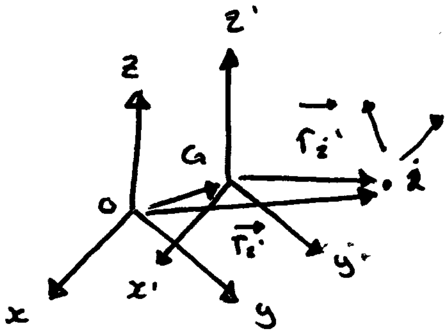
OCT. 24/17

DYNAMICS II



$$\vec r_i = \vec r_G + \vec r_i'$$

$$\vec v_i = \vec v_G + \vec v_i'$$

$$\vec H_G' = \sum \vec r_i' \times m_i \vec v_i'$$

$$= \sum \vec r_i' \times (m_i \vec v_i - m_i \vec v_G)$$

$$\Rightarrow \vec H_G' = \sum \vec r_i' \times$$

$$= \sum \vec r_i' \times m_i v_i - (\sum m_i r_i') \times \vec v_G$$

$$\boxed{H_G = \sum \vec r_i' \times m_i \vec v_i}$$

$$d/dx\ \vec H_G' = d/dt \sum \vec r_i' \times m_i \vec v_i$$

$$= \sum \dot{\vec r}_i' \times m_i \vec v_i + \sum \vec r_i' \times m_i \dot{\vec v}_i$$

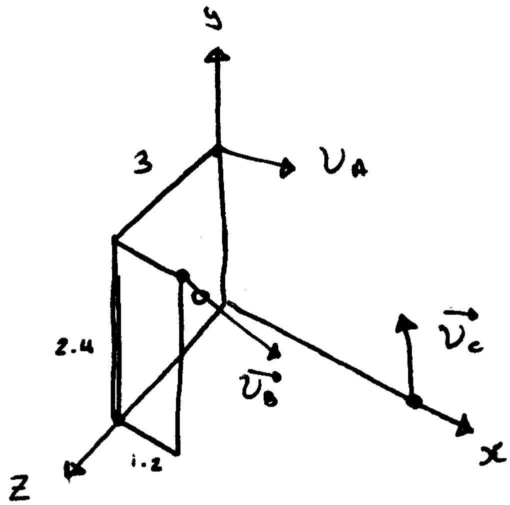
$$\boxed{\dot{\vec H}_G = \sum \dot{\vec M}_G}$$

14.6 Conservation of momentum

If no external forces act on the particles of the system, then the linear momentum and angular momentum about the fixed point are conserved.

$$\sum \vec F = \frac{d\vec L}{dt} \Rightarrow \boxed{\vec L = \text{const}}$$

$$\sum \vec m = \frac{d\vec H}{dt} = \boxed{\vec H = \text{const}}$$



$m_A = 3kg$

$m_B = 2kg$

$m_C = 9kg$

$\vec{V_A} = 4i + 2j + 2k$

$V_B = 4 \ldots + 3 \ldots$

...

$3 + 2 + 4 = 9$

Find H_G: Solution

$m = m_A + m_B + m_C$

$$r_G = \frac{m r_A + m r_B + m r_C}{m}$$

$$= (3)(3j) + (2)(1.2j + 2.4i + 3k) + (9) \ldots$$

$$= \ldots$$

$\vec{r_A}' = \vec{r_A} - \vec{r_G}$, $\vec{r_B}' = \vec{r_B} - \vec{r_G}$, $\vec{r_c}' = \vec{r_c} - \vec{r_G}$

$\vec{H_G}' = \sum \vec{r_i}' \times m_i \vec{v_i}$

$$\left(\vec{v_G} = \frac{m_A v_A + m_B v_B + m_c v_c}{m} \right)$$

$\vec{v_G} = 1.3333\vec{i} + 3.1111\vec{j} + 1.5556\vec{k}$

$$H_G' = \vec{r_A}' \times m_A (\vec{v_A} - \vec{v_G}) + \vec{r_B}' \times m_B (\vec{v_B} - \vec{v_G}) + \vec{r_c}' \times m_c (\vec{v_c} - \vec{v_G})$$

$$= -28\vec{i} + 13.333\vec{j} - 24.267\vec{k}$$

$\vec{H_G}' = \sum \vec{r_i}' \times m_i \vec{v_i}$

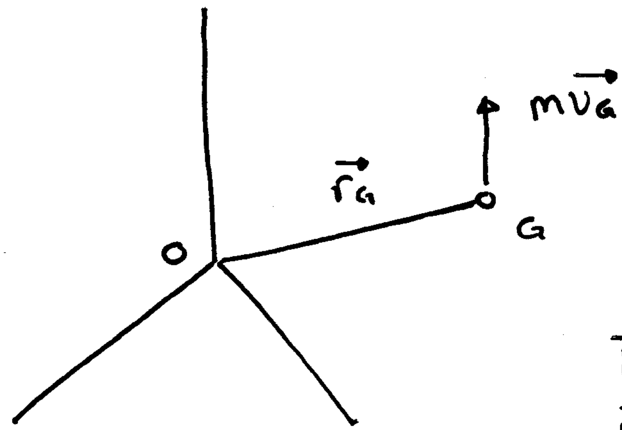
$$= \vec{r_A}' \times m_A \vec{v_A} + \vec{r_B} \times m_B \vec{v_B} + \vec{r_c} \times m_c \vec{v_B}$$

$$= 12.8\vec{i} + 3.20\vec{j} - 28.8\vec{k}$$

$$- 14\vec{i} + 18.6667\vec{j} - 10.9333\vec{k}$$

$$- 1.6\vec{j} - 8.5333j + 15.4667\vec{k}$$

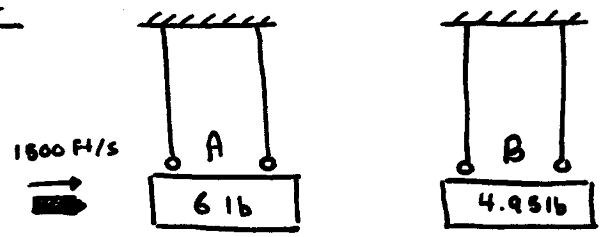
$$= -2.8 i + 13.333j - 24.267\vec{k}$$



$$\vec{H}_o = -4.8\vec{j} + 9.6\vec{k}$$

$$\vec{H}_o = \vec{H}_G + \vec{r}_G \times m\vec{v}_G$$

Example



Blocks A and B start moving with velocities of 5 ft/s and 9 ft/s.

a) Find the weight of the bullet

b) Find the velocity of the bullet when it travels from A to B

Solution: The bullet, A, B as a system, there is no external horizontal forces, conservation of the linear momentum of the system in the horizontal direction.

before impact: $v_0 = 1500$ ft/s

$$v_A = v_B = 0$$

$$L_o = m v_0 + m_A v_A + m_B v_B = 1500 m$$

After the bullet embedded in block B:

$v_A' = 5$ ft/s $\qquad v_B' = v_0' = 9$ ft/s

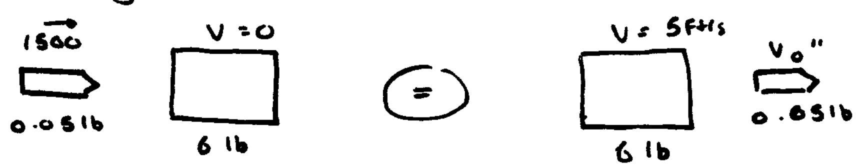
$$L_0' = m v_0' + m_A v_A' + m_B v_B'$$

$$L_0' = (m)(9) + (6/32.2)(5) + (4.95/32.2)(9) = 1500$$

$$L_0 = L_0' \qquad \hookrightarrow W = mg = 0.05 \text{ lb}$$

b) The bullet and block A as a system,

conservation of linear momentum of the system.



$$\left(\frac{0.05}{32.2}\right)(1500) + 0 = \left(\frac{6}{32.2}\right)(5) + \left(\frac{0.05}{32.2}\right) v_0''$$

$$v_0'' = 900 \text{ ft/s}$$

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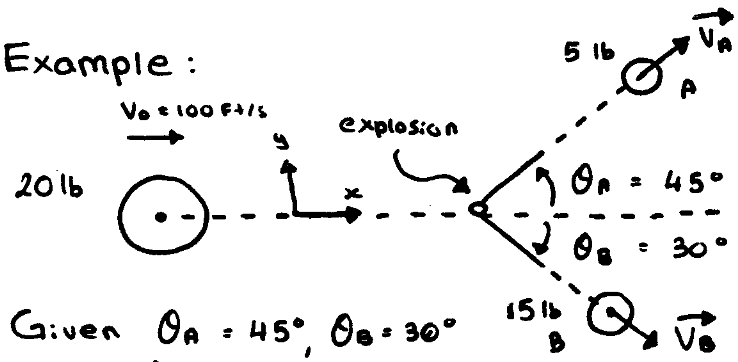
if $\Delta t = 0.005$ s $\qquad \vec{F}$?

using Bullet and B

Example :



Given $\theta_A = 45°$, $\theta_B = 30°$

Find $\vec{v_A}$ and $\vec{v_B}$

Solution : $(m_A + m_B)\vec{v_0} = m_A\vec{v_A} + m_B\vec{v_B}$

x : $(m_A + m_B)v_0 = m_A v_A \cos\theta_A + m_B v_B \cos\theta_B$

y : $0 = m_A v_A \sin\theta_A + m_B v_B \sin\theta_B$

$\Rightarrow$ $v_A = 207$ ft/s

$v_B = 97.6$ ft/s

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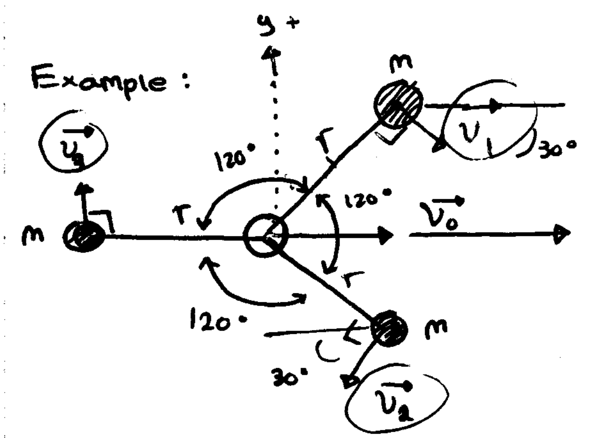
Dynamics II

Example:



$\rightsquigarrow \vec{v}_1 = \vec{v}_2 = \vec{v}_3$

a) the linear momentum of the system is in the positive x-direction ; correct, (if $\vec{v}_0 > 0$)

b) the angular momentum of the system is in the positive y-direction ; x - direction

c) the angular momentum of the system about G is zero ; x - $3mv_1r\,\hat{k}$

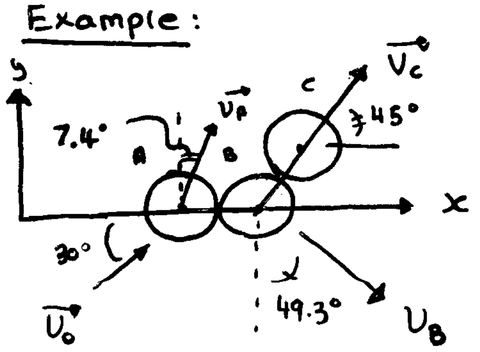
d) the linear momentum of the system is zero : False

$$m(\vec{v}_1 + \vec{v}_0) + m(\vec{v}_2 + \vec{v}_0) + m(\vec{v}_3 + \vec{v}_0) = \ldots$$

$$\ldots \; 3m\vec{v}_0 + \underbrace{m(\vec{v}_1 + \vec{v}_2 + \vec{v}_3)}_{\Sigma = 0} \stackrel{?}{\neq} 0$$

if $v_0 \neq 0$, this is not true

Example:



Given $\vec{v}_0$ = 12 ft/s

$\vec{v}_C$ = 6.29 ft/s

Find $\vec{v}_A$ and $\vec{v}_B$

Solution : No external impulsive forces in the horizontal direction

=> Conservation of linear momentum of the system.

$$\Rightarrow \quad m\vec{v_0} + \varnothing + \varnothing = m\vec{v_A} + m\vec{v_B} + m\vec{v_C}$$

$$x: \quad \cancel{m}\vec{v_0}\cos(30°) = \cancel{m}\vec{v_A}\sin(7.4°) + \cancel{m}\vec{v_B}(\sin 49.3°) + \cancel{m}\vec{v_C}(\cos 45°)$$

$$y: \quad \cancel{m}\vec{v_0}\sin(30°) = \cancel{m}\vec{v_A}\cos(7.4°) + \cancel{m}\vec{v_B}(\cos 49.3°) + \cancel{m}\vec{v_C}(\sin 45°)$$

Mass is constant : $v_A = 6.\varnothing5$ ft/s

(all have same mass) $v_B = 6.81$ ft/s

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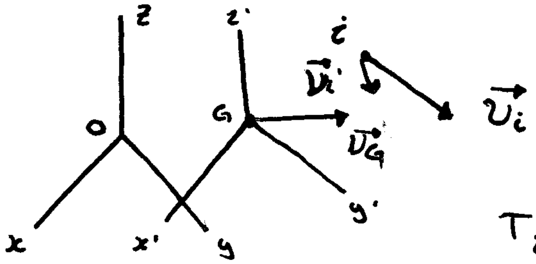
DYNAMICS II

14.7 Kinetic Energy



$$T_i = \frac{1}{2} m_i v_i^2$$
$$= \frac{1}{2} m_i \vec{v}_i \cdot \vec{v}_i$$
$$\vec{v}_i = \vec{v}_i' + \vec{v}_G$$
$$T_i = \frac{1}{2} m_i (\vec{v}_i' + \vec{v}_G) \cdot (\vec{v}_i' + \vec{v}_G')$$
$$\therefore T = \sum_{i=1}^{n} T_i = \sum \frac{1}{2} m_i \vec{v}_i' \cdot \vec{v}_i' + \sum \frac{1}{2} m_i \vec{v}_G \cdot \vec{v}_G + \sum m_i \vec{v}_i' \cdot \vec{v}_G$$

$$= \frac{1}{2} \sum m_i v_i'^2 + \frac{1}{2} \left(\sum m_i\right) v_G^2 + \left(\cancel{\sum m_i v_i'}\right)^0 \cdot v_G$$

$$\boxed{T = \frac{1}{2} \sum m_i v_i'^2 + \frac{1}{2} m v_G^2}$$

$$\sum \vec{F}_i = \dot{\vec{L}} \qquad \sum \vec{M}_O = \dot{\vec{H}}_O$$

14.8 Work-Energy Principle

Particle $\underline{\underline{i}}$:

$$T_1^{(i)} + \sum U_{1\to 2} = T_2^{(i)}$$

System :

$$\sum T_1^{(i)} + \sum U_{1\to 2}^{(i)} = \sum T_2^{(i)}$$ general case

$$\Rightarrow T_1 + U_{1\to 2} = T_2$$ (work energy principle)

If the forces acting on the particles are conservative :

$$T_1 + V_1 = T_2 + V_2$$ (conservation of energy)

special case

14.9 Principle of Impulse and Momentum

$$\sum \vec{F}_i = \dot{\vec{L}}$$
$$\int_{t_1}^{t_2} \sum \vec{F}_i \, dt = \int_{t_1}^{t_2} \dot{\vec{L}} \, dt = \vec{L}_2 - \vec{L}_1$$

impulse

$$\Rightarrow \boxed{\vec{L}_1 + \Sigma \int_{t_1}^{t_2} \vec{F}_i \, dt = \vec{L}_2}$$

$$\Sigma \vec{M}_o = \dot{\vec{H}}_o$$

$$\int_{t_1}^{t_2} \Sigma \vec{M}_o \, dt = \int_{t_1}^{t_2} \dot{\vec{H}}_o \, dt = \vec{H}_2 - \vec{H}_1$$

$$\Rightarrow \boxed{\vec{H}_1 + \Sigma \int_{t_1}^{t_2} \vec{M}_o \, dt = \vec{H}_2}$$

Example:

$W_A = 25 \text{ lb}$

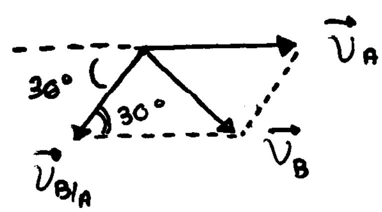
$W_B = 15 \text{ lb}$

Determine : a) the velocity of B relative to A after it has slid 3 ft down the machined surface of the wedge from rest.

b) the corresponding velocity of A

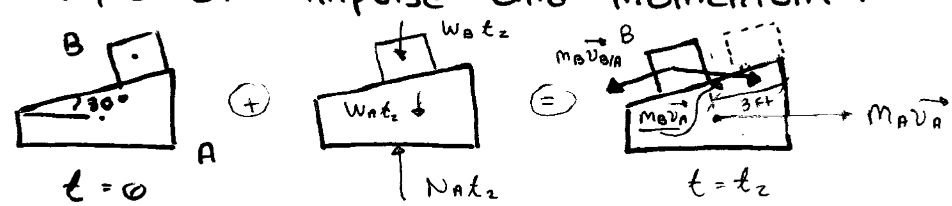
Solution : Relative velocity analysis :

$$\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$



$$v_B^2 = v_A^2 + v_{B/A}^2 - 2 v_A v_{B/A} \cos(30°) \quad \text{— (1)}$$

Principle of impulse and momentum :

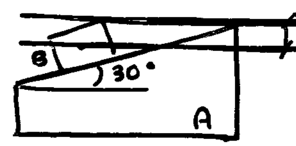


$$x: \quad 0 + 0 = m_A v_A + m_B v_B - m_B v_{B/A} \cos 30° \quad (2)$$

$$\Rightarrow v_A = \frac{m_B}{m_A + m_B} v_{B/A} \cos(30°) = 0.32476\, v_{B/A} \quad (3)$$

$$(3) \rightarrow (1): \quad v_B^2 = 0.54297\, v_{B/A}^2 \quad \text{—} \quad (4)$$

Conservation of Energy :



$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = 0 \,, \quad V_1 = 0$$

$$T_2 = \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2$$

$$V_2 = -W_B d \sin 30°$$

$$0 + 0 = \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 - W_B d \sin 30°$$

$(3)(4) \rightarrow (5)$:

$$(\tfrac{1}{2})(\tfrac{25}{32.2})(0.32476\, v_{B/A})^2 + (\tfrac{1}{2})(\tfrac{15}{32.2})(0.5429\, v_{B/A}^2)$$
$$= 15(3) \sin 30°$$

$$\Rightarrow v_{B/A} = 11.59 \text{ ft/s}$$

$$\therefore \vec{v}_{B/A} = 11.59 \text{ ft/s} \text{ " } 30° \text{ "}$$

$$(3): \quad v_A = 0.32476\, v_{B/A} = 3.76 \text{ ft/s}$$

$$\therefore \vec{v}_A = 3.76 \text{ ft/s} \text{ " } \longrightarrow \text{ "}$$