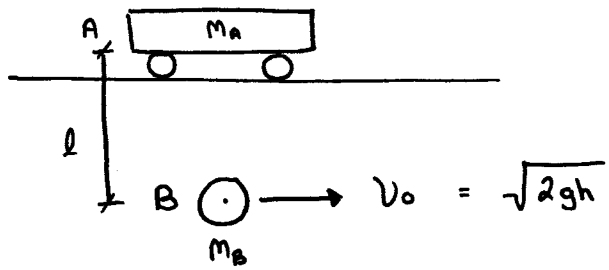
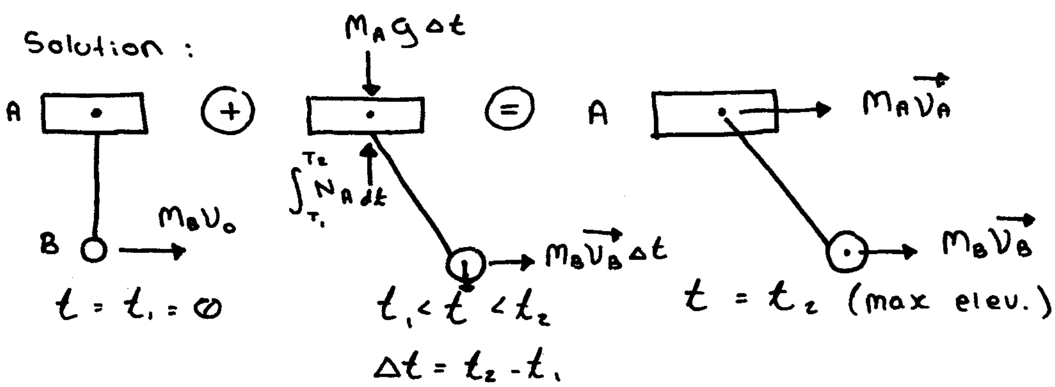
Example :



Determine a) the velocity of B as it reaches its max elevation.

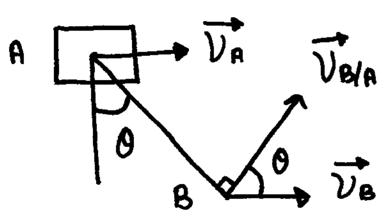
b) the max vertical distance h through which B will rise.

Solution :



$\underline{x}$: $\quad \emptyset + m_B v_0 = m_A v_A + m_B v_B$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

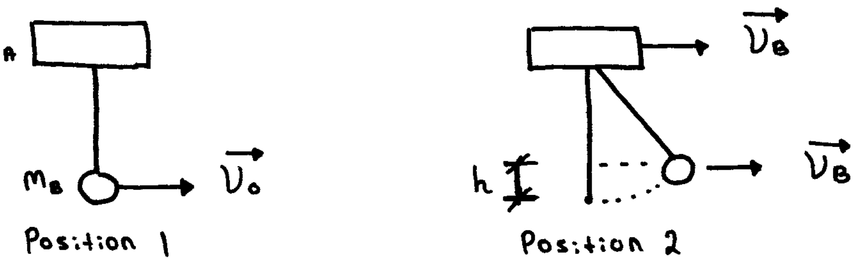


$v_{B/A} = \emptyset$

$\Rightarrow \quad v_B = v_A$

$$\Rightarrow \quad v_B = \frac{m_B}{m_A + m_B} v_0$$

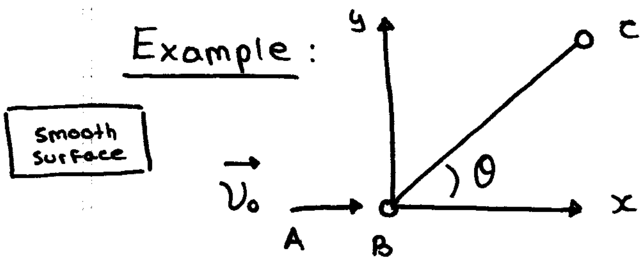
b)



$$\tfrac{1}{2} m_B v_0^2 + 0 + m_A g l = \tfrac{1}{2} m_A v_B^2 + \tfrac{1}{2} m_B v_B^2 + m_B g h + m_A g l$$

$$h = \frac{m_A}{m_A + m_B} \frac{v_0^2}{2g} = \frac{m_A}{m_A + m_B} \cdot l$$

Example:



$W_A = W_B = W_C = 2\text{ lb}$

$BC = 1.5 \text{ ft}$

$\vec{v}_0 = 8\vec{i} \text{ ft/s}$

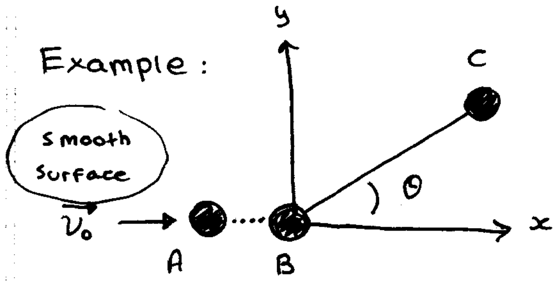
$\theta = 45°$

After impact : $\vec{v}_A = 0 \quad \vec{v}_B = 6i + v_{By}\vec{j}$ ft/s

determine $\boxed{v_{By}}$ and $\boxed{\vec{v}_C}$ immediately after impact

Nov. 1/17
Dynamics

Example:



$W_A = W_B = W_C = 2$ lb

$BC = 1.5$ ft

$\vec{v}_0 = 8\vec{i}$ ft/s

$\theta = 45°$

After impact : $\vec{v}_A = 0$; $\vec{v}_B = 6\vec{i} + v_{By}\vec{j}$ ft/s

Determine $\vec{v}_{By}$ and $\vec{v}_C$ immediately after impact

Solution : FBD - No external horizontal Force

=> Conservation of linear momentum :

$$\cancel{m}\vec{v}_0 = m(0) + \cancel{m}\vec{v}_B + \cancel{m}\vec{v}_C$$

$\underline{\underline{x}}$: $8 = 6 + v_{cx}$ => $v_{cx} = 2$

$\underline{\underline{y}}$: $0 = v_{By} + v_{cy}$ => thus, $v_{By} = -2$

Moment about z-axis

$$M_z = (\vec{r} \times \vec{F}) \cdot \vec{k} = 0$$

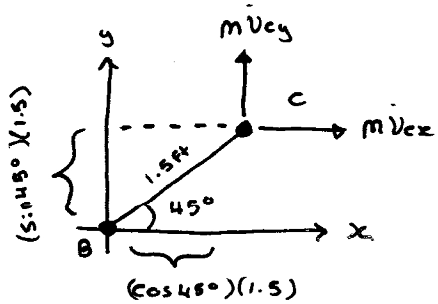
$$\Sigma M_z = 0$$

=> Conservation of Angular Momentum about z-axis

$$0 = 0 + 0 + (\vec{BC} \times m\vec{v}_C)\vec{k}$$

$$0 = -m v_{cx}(1.5)\sin 45° + m v_{cy}(1.5)\cos(45°)$$

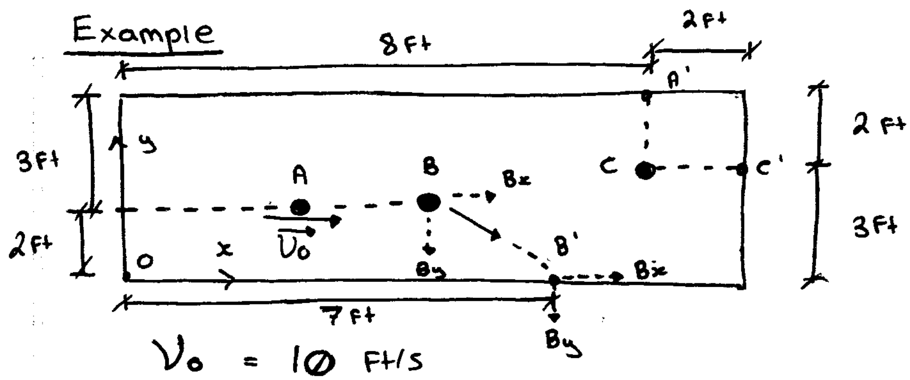
$$\Rightarrow v_{cx} = v_{cy} = 2$$



$$\therefore \vec{v}_B = 6\vec{i} - 2\vec{j} \text{ (ft/s)}$$

$$\vec{v}_C = 2\vec{i} + 2\vec{j} \text{ (ft/s)}$$

Example



$V_0 = 10$ ft/s

* Perfectly elastic collisions

Find the velocities of A, B, C after the collisions

Solution :

$$\vec{V_A} = V_A \vec{j}$$

$$\vec{V_B} = V_{Bx}\vec{i} - V_{By}\vec{j}$$

$$\vec{V_C} = V_C\vec{i}$$

Linear

$$\underline{\underline{x}} : \quad mV_0 = mV_{Bx} + mV_C \qquad (1)$$

$$\underline{\underline{y}} : \quad 0 = mV_A - mV_{By} \qquad (2)$$

Energy :

$$\tfrac{1}{2} m V_0^2 = \tfrac{1}{2} m V_A^2 + \tfrac{1}{2} m (V_{Bx}^2 + V_{By}^2) + \tfrac{1}{2} m V_C^2 \qquad (3)$$

Angular Momentum about x-axis :

→ through O :

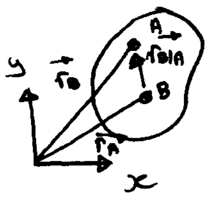
$$-(2)(mV_0) = -(3)(mV_C) + (8)(mV_A) - (7)(mV_{By}) \qquad (4)$$

Nov. 2/17
Dynamics II

Chapter 17 - Plane motion of rigid bodies - energy and momentum method

17.0 - review

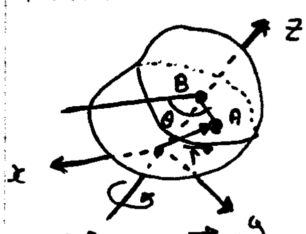
<u>Translation</u> : direction of any straight line inside the body is constant.



$$\vec{r}_B = \vec{r}_{B/A} + \vec{r}_A$$

$$\begin{cases} \vec{v}_B = \vec{v}_A \\ \vec{a}_B = \vec{a}_A \end{cases}$$

Rotation about a fixed axis :



$\vec{\omega} = \omega \vec{k}$

$$\vec{v}_A = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

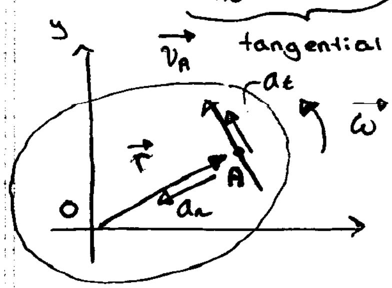
$$\vec{\omega} = \omega \vec{k} = \frac{d\theta}{dt} \vec{k}$$

Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$= \underbrace{\frac{d\vec{\omega}}{dt} \times \vec{r}}_{\text{tangential}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})}_{\text{radial (normal)}}$$



$\circlearrowleft \vec{\omega} = \omega \vec{k}$

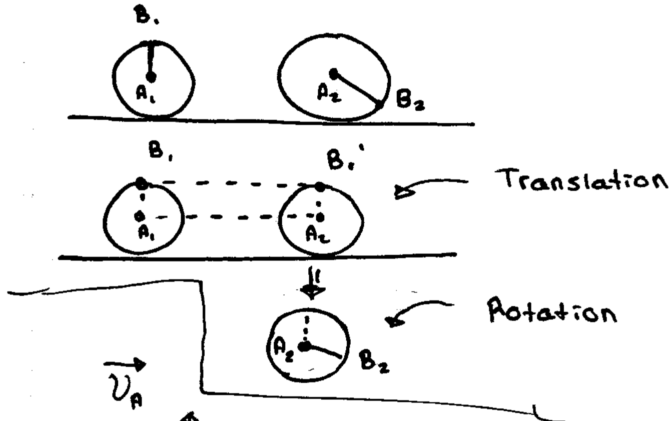
$\circlearrowleft \alpha = \dot{\omega} \vec{k}$

$\vec{a} = \vec{a}_t + \vec{a}_n$

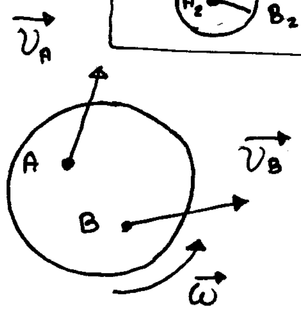
$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \vec{k}$$

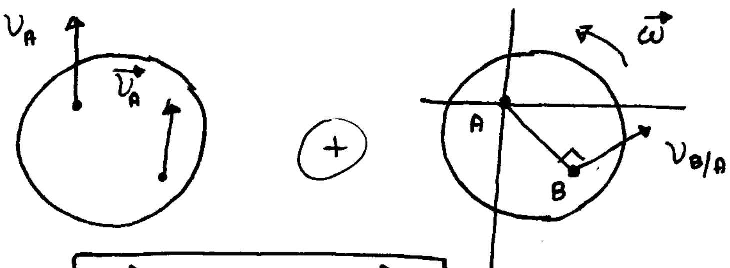
$$\begin{cases} a_t = r\alpha \\ a_n = r\omega^2 \end{cases}$$

General plane motion



General plane motion is the sum of a translation and a rotation (in general)





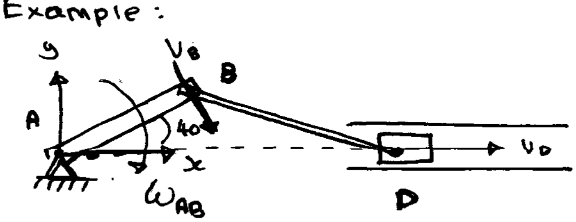
$$\rightarrow \boxed{\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}}$$

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{AB} = \vec{\omega} \times \vec{r}_{B/A}$$

$$v_{B/A} = r_{B/A}\,\omega$$

$$\Rightarrow \boxed{\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}}$$

Example :



$\omega_{AB} = 2000$ rpm

$AB = 3$ in

$BD = 8$ in

1) Find the angular velocity of the bar BD

2) Find the velocity of the piston D.

Solution : $\triangle ABD$

$$\frac{AB}{\sin\beta} = \frac{BD}{\sin 40°} \Rightarrow \sin\beta = \frac{AB}{BD}(\sin 40°) \Rightarrow \beta = 13.95°$$

$$\vec{V_B} = \vec{\omega} \times \vec{r_{B/A}}$$

$$\vec{\omega_{AB}} = 2000 \text{ rpm} = 2000 \times \frac{2\pi \text{ radians}}{60 \text{ sec}} \rightarrow 209.4 \text{ rad/s}$$

$$\vec{\omega_{AB}} = -209.4\ \vec{k}$$

$$\vec{r_{B/A}} = AB\cos(40°)\,\vec{i} + AB\sin(40°)\,\vec{j}$$

i j k i j

$$\therefore \vec{V_B} = \vec{\omega_{AB}} \times \vec{r_{B/A}}$$

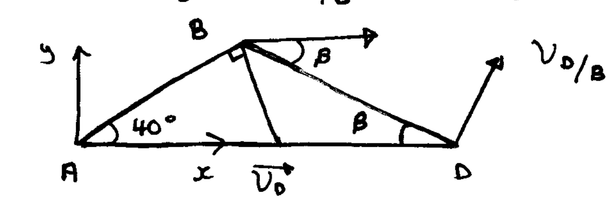
$$= -209.4\,\vec{k} \times (3\cos(40°)\,\vec{i} + 3\sin(40°)\,\vec{j})$$

$$= -209.4 \times 3\cos(40°)\,\vec{j} + 209.4 \times 3\sin(40°)\,\vec{i}$$

$$\vec{V_D} = \vec{V_{D\,i}}$$

rigid bar $\overline{BD}$

$$\vec{V_D} = \vec{V_B} + \vec{V_{D/B}} = \vec{V_B} + \vec{\omega_{BD}} \times \vec{r_{D/B}}$$



$$\vec{r_{D/B}} = BD\cos\beta\,\vec{i} - BD\sin\beta\,\vec{j}$$

$$\vec{\omega_{BD}} = \omega_{BD}\,\vec{k}$$

$$V_D\,\vec{i} = -209.4 \times 3\cos 40°\,\vec{j} + 209.4 \times 3\sin 40°\,\vec{i} \ \ldots$$

$$\ldots + \omega_{BD}\,\vec{k} \times (8\cos 13.95\,\vec{i} - 8\sin 13.95°\,\vec{j})$$

$$= -209.4 \times 3\cos 40°\,\vec{j} + 209.4 \times 3\sin 40°\,\vec{i} \ \ldots$$

$$\ldots + 8\,\omega_{BD}\cos 13.95°\,\vec{j} + 8\,\omega_{BD}\sin 13.95°\,\vec{i}$$

$$\Rightarrow \begin{cases} V_D = 209.4 \times 3\sin 40° + 8\,\omega_{BD}\sin 13.95° \\ 0 = -209.4 \times 3\cos 40° + 8\,\omega_{BD}\cos 13.95° \end{cases}$$

$$\Rightarrow \begin{cases} \omega_{BD} = \dfrac{209.4 \times 3\cos 40^\circ}{8\cos 13.95^\circ} = 61.98 \text{ rad/s} \\ V_D = 523.41 \text{ in/s} \end{cases}$$