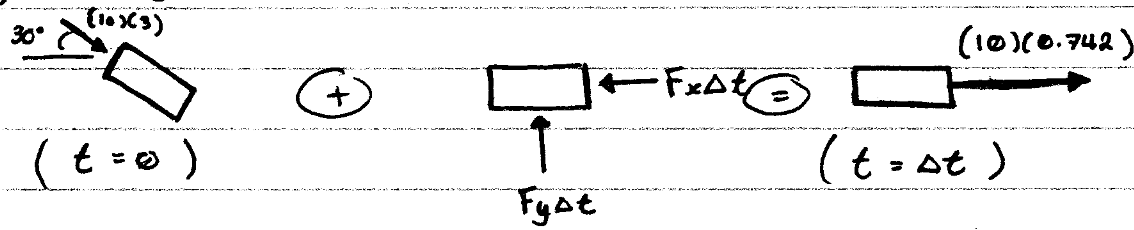
Oct. 3/17
DYNAMICS II

From previous problem:

x: $(10)(3)\cos(30°) = (10 + 24)v_2$

$v_2 = 0.742$ m/s

b) Package



$(0 < t < \Delta t)$

x: $(10)(3)\cos 30° - F_x\Delta t = (10)(0.742)$

$F_x\Delta t = 18.56$ N·s

y: $-(10)(3)\sin 30° + F_y\Delta t = 0$

$F_y\Delta t = 15$ N·s

$\therefore \vec{F}\Delta t = -18.56\,\vec{i} + 15\,\vec{j}$ N·s

$\therefore F\Delta t = \sqrt{(18.56)^2 + (15)^2} = 23.9$ N·s

(only considering impulsive forces)

c) $\Delta T = \frac{1}{2}(10)(3^2) - \frac{1}{2}(10+24)(0.742^2) = 45 - 9.63$

$\frac{\Delta T}{T_1} = \frac{45 - 9.63}{45} = 78.6\,\%$

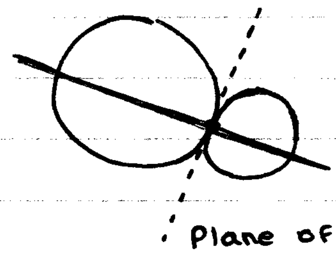
13.12 Impact

Impact (collision between two bodies) is an event, that usually occurs in a <u>very brief</u> interval of time.

<u>Impulsive Force</u>

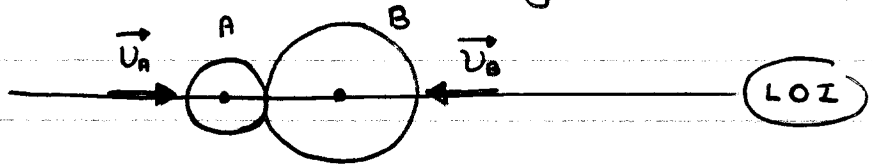
<u>deformation</u> (elastic, plastic)

Energy (mechanical) converted to Sound, heat

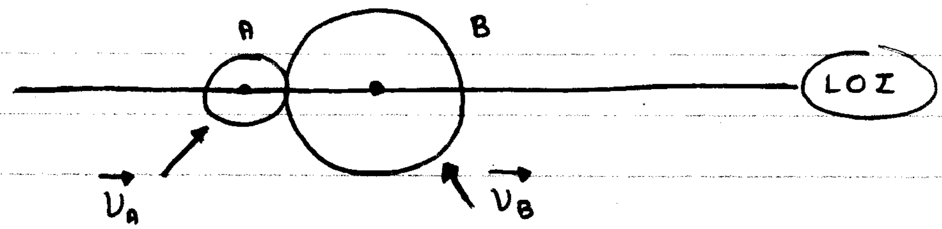


Line of impact : normal to the contacting surface, at the point of impact.

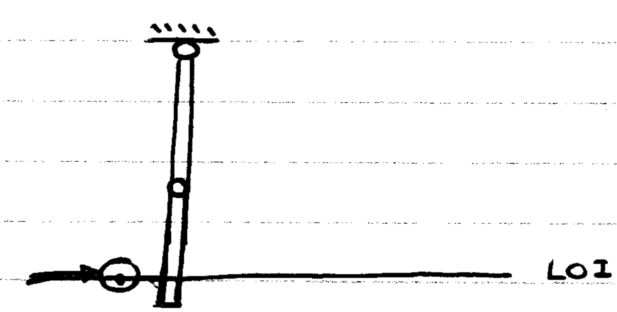
$\boxed{\text{Central impact :}}$ the mass center of both body are on the line of impact ; 2) the initial velocities of the bodies are along the line of impact.



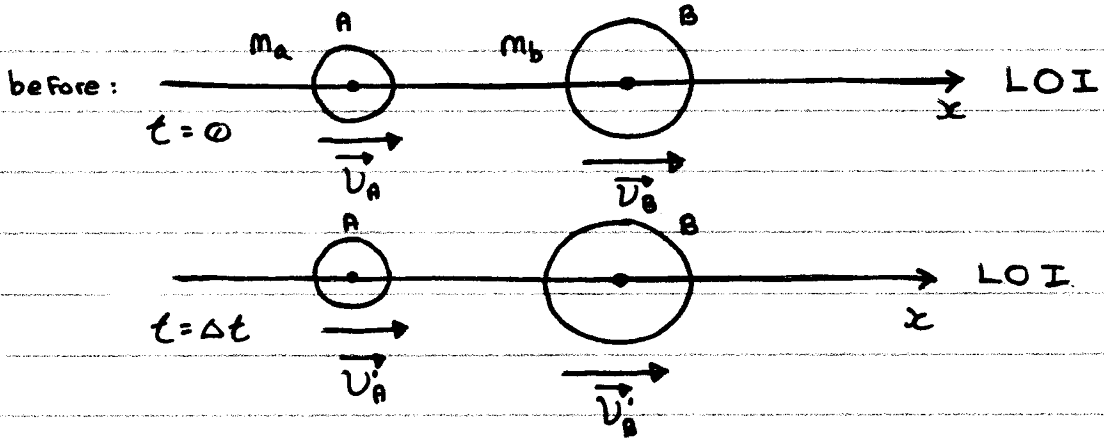
$\boxed{\text{Oblique Impact :}}$ 1) the mass center of both bodies are on the line of impact (LOI); 2) the initial velocities of the impacting bodies are not along the LOI.



Eccentric Impact : the mass center of one or both bodies does not lie on the LOI.



13.13 Direct Central Impact



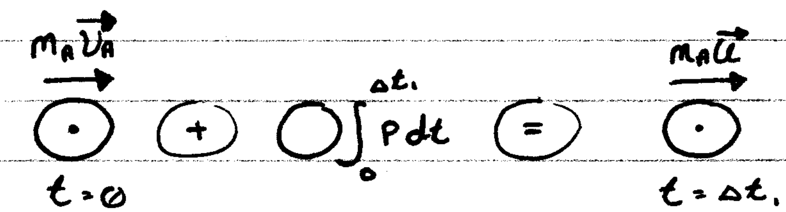
$$(0, \Delta t) \longrightarrow (0, \Delta t_1) + (\Delta t_1, \Delta t)$$

At $t = \Delta t_1$, max deformation, both particles have the same velocity $\vec{u}$.

<u>Particle A</u> :

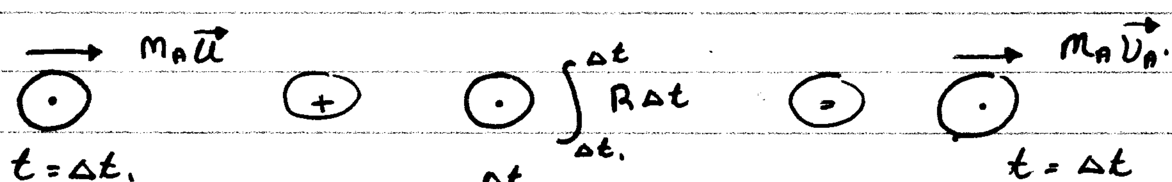
$0 \le t \le \Delta t_1$

deformation



$$m_A v_A - \int_0^{\Delta t_1} P dt = m_A u$$

$\Delta t_1 \le t \le \Delta t$



$$m_A u - \int_{\Delta t_1}^{\Delta t} R dt = m_A v_A'$$

The coefficient of restitation, e, :

$$e = \frac{\int_{\Delta t_1}^{\Delta t} R dt}{\int_0^{\Delta t_1} P dt} \quad ; \quad 0 \le e \le 1 \quad \Rightarrow \quad \boxed{e = \frac{u - v_A'}{v_A - u}}$$

Particle B

$$\boxed{e = \frac{v_B' - u}{u - v_B}}$$

$$\Rightarrow \boxed{e = \frac{v_B' - v_A'}{v_A - v_B}} = \boxed{- \frac{v_A' - v_B'}{v_A - v_B}}$$

Conservation of Linear Momentum :

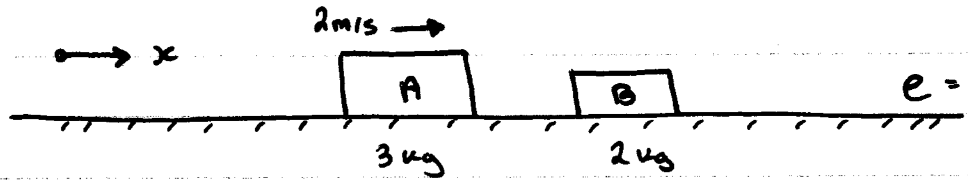
$$\boxed{m_A v_A + m_B v_B = m_A v_A' + m_B v_B'}$$

$e = 1$: Elastic impact
Conservation of energy

$$\Delta t = \tfrac{1}{2} m_A v_A'^{\,2} + \tfrac{1}{2} m_B v_B'^{\,2} - \tfrac{1}{2} m_A v_A^2 - \tfrac{1}{2} m_B v_B^2 = 0$$

$e = 0$: $v_A' = v_B'$ Plastic impact

Example :



$e = 1$, $\mu_k = 0.3$

1° Find the velocity of each block after collision.

2° Find the distance between the blocks when they stop sliding.

Solution 1° $v_A = 2$; $v_B = 0$

After the collision, v_A', v_B' $(\Rightarrow)$

→ Conservation of linear momentum

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$(3)(2) + (2)(0) = 3 v_A' + 2 v_B' \quad (1)$$

→ Using the relation

$$e = -\frac{v_A' - v_B'}{v_A - v_B} = \frac{v_B' - v_A'}{v_A - v_B}$$

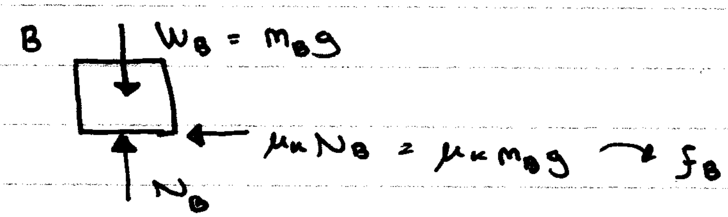
$$1 = -\left(\frac{v_A' - v_B'}{2 - 0}\right) \quad (2)$$

$\Rightarrow v_A' = 0.400$ m/s $\quad \Rightarrow v_B' = 2.40$ m/s

Solution 2°

Position 1 : $v_B' = 2.40$ m/s

Position 2 : $v_{B2} = 0$



$$T_1 + U_{1\to2} = T_2$$

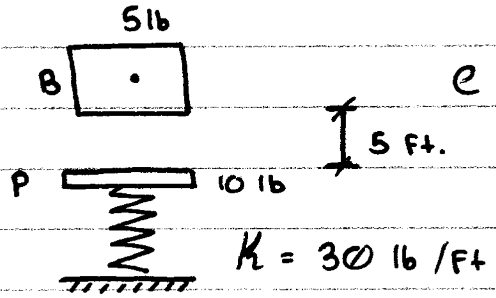
$$\tfrac{1}{2}\cancel{m_B} v_B'^2 + (-\mu_k \cancel{m_B} g\, d_B) = 0$$

$$\boxed{d_B = \frac{v_B'^2}{2\mu_k g}}$$

$$d_A = \frac{v_A'^2}{2\mu_k g} = \frac{0.400^2}{2 \times 0.3 \times 9.81} = 0.0272$$

$$d = d_B - d_A = 0.8786 - 0.0272 = \underline{0.851 \text{ m}}$$

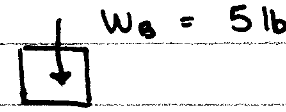
Example :



$e = 0.6$

Find the max. compression imparted to the spring.

Solution : 1° Free Falling

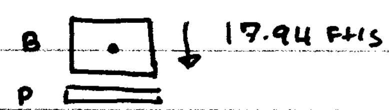


$$\underset{(mgh)}{0 + 5 \times 5} = \tfrac{1}{2}\left(5/32.2\right)(v_{B1})^2 + 0$$

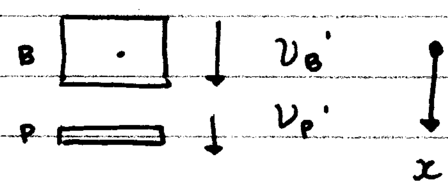
$$\downarrow$$

$$v_{B1} = 17.94 \text{ Ft/s}$$

2° Impact



$\Rightarrow$

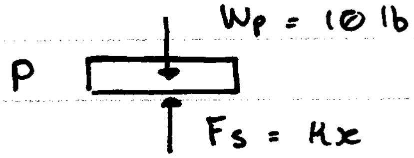


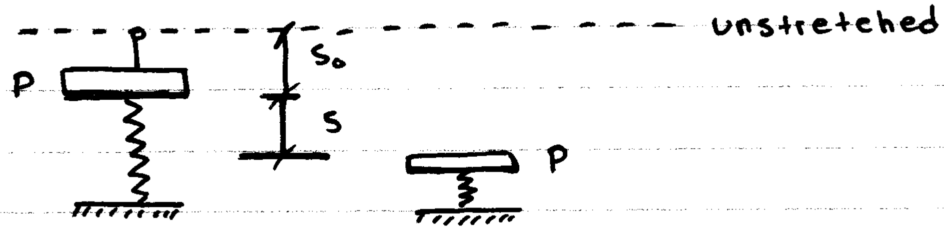
$$\left(\frac{5}{32.2}\right)(17.94) + 0 = \left(\frac{5}{32.2}\right)v_B' + \left(\frac{10}{32.2}\right)v_P'$$

$$e = 0.6 = -\left(\frac{v_B' - v_P'}{17.94 - 0}\right) \qquad \therefore\ v_B' = -1.196$$

$$v_P' = 9.568$$

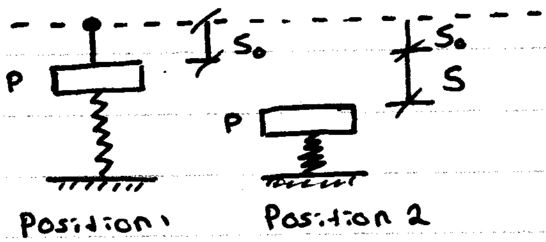
3° Plate + Spring

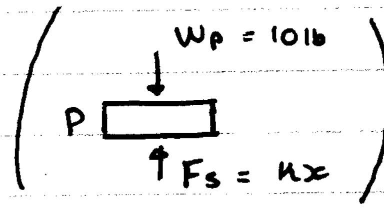




Oct. 5/17

DYNAMICS





$KS_o = mg \Rightarrow S_o = mg/k = 10/30 = 0.3333$ ft

$T_1 + V_1 = T_2 + V_2$

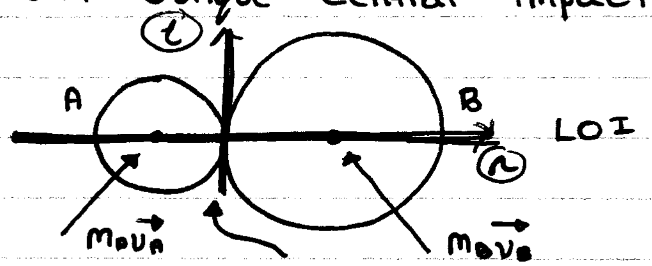
$\frac{1}{2}\left(\frac{10}{32.2}\right)(9.568)^2 + (-10)(0.3333) + \frac{1}{2}(30)(0.3333)^2 = \cdots$

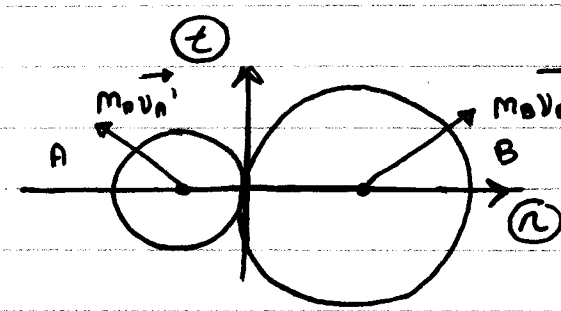
$\cdots \; 0 + (-10)(0.3333 + S) + \frac{1}{2}(30)(0.3333 + S)^2$

$\Rightarrow S = 0.9740$ ft

the max compression $= 0.3333 + 0.9740 = 1.31$ ft

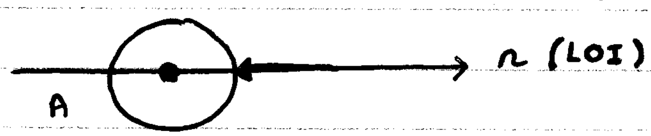
13.14 Oblique Central Impact





$$e = -\left(\frac{v_{A'n} - v_{B'n}}{v_{An} - v_{Bn}}\right)$$

$$m_A v_{An} + m_b v_{Bn} = m_A v_{A'n} + m_b v_{B'n}$$



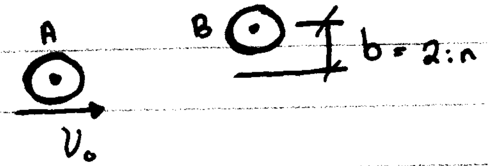
$m_A v_{At} = m_A v_{At}'$

$\Rightarrow v_{A't} = v_{At}$

So; $m_B v_{Bt} = m_B v_{B't}$

$\Rightarrow v_{B't} = v_{Bt}$

Example :

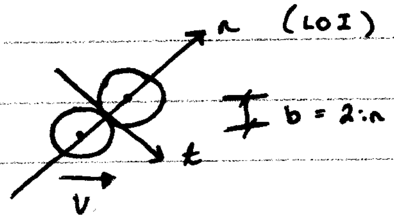


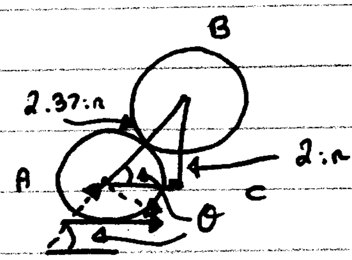
$d = 2.37$ in

$e = 0.7$

Find the velocity of each ball after impact.

Solution:





$\triangle ABC: \quad \sin\theta = \dfrac{BC}{AB} = \dfrac{2}{2.37} \Rightarrow \theta = 57.552$

$V_{An} = V_0 \cos\theta \qquad\qquad V_{At} = V_0 \sin\theta$

$V_{At}' = V_{At} = V_0 \sin\theta$

$V_{Bt}' = V_{Bt} = 0$

$$e = 0.7 = -\left(\frac{V_{An}' - V_{Bn}'}{V_{An} - V_{Bn}}\right) = -\left(\frac{V_{An}' - V_{Bn}'}{V_0\cos\theta - 0}\right)$$

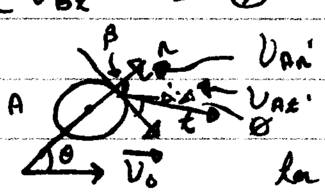
$m_A V_{An} + m_B V_{Bn} = m_A V_{A'n} + m_B V_{B'n}$

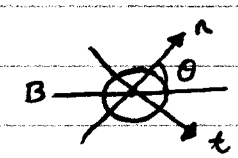
$m_A V_0 \cos\theta + 0 = m_A V_{A'n} + m_B V_{B'n}$

$V_0 \cos\theta = V_{A'n} + V_{B'n}$

$\Rightarrow \begin{cases} V_{A'n} = 0.08048\, V_0 \\ V_{B'n} = 0.45665\, V_0 \end{cases}$

$\Rightarrow \begin{cases} V_{At}' = V_0 \sin\theta = 0.84388\, V_0 \\ V_{Bt}' = 0 \end{cases}$





$\tan\beta = \dfrac{V_{A't}}{V_{A'n}}$

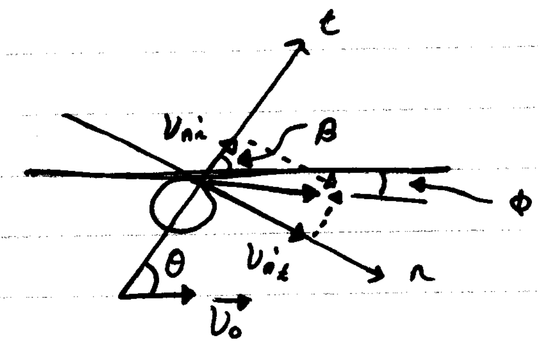
$\beta = 84.552°$

$$\tan\beta = \frac{v'_{At}}{v'_{An}} = \frac{0.84333\,v_0}{0.08048\,v_0}$$

$\beta = 84.552°$

$\phi = \beta - \theta$

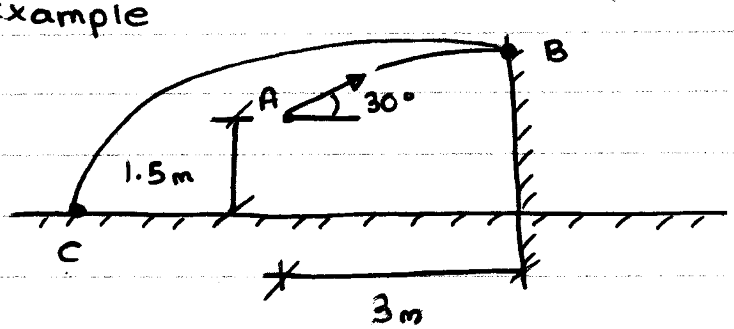
$\phi = 27.0°$



Ch. 13 – Midterm (covers assignments)

– Formula Sheet to be provided

Example



$v_A = 10\,m/s$

$m = 0.5\,kg$

$e = 0.5$

1) The velocity at which it strikes the wall at B

2) The velocity at which it rebounds from the wall.

3) The distance s from the wall to where it strikes the ground at C.

Solution: 1) Horizontal

$v_{Ax} = v_A \cos 30° = 10 \cos 30° = 8.66\,m/s$

$s = v_{Ax} t$

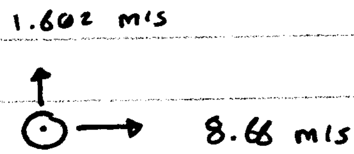
$t = \frac{s}{v_{Ax}} = \frac{3}{8.66} = 0.3464\,s$

$$v_{Bx} = v_{Ax} = 8.66 \text{ m/s}$$

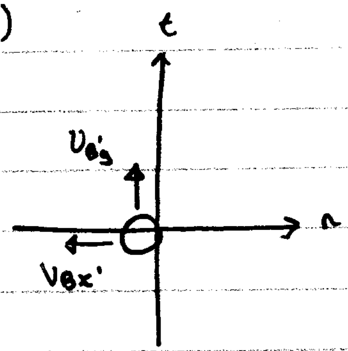
$$v_{By} = v_{Ay} - gt$$

$$= 10\sin 30^\circ - 9.81\,(0.3464)$$

$$= 1.602 \text{ m/s}$$



2)



$$v_{By}' = 1.602 \text{ m/s}$$

$$v_{Bx}' = ?$$

$$m_B(8.66) + 0 = m_B(-v_{Bx}') + (0 \cdot \infty)$$

↳ CAN'T USE

$$e = 0.5 = -\left(\frac{-v_{Bx}' - v_w'}{v_{Bx} - v_w}\right)$$

$$= \frac{v_{Bx}'}{v_{Bx}}$$

$$v_{Bx}' = e\,v_{Bx}$$

3) $S = 3.96$ m