular bars $b/c \geq 10$; $\alpha_i = 1/3$

$$\tau_{max} = \frac{3T}{bc^2} \quad \text{at } A_1$$

In general $\tau = \frac{T}{\alpha_1 bc^2}$ for point A_1

$\tau = \frac{T}{\alpha_2 bc^2}$ for point A_2

$\theta_1 = \frac{T}{\beta G b c^3}$ ang. def. rad/in of length

Table 3-3 (Pg. 228)