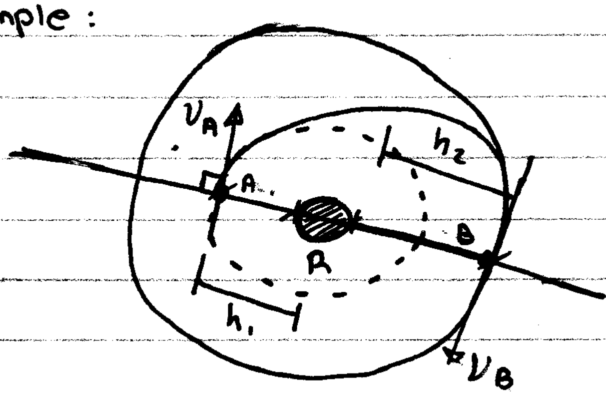
Sept. 26/17
Dynamics

Example :



$h_1 = 200$ mi
$h_2 = 500$ mi
$R = 6370$ km

Determine : a) the required increases in speed at A and B
b) the total energy per unit mass required to execute the transfer

Solution : $h_1 = 200\text{ mi} = 320\text{ km}$

$h_2 = 500\text{ mi} = 800\text{ km}$

$r_A = h_1 + R = 320 + 6370 = 6690\text{ km}$

$\Rightarrow 6690 \times 10^3\text{ m}$

$r_B = h_2 + R = 7170 \times 10^3\text{ m}$

Conservation of angular momentum

$m r_A v_A = m r_B v_B \quad (1)$

Since: $T_A = \frac{1}{2} m v_A^2 \qquad T_B = \frac{1}{2} m v_B^2$

$V_A = \dfrac{-GMm}{r_A} \qquad V_B = \dfrac{-GMm}{r_B}$

On the Earth's surface : $W = m \cdot g = \dfrac{GMm}{R^2}$

$\boxed{\Rightarrow GM = R^2 g}$

$\Rightarrow V_A = \dfrac{-R^2 mg}{r_A} \qquad V_B = \dfrac{-R^2 mg}{r_B}$

Conservation of energy

$$\frac{1}{2}\cancel{m} v_A^2 - \frac{R^2 \cancel{m} g}{r_A} = \frac{1}{2}\cancel{m} v_B^2 - \frac{R^2 \cancel{m} g}{r_B}$$

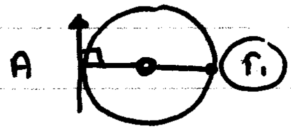
$$\Rightarrow \tfrac{1}{2} v_A^2 - \frac{(6370 \times 10^3)^2 (9.81)}{(6690 \times 10^3)} = \tfrac{1}{2} v_B^2 - \frac{(6370 \times 10^3)^2 (9.81)}{(7180 \times 10^3)} \qquad (2)$$

$$\Rightarrow \cancel{m}\, r_A v_A = \cancel{m}\, r_B v_B$$

$$\Rightarrow v_A = 7861 \text{ m/s}$$

$$v_B = 7334 \text{ m/s}$$

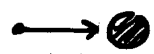
At A:



FBD

$$F = \frac{GMm}{r_1^2}$$

KD



$$ma_n$$

$$F = ma_n$$

$$\frac{GM\cancel{m}}{r_1^2} = \cancel{m} a_n = \cancel{m}\frac{v^2}{r_1}$$

$$\frac{R^2 g}{r_1^{\cancel{2}}} = \frac{v^2}{\cancel{r_1}} \rightarrow v = \sqrt{\frac{R^2 g}{r_1}} = v = R\sqrt{\frac{g}{r_1}}$$

$$v_{cir,A} = R\sqrt{g/r_1}$$

$$= (6370 \times 10^3)\left(\frac{9.81}{6690 \times 10^3}\right)$$

$$v_{cir,A} = 7714 \text{ m/s}$$

$$v_{cir,B} = R\sqrt{g/r_2}$$

$$= 7451 \text{ m/s}$$

Increase in speed at A : $v_A - v_{cir,A} = 7861 - 7714$

$= 147$ m/s

B : $v_{cir,B} - v_B = 7451 - 7334$

$= 117$ m/s

b) $\Delta T = \frac{1}{2} m v_A^2 - \frac{1}{2} m v_{cir,A}^2 + \frac{1}{2} m v_{cir,B}^2 - \frac{1}{2} m v_B^2$

$$\frac{\Delta T}{m} = (1/2)\left[v_A^2 - v_{cir,A}^2 + v_{cir,B}^2 - v_B^2 \right]$$

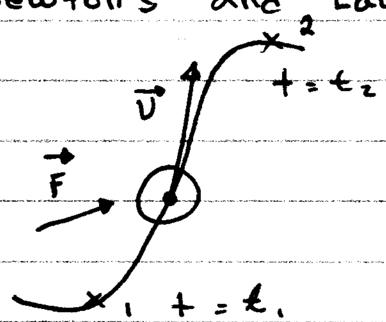
$$= 2.01 \times 10^6 \ (J/kg)$$

13.10 : Principle of Impulse and Momentum

* Force, Velocity and Time

* Impact

Newton's 2nd Law



$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\int_{t_1}^{t_2} \vec{F}\,dt = \int_{\vec{v}_1}^{\vec{v}_2} m\,d\vec{v}$$

m = const.

$$\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$$

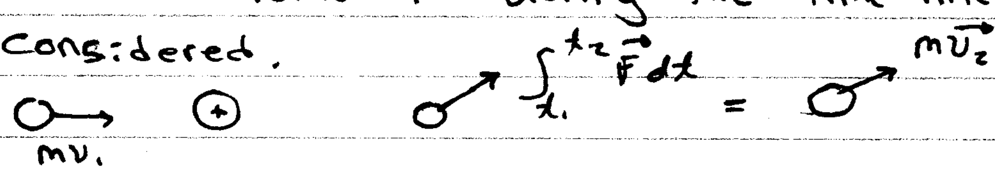
$$\Rightarrow \quad \boxed{m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2}$$

$m\vec{v}$: Linear momentum of the particle

$\vec{Imp}_{1-2} = \int_{t_1}^{t_2} \vec{F}\,dt \rightarrow$ the linear impulse

Principle of Impulse and Momentum :

The final momentum $m\vec{v}_2$ of the particle can be obtained by adding vectorially its initial momentum $m\vec{v}_1$ and the impulse of the force $\vec{F}$ during the time interval considered.



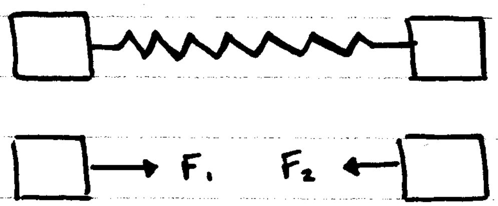
$\vec{F} = \text{Const.}$

$$\int_{t_1}^{t_2} \vec{F}\, dt = \vec{F}(t_2 - t_1)$$

Components Form:

$$\begin{cases} (mV_x)_1 + \int_{t_1}^{t_2} F_x\, dt = (mV_x)_2 \\ (mV_y)_1 + \int_{t_1}^{t_2} F_y\, dt = (mV_y)_2 \\ (mV_z)_1 + \int_{t_1}^{t_2} F_z\, dt = (mV_z)_2 \end{cases}$$

Internal Forces :



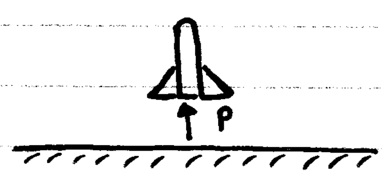
The impulse from the internal forces will cancel out due to Newton's third law.

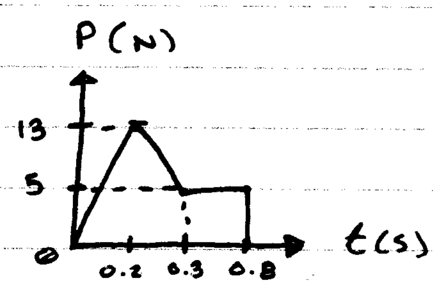
Only the impulse from the external forces will be considered.

Conservation of momentum: If no external forces are exerted on the particles or if the sum of the external forces is zero :

$$\sum m\vec{V_1} = \sum m\vec{V_2}$$

Example :





$m = 60\,g$

Find a) The max speed of the rocket as it goes up

b) Time for rocket to reach max elevation

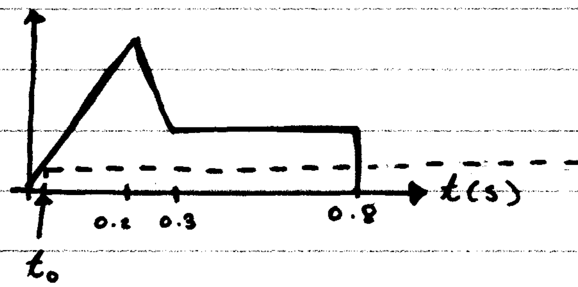
$$(mv_x)_1 + \int_{t_1}^{t_2} F_x\,dt = (mv_x)_2$$

Solution : FBD

$W = mg$

$\hookrightarrow W = (0.06)(9.81)$

$= 0.5886 \text{ N}$



Find $\displaystyle\int_0^{t_2} (P - W)\,dt = ?$

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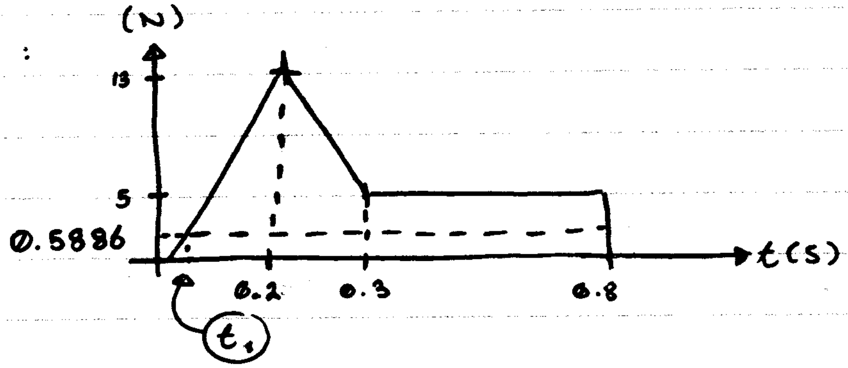
Dynamics II

Solution :



$$mv_1 + Imp_{1\to 2} = mv_2$$

At t_1 ; $v_1 = 0$ $\longrightarrow$ At t_2 ; $v_2 = max$

$$t_2 = 0.8s$$

$0 \le t \le 0.2s$ $\qquad P(t) = \frac{13}{0.2}t = 0.5886$

$$t = 0.009055 \text{ sec} = t_1$$

$$Imp_{1\to 2} = \int_{t_1}^{t_2} [P(t) - W]\,dt$$

$$= \int_{0.009055}^{0.8} P(t)\,dt - \int_{0.009055}^{0.8} 0.5886\,dt$$

$$= \frac{[0.5886 + 13](0.2 - 0.009055)}{2} + \frac{(13+5)(0.3-0.2)}{2} + \ldots$$

$$\ldots + 5x(0.8 - 0.3) - 0.5886(0.8 - 0.009055)$$

$$= (\ldots)$$

$$\Rightarrow (0.06)(0) + Imp_{1\to 2} = 0.06v_2$$

$$\Rightarrow v_2 = 70.5 \text{ m/s}$$

b) At it's highest elevation, $t = t_3$, $v_3 = 0$

$* \; mv_1 + Imp_{1\to 3} = mv_3$

$* \; mv_2 + Imp_{2\to 3} = mv_3$

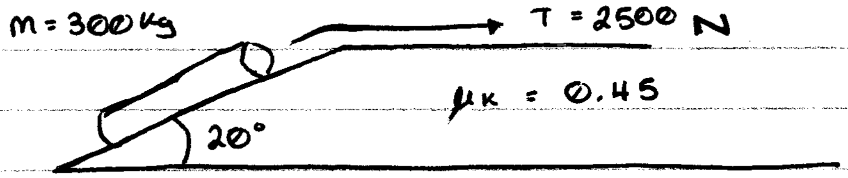
$$(0.06)(70.5) + \int_{0.8}^{t_3} (-0.5886)\,dt = 0$$

2

$0.06\,(70.5) - 0.5886\,(t_3 - 0.8) = 0$

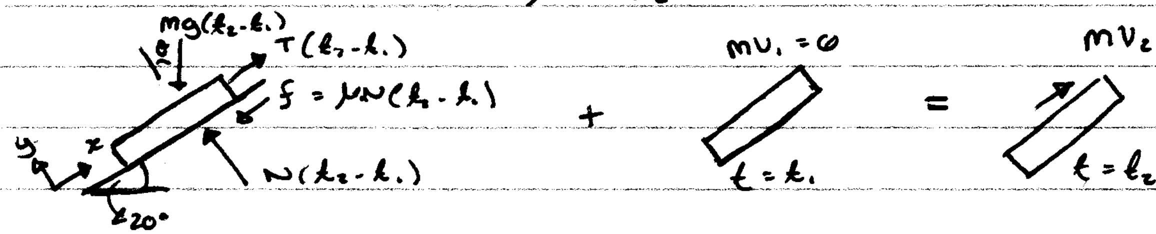
$t_3 = 7.99\,s$

Example :



Determine the time for the log to reach a speed of 0.5 m/s. Starting from rest.

Solution : $t_1 = 0$, $v_1 = 0$
$t_2 = ?$, $v_2 = 0.5\,m/s$



<u>In the y-direction</u> : $0 + N(t_2 - t_1) - mg(\cos 20^\circ)(t_2 - t_1) = 0$

$N = mg\cos\theta$

<u>In the x-direction:</u> $0 + T(t_2 - t_1) - \mu_K N(t_2 - t_1) - mg(\sin 20^\circ)\cdots$

$\cdots(t_2 - t_1) = mv_2$

$\Rightarrow T t_2 - \mu_K mg\cos\theta \cdot t_2 - mg\sin\theta \cdot t_2 = mv_2$

$\Rightarrow 2500\,t_2 - 0.45(300)(9.81)\cos 20^\circ\,t_2 - (300)(9.81)\sin 20^\circ \cdot t_2$

$\Rightarrow 300(0.5)$

$\Rightarrow t_2 = 0.603\,s$

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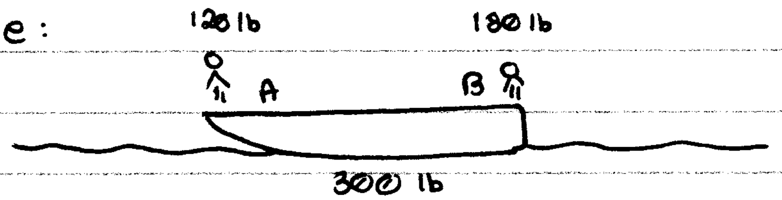
Dynamics

Example:



Each dive with a 16 ft/s velocity relative to the boat. Determine the velocity of the boat after they both dived.

a) A dives first b) B dives first

Solution: All motion occurs in the horizontal direction

FBD - no external horizontal force

- Conservation of linear momentum in the horizontal

$$\sum m v_1 = \sum m v_2$$

a)

$v_{A/BOAT} = -16$ ft/s

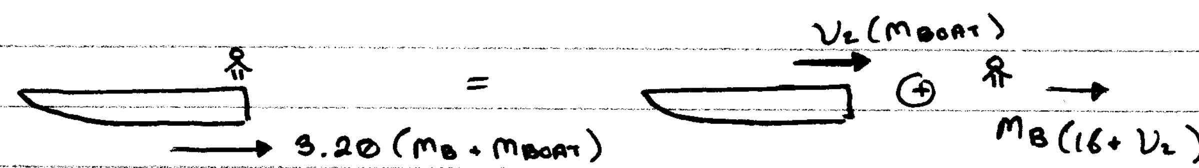
$$\boxed{v_{A/water} = \vec{v}_{A/BOAT} + \vec{v}_1}$$

$v_{A/water} = v_1 - 16$

$0 = m_A v_{A/water} + (m_B + m_{BOAT}) v_1$

$0 = \frac{120}{32.2}(v_1 - 16) + \left(\frac{180 + 30}{32.2}\right) v_1$

$v_1 = 3.20$ ft/s $(\rightarrow)$



$$\boxed{\vec{v}_{B/water} = \vec{v}_{B/BOAT} + \vec{v}_2}$$

$v_{B/water} = 16 + v_2$

$\Rightarrow (m_B + m_{BOAT})(3.20) = m_{BOAT} v_2 + m_B (16 + v_2)$

$$\frac{180 + 300}{g}(3.20) = \frac{300}{g}v_2 + \frac{180}{g}(16 + v_2)$$

$\Rightarrow v_2 = -2.80$ ft/s

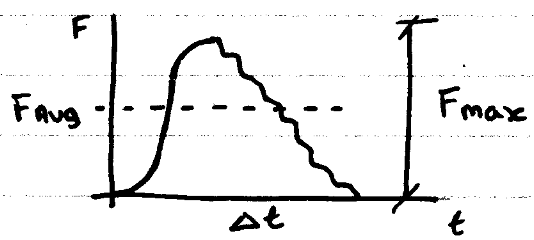
$= 2.80$ ft/s $(\leftarrow)$

b) The final velocity of the boat $= 0.229$ ft/s $(\leftarrow)$

↳ order makes a difference.

B.11): Impulsive Force

Force acting on a particle during a very short time interval that is large enough to cause a significant change in momentum is called a impulse force



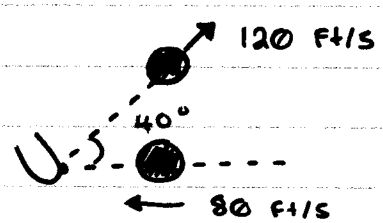
$$\int_0^{\Delta t} F(t)\,dt = F_{avg} \cdot \Delta t$$

$$m\vec{v}_1 + \vec{F}_{AVG}\,\Delta t = m\vec{v}_2$$

Non impulsive forces : $\vec{F}\Delta t$ is small

Weight

Example :



If the bat and the ball are in contact for 0.015 s, Find the average impulse force exerted on the ball during the impact ?

$W = 4$ oz.

1 oz. $= 0.0625$ lb.

Solution :

<u>x - component</u>

$$mv_{1x} + F_{AVG,x}\Delta t = mv_{2x}$$

$$\frac{4\times(0.0625)}{32.2}(-80) + F_{AVG,x}(0.015) = \frac{4\times(0.0625)}{32.2}$$

$$F_{AVG,x} = 89 \text{ lb}$$

y - component

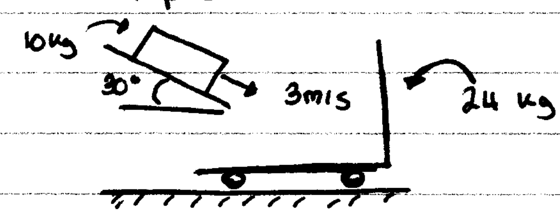
$$mv_{1y} + F_{AVG,y}\Delta t = mv_{2y}$$

$$0 + F_{AVG,y}(0.015) = \frac{4(0.0625)}{32.2}(120 \sin 40°)$$

$$F_{AVG,y} = 39.9 \text{ lb}$$

$$\therefore \vec{F}_{AVG} = 89\vec{i} + 39.9\vec{j} \text{ lb}$$

Example :

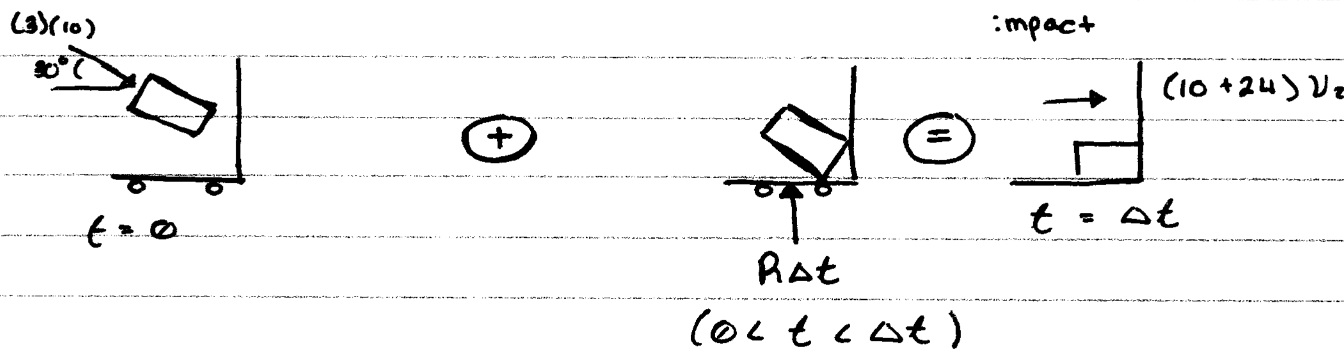


Find a) the final velocity of the cart

b) the impulse exerted by the cart on the package

c) the energy lost in the impact

Solution A)



$$\underline{x}: (10)(3)\cos 30° = (10+24)v_2$$

$$v_2 = 0.742 \text{ m/s}$$