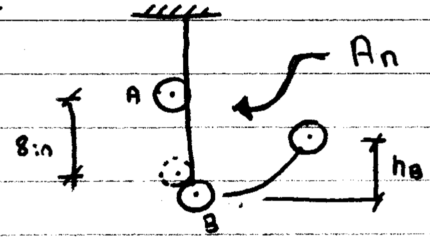
OCT. 18/17

DYNAMICS

EXAMPLE :




$e = 0.9$

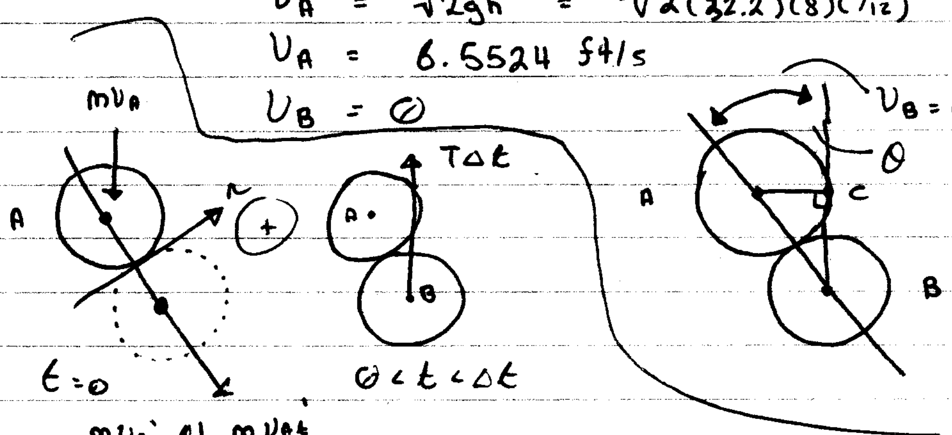
Find the resulting max. vertical distance of B.

Solution: Just before the impact

$v_A = \sqrt{2gh} = \sqrt{2(32.2)(8)(1/12)}$

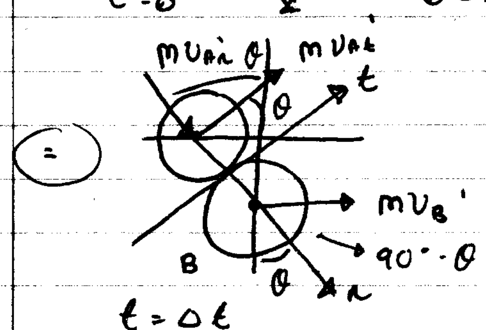
$v_A = 6.5524$ ft/s

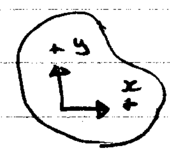
$v_B = 0$




$\sin\theta$ - AB - 2r

$\theta = 30°$




Conservation of the linear momentum in the x-direction :

$$0 + 0 = mv_B' + mv_{At}' \cos\theta + mv_{An}' \sin\theta \quad (1)$$

A : In the tangential direction :

$$-mv_A \sin\theta = mv_{At}' \quad (2)$$

A and B :

$$e = -\left(\frac{v_{An}' - v_{Bn}'}{v_{An} - v_{Bn}}\right)$$

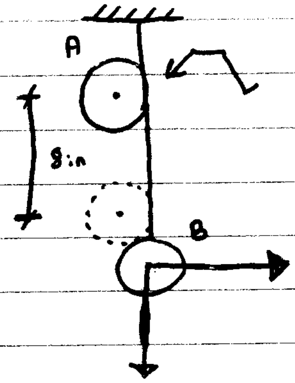
$$0.9 = \frac{v_{An}' - v_B' \cos(90° - \theta)}{v_A \cos\theta - 0} \quad (3)$$

$v_B' = 4.3127$ ft/s

$v_{An}' = -2.9508$ ft/s ; $v_{At}' = -3.2762$ ft/s

$h_B = 0.2888$ ft

Work energy principle

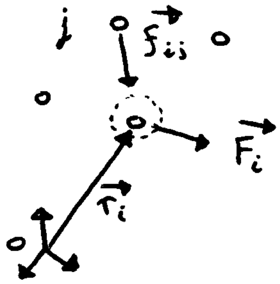



$W_A = W_B = 2\ \text{lb}$

$k = 10\ \text{lb/in}$

$e = 0.9$

## 14.2 APPLICATION OF NEWTON'S LAW




Particle $i$ :

$$\vec{F}_i + \vec{f}_{i1} + \vec{f}_{i2} + \ldots + \vec{f}_{in} = m_i\vec{a}_i$$

$$\vec{f}_{ii} = \emptyset$$

$$\vec{F}_i + \sum_{j=1}^{n}\vec{f}_{ij} = m_i\vec{a}_i \quad \text{—— } 1$$

$$\vec{r}_i \times \vec{F}_i + \vec{r}_i \times \sum_{j=1}^{n}\vec{f}_{ij} = \vec{r}_i \times m_i\vec{a}_i$$

$$\Rightarrow \vec{r}_i \times \vec{F}_i + \sum_i(\vec{r}_i \times \vec{f}_{ij}) = \vec{r}_i \times m_i\vec{a}_i \quad 2$$

$m_i\vec{a}_i$ : effective Force

Summing over all the equations

$$\sum_{i=1}^{n}\vec{F}_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\vec{f}_{ij} = \sum_{i=1}^{n} m_i a_i \quad \text{—— } 3$$

$$\vec{f}_{12} + \vec{f}_{21} = \vec{f}_{34} + \vec{f}_{43} = \emptyset$$

$$\Rightarrow \boxed{\sum_{i=1}^{n}\vec{F}_i = \sum_{i=1}^{n} m_i\vec{a}_i}$$

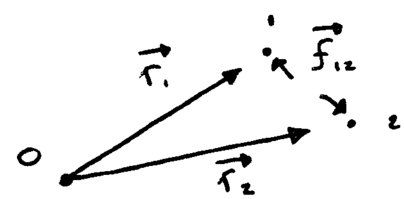
$$\sum_{i=1}^{n}\vec{r}_i \times \vec{F}_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\vec{r}_i \times \vec{f}_{ij} = \sum_{i=1}^{n}\vec{r}_i \times m_i\vec{a}_i$$

$$\vec{r}_1 \times \vec{f}_{12} + \vec{r}_2 \times \vec{f}_{21}$$

$$= \vec{r}_1 \times \vec{f}_{12} - \vec{r}_2 \times \vec{f}_{12}$$

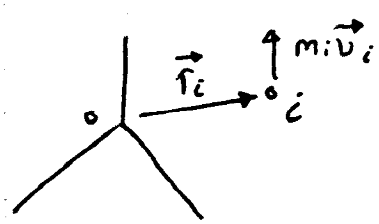
$$= (\vec{r}_1 - \vec{r}_2) \times \vec{f}_{12}$$

$$= \emptyset$$




$$\Rightarrow \boxed{\sum_{i=1}^{n}\vec{r}_i \times \vec{F}_i = \sum_{i=1}^{n}\vec{r}_i \times m_i\vec{a}_i}$$

## 14.3 Linear and Angular Momentum




linear momentum

$$\vec{L}_i = m_i\vec{v}_i$$

Linear momentum of the system

$$L = \sum_{i=1}^{n}\vec{L}_i = \sum_{i=1}^{n} m_i\vec{v}_i$$

Angular momentum about the fixed point O

$$\vec{H}_{i,O} = \vec{r}_i \times m_i\vec{v}_i = \vec{r}_i \times \vec{L}_i$$

For the System:

$$\vec{H}_o = \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{v}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{L}_i$$

Since: $\vec{L} = \sum_{i=1}^{n} m_i \vec{v}_i$

$$\Rightarrow \dot{\vec{L}} = \sum_{i=1}^{n} m_i \dot{\vec{v}}_i = \sum_{i=1}^{n} m_i \vec{a}_i = \sum_{i=1}^{n} \vec{F}_i$$

$$\boxed{\Rightarrow \sum \vec{F}_i = \dot{\vec{L}}}$$

$$\dot{\vec{H}}_o = \frac{d}{dt}\left( \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{v}_i \right)$$

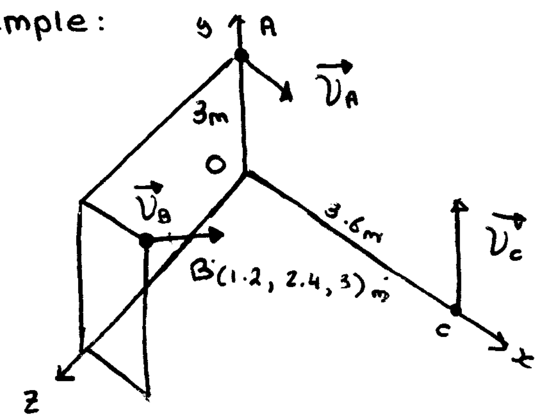
$$= \sum_{i=1}^{n} \left( \cancelto{0}{\dot{\vec{r}}_i \times m_i \vec{v}_i} + \vec{r}_i \times m \dot{\vec{v}}_i \right)$$

$$= \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{a}_i$$

$$= \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i$$

$$\boxed{\begin{aligned} \sum \vec{r}_i \times \vec{F}_i &= \dot{\vec{H}}_o \\ \sum \vec{M}_o &= \dot{\vec{H}}_o \end{aligned}}$$

Example:




$m_A = 3$ kg

$m_B = 2$ kg

$m_c = 4$ kg

$\vec{v}_A = 4\vec{i} + 2\vec{j} + 2\vec{k}$

$\vec{v}_B = 4\vec{i} + 3\vec{j}$

$\vec{v}_c = -2\vec{i} + 4\vec{j} + 2\vec{k}$

Determine $\vec{L}$ and $\vec{H}_o$

Solution

$$\vec{L} = m_A \vec{v}_A + m_B \vec{v}_B + m_c \vec{v}_c$$

$$= 3(4\vec{i} + 2\vec{j} + 2\vec{k}) + 2(4\vec{i} + 3\vec{j}) + 4(-2\vec{i} + 4\vec{j} + 2\vec{k})$$

$$= 12\vec{i} + 28\vec{j} + 14\vec{k}$$

$$\vec{H}_o = \vec{H}_{A,O} + \vec{H}_{B,O} + \vec{H}_{C,O}$$

$$= \vec{r}_A \times m_A \vec{v}_A + \vec{r}_B \times m_B \vec{v}_B + \vec{r}_c \times m_c \vec{v}_c$$

Here, $\vec{r}_A = 3\vec{j}$, $\vec{r}_B = 1.2\vec{i} + 2.4\vec{j} + 3\vec{k}$, $\vec{r}_c = 3.6\vec{i}$

$$\vec{H}_{A;O} = \vec{r}_A \times m_A \vec{v}_A$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 0 \\ 12 & 6 & 6 \end{vmatrix} = 18\vec{i} - 36\vec{k}$$

$$\vec{H}_{B;O} = \vec{r}_B \times m_B \vec{v}_B$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.2 & 2.4 & 3 \\ 8 & 6 & 0 \end{vmatrix} = -18\vec{i} + 24\vec{j} - 12\vec{k}$$

$$\vec{H}_{C;O} = -28.8\vec{j} + 57.6\vec{k}$$

$$\therefore \vec{H}_O = \dots = -4.8\vec{j} + 9.6\vec{k}$$

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$$\vec{H}_{A;O} = \vec{r}_A \times m_A \vec{v}_A$$

(Another method for cross-product)

$$= 3\vec{j} \times (12\vec{i} + 6\vec{j} + 6\vec{k})$$

$$= 36\vec{j} \times \vec{i} + \cancel{18\vec{j} \times \vec{j}}^{\,0} + 18\vec{j} \times \vec{k}$$

$$\boxed{\vec{i} \times \vec{j} = \vec{k} \qquad \vec{j} \times \vec{k} = \vec{i} \qquad \vec{k} \times \vec{j} = \vec{i}}$$

$i \; j \; k \; i \; j$ $\qquad \Rightarrow 18\vec{i} - 36\vec{k}$

## 14.4 Motion of a mass centre of a system of Particles

Mass Centre :

$$\vec{r}_G = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}$$


Define $\quad m = \sum_{i=1}^{n} m_i$

$$\boxed{\vec{r}_G = 1/m \sum_{i=1}^{n} m_i \vec{r}_i = \sum_{i=1}^{n} \left(\frac{m_i}{m}\right) \vec{r}_i}$$

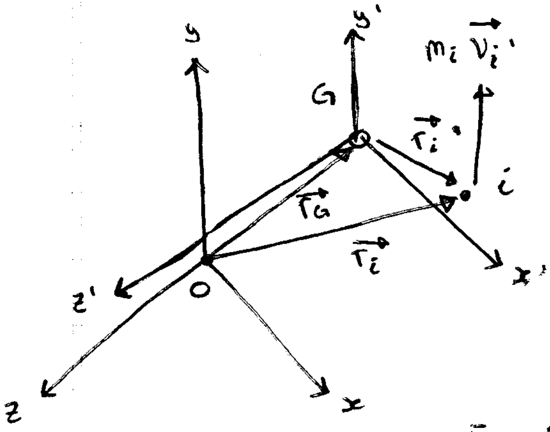
$$\Rightarrow m\vec{r}_G = \sum_{i=1}^{n} m_i \vec{r}_i$$

$$\Rightarrow m\vec{v}_G = \sum_{i=1}^{n} m_i \vec{v}_i = \vec{L}$$

$$\boxed{\vec{L} = m\vec{v}_G}$$

$$\Rightarrow \dot{\vec{L}} = m\vec{a}_G = \sum_{i=1}^{n} \vec{F}_i$$

## 14.5 Angular momentum about the mass centre




$$\vec{H}_G' = \sum_{i=1}^{n} \left(\vec{r}_i' \times m_i \vec{v}_i'\right)$$

$$\dot{\vec{H}}_G' = \sum_{i=1}^{n} \left(\dot{\vec{r}}_i' \times m_i \vec{v}_i' + \vec{r}_i' \times m_i \dot{\vec{v}}_i'\right)$$

$$\vec{r}_i = \vec{r}_G + \vec{r}_i' \qquad = \sum_{i=1}^{n} \vec{r}_i' \times m_i \vec{a}_i'$$

$$\Rightarrow \vec{a}_i = \vec{a}_G + \vec{a}_i' \qquad = \sum_{i=1}^{n} \vec{r}_i' \times m_i \left(\vec{a}_i - \vec{a}_G\right)$$

$$= \sum_{i=1}^{n} \vec{r}_i' \times m_i \vec{a}_i - \left(\sum_{i=1}^{n} m_i \vec{r}_i'\right) \times \vec{a}_G$$

$$= \sum_{i=1}^{n} \vec{r}_i' \times m_i \vec{a}_i = \sum_{i=1}^{n} \vec{r}_i' \times \vec{F}_i$$

$$\boxed{\dot{\vec{H}}_G' = \sum \vec{M}_G}$$