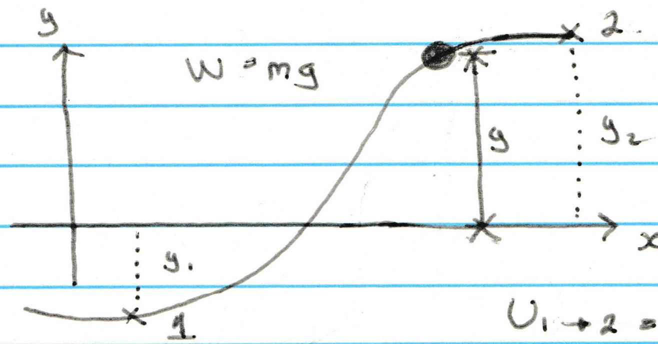
13.6 Potential Energy

Gravitational potential energy

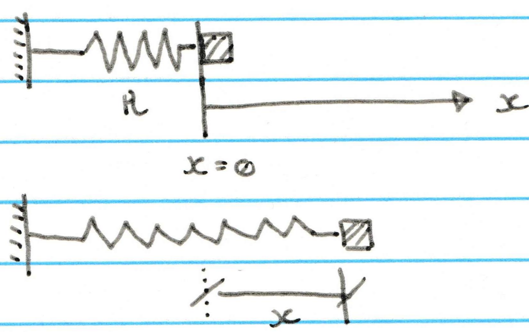


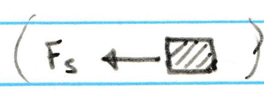
$V_g = Wy$

($y = 0$, datum)

$$U_{1\to2} = -W\Delta y$$
$$= -W(y_2 - y_1)$$
$$= Wy_1 - Wy_2$$
$$\boxed{U_{1\to2} = V_{g1} - V_{g2}}$$

Elastic Potential Energy





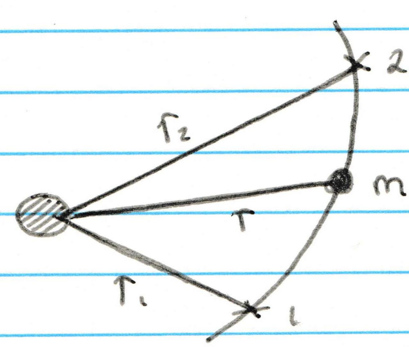
Define $V_e = \frac{1}{2} kx^2$

$$U_{1\to2} = \tfrac{1}{2} kx_1^2 - \tfrac{1}{2} kx_2^2$$
$$\boxed{U_{1\to2} = V_{e1} - V_{e2}}$$

Gravitational Potential Energy (Again)

$$V_g = \frac{-GMm}{r}$$

$$\boxed{U_{1\to2} = V_{g1} - V_{g2}}$$



13.8 Conservation of Energy

When the work done on a particle is due to the conservative forces, the work-energy principle :

$$T_1 + U_{1\to 2} = T_2$$

Since

$$U_{1\to 2} = V_1 - V_2$$

Here $V = V_g + V_e$

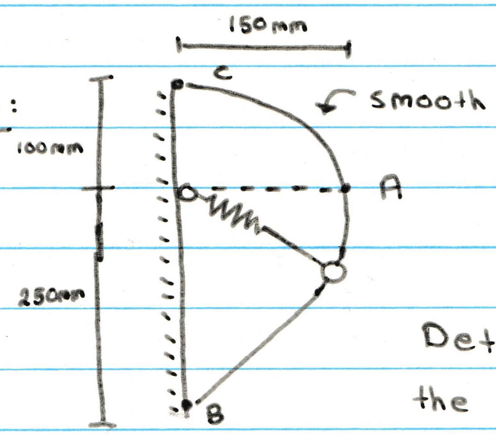
$$\Rightarrow \boxed{T_1 + V_1 = T_2 + V_2}$$

(From $T_1 + V_1 - V_2 = T_2$)

Mechanical Energy

$$E = T + V$$

Example:



$m = 500\ g$

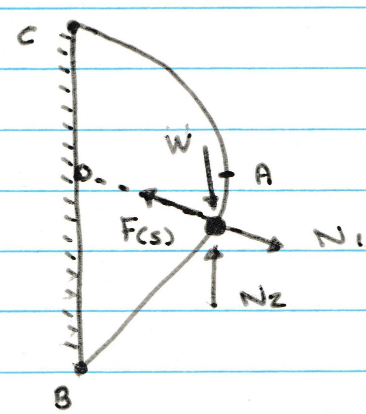
$k = 400\ KN/m$

Undeformed length of the spring is 80mm.

Determine 1.) the velocity that the collar should be given at A to reach B with zero velocity

b) the velocity of the collar when it eventually reaches C.

Solution : FBD



$U_{(N_1)} = \emptyset$

$U_{(N_2)} = \emptyset$

$U_{(w)} = \emptyset$

* only spring force does the work.

$$T_1 + V_1 = T_2 + V_2$$

1) Collars move from A to B

$$T_1 = \tfrac{1}{2} m v^2 = \tfrac{1}{2} \frac{500}{1000} v_A^2$$

$$V_1 = \tfrac{1}{2} k x^2 = \tfrac{1}{2}\, 400 (10^3) \left(\frac{150 - 80}{1000} \right)^2$$

$$T_2 = \tfrac{1}{2} m v^2 = 0$$

$$V_2 = \tfrac{1}{2} k x^2 = \tfrac{1}{2}\, 400 (10^3) \left(\frac{250 - 80}{1000} \right)^2$$

$$\Rightarrow \frac{1}{2} \left(\frac{500}{1000} \right) v_A^2 + \frac{1}{2}\, 400 (10^3) \left(\frac{150 - 80}{1000} \right)^2$$

$$= 0 + \tfrac{1}{2}\, 400\, (10^3) \left(\frac{250 - 80}{1000} \right)^2$$

$$\Rightarrow v_A = 87.2 \text{ m/s}$$

2) From B to C

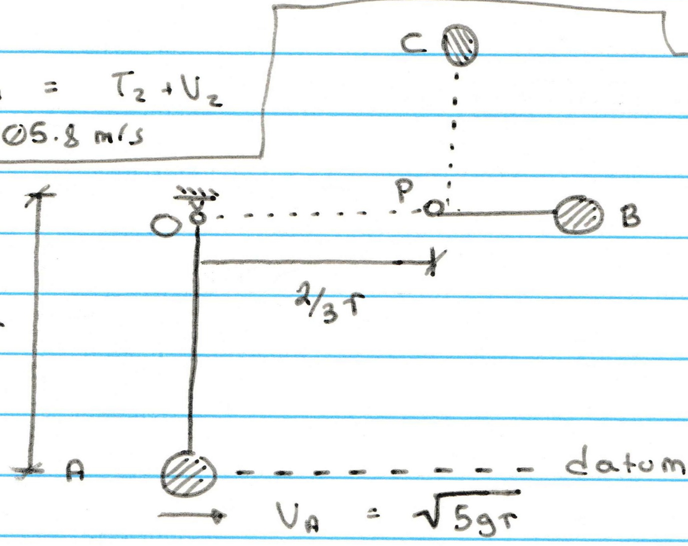
$$T_1 = 0 \; ; \; V_1 = \tfrac{1}{2}\, 400\, (10^3) \left(\frac{250 - 80}{1000} \right)^2$$

$$T_2 = \tfrac{1}{2} \frac{500}{1000} v_2^2 \Rightarrow V_2 = \tfrac{1}{2} (400)(10^3) \left(\frac{100 - 80}{1000} \right)^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$v_2 = 105.8 \text{ m/s}$$

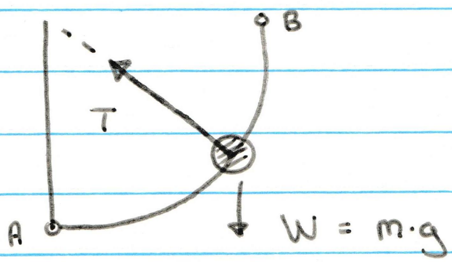
Example:



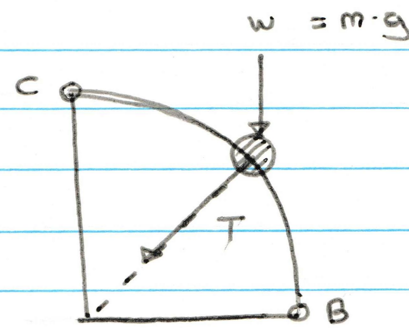
Determine the speed of the ball and the tension in the cord when it is at the highest point C.

Solution FBD

From A to B:



From B to C:



only weight does the work

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2} m \left(\sqrt{5gr}\right)^2 \Rightarrow 5/2\, mgr$$

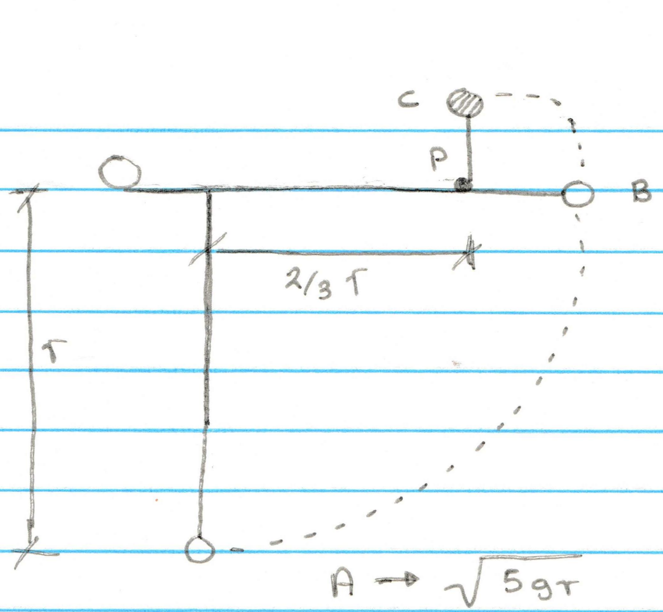
$$V_1 = 0$$

$$T_2 = \tfrac{1}{2} m v_2^2$$

$$V_2 = wy = mg(r + r/3) = \frac{4mgr}{3}$$

$$\Rightarrow 5/2\, \cancel{m}gr + 0 = \tfrac{1}{2} \cancel{m} v_2^2 + \frac{4\cancel{m}gr}{3}$$

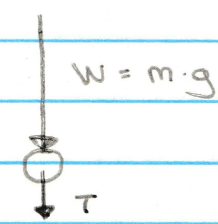
$$\Rightarrow v_2 = \sqrt{7/3\, gr}$$



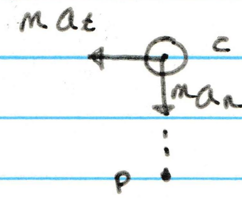
$V_c = \sqrt{7/3\, g r}$

b) Tension at C

FBD



KD



$\sum \vec{F} = m\vec{a}$

Normal direction : $\sum F_n = m a_n$

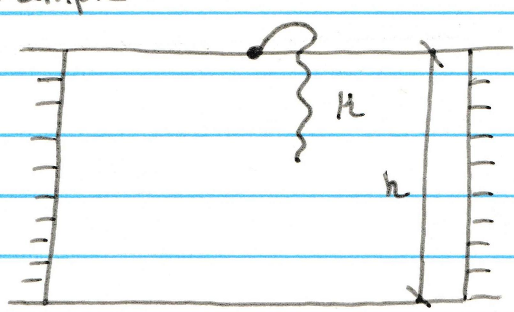
$T + mg = m a_n$

Since $a_n = \frac{v^2}{\rho} = \frac{\left(\sqrt{7/3\, g r}\right)^2}{(1/3\, r)} \Rightarrow 7g$

$\frac{\sqrt{(7)(\cancel{3})\, g \cancel{r}}^{\,2}}{(\cancel{3})\cancel{r}}$

$\Rightarrow T = 6mg$

Example :



$h = 120$ ft
$k = 80$ lb/ft
$W = 150$ lb

1) Find length, l_0, so they just touch the water's surface.

cont. →

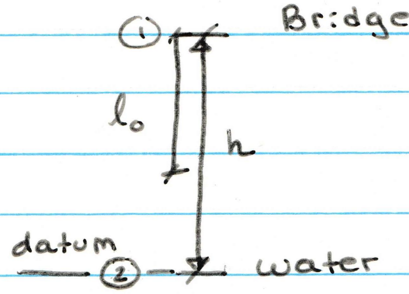
2) At the surface, B jumps off and A will move upward. What is the max acceleration A will have? What is the maximum height A can reach?

Solution FBD

conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

At position 1, $T_1 = 0$, $V_1 = 2wh$



At position 2, $T_2 = 0$

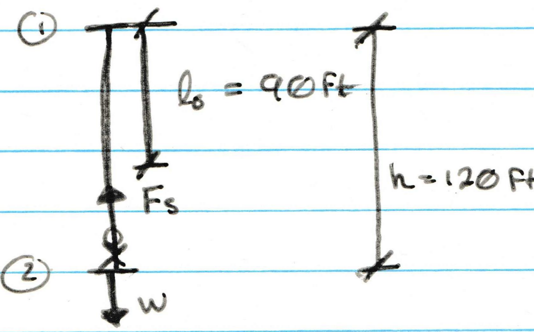
$$V_2 = \tfrac{1}{2} k (h - l_o)^2$$

$$\Rightarrow 0 + 2wh = 0 + \tfrac{1}{2} k (h - l_o)^2$$

$$\Rightarrow 2 \cdot 150 \cdot 200 = \tfrac{1}{2}(80)(120 - l_o)^2$$

$$l_o = 90 \text{ ft}$$

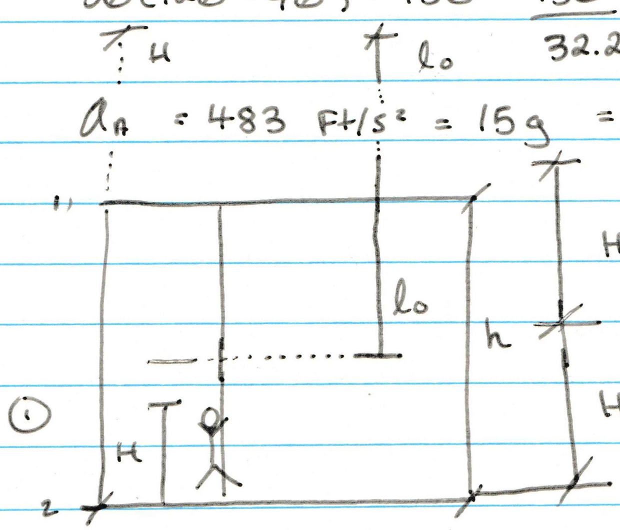
b) FBD of A



$$F_s - W = ma_A = \frac{W}{g} a_A$$

$$80(120 - 90) - 150 = \frac{150}{32.2} a_A$$

$$a_A = 483 \text{ ft/s}^2 = 15g = a_{max}$$



Case 1 $H < 30$ ft

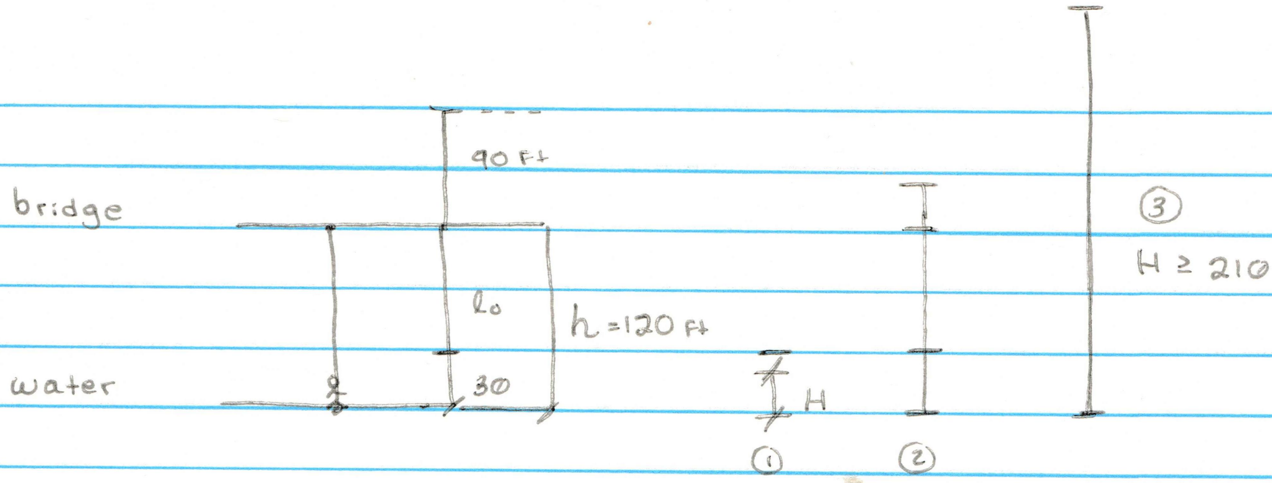
Case 2 $30 < H < 120$ ft

Case 3 $H > 210$

$$H_{max} = 218.93 \text{ ft}$$

Sept. 21/17

Dynamics II



$0 \le H \le 30$ $\quad$ $30 \le H \le 3 + 90 + 90$

$30 \le H \le 210$

Case ①: $0 \le H \le 30$

$T_1 = 0 \qquad V_1 = V_{g1} + V_{e1}$
$$= 0 + \tfrac{1}{2}k(h - l_0)^2$$
$$= \tfrac{1}{2}(80)(120 - 90)^2$$
$$= 36000$$

$T_2 = 0 \qquad V_2 = V_{g2} + V_{e2}$
$$= WH + \tfrac{1}{2}k(30 - H)^2$$
$$= 150H + \tfrac{1}{2}(80)(30 - H)^2$$

$T_1 + V_1 = T_2 + V_2$

$0 + 36000 = 150H + \tfrac{1}{2}(80)(30 - H)^2 + 0$

$\Rightarrow \quad H = 0, \quad H = 56.25 > 30$

Case ②: $30 \le H \le 210$

$T_2 = 0 \qquad V_2 = V_{g2} + V_{e2} = 150H$

$0 + 36000 = 150H$

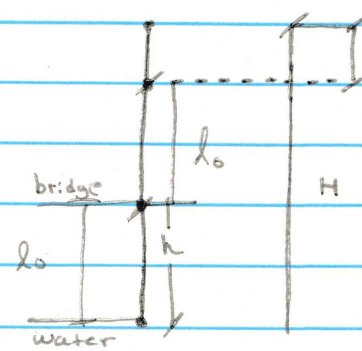
$H = 240 > 210$

Case ③: $H \ge 210$

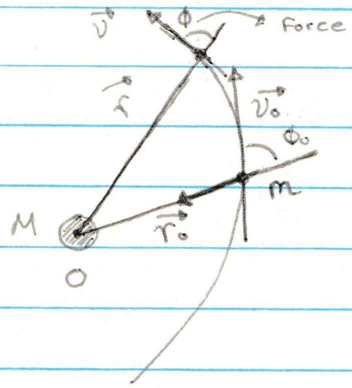
$T_2 = 0 \qquad V_2 = V_{g2} + V_{e2}$
$$= 150H + \tfrac{1}{2}(80)(H + 210)^2$$

$\therefore \; 0 + 36000 = 150H + 40(H - 210)^2$

$H = 218.9 > 210$ ✓



13.9 Motion Under a conservative Central Force



$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = \tfrac{1}{2} m v_0^2$$

$$V_1 = \frac{-GMm}{r_0}$$

$$T_2 = \tfrac{1}{2} m v^2$$

$$V_2 = -\frac{GMm}{r}$$

$$\Rightarrow \tfrac{1}{2} m v_0^2 - \frac{GMm}{r_0} = \tfrac{1}{2} m v^2 - \frac{GMm}{r}$$

Newton's 2nd Law:

$$\vec{F} = m\vec{a} \Rightarrow m\frac{d\vec{v}}{dt}$$

$$\vec{r} \times \vec{F} = \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$\vec{r} \times m\frac{dv}{dt} = 0$$

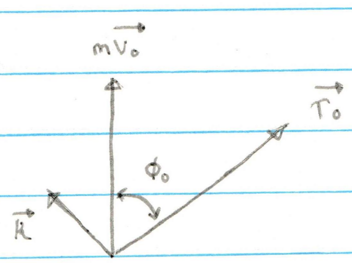
$$m\,{}^{d}\!/\!{}_{dt}(\vec{r} \times \vec{v}) = m\,\cancel{\frac{d\vec{r}}{dt} \times \vec{v}}^{\,0} + m\,\cancel{\vec{r} \times \frac{d\vec{v}}{dt}}^{\,0}$$

$$= 0$$

$$\vec{r} \times m\vec{v} = \text{const.}$$

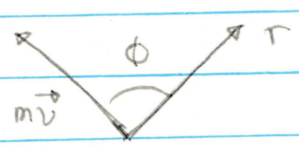
$$\boxed{\vec{r}_0 \times m\vec{v}_0 = \vec{r} \times m\vec{v}}$$

conservation of angular momentum about O



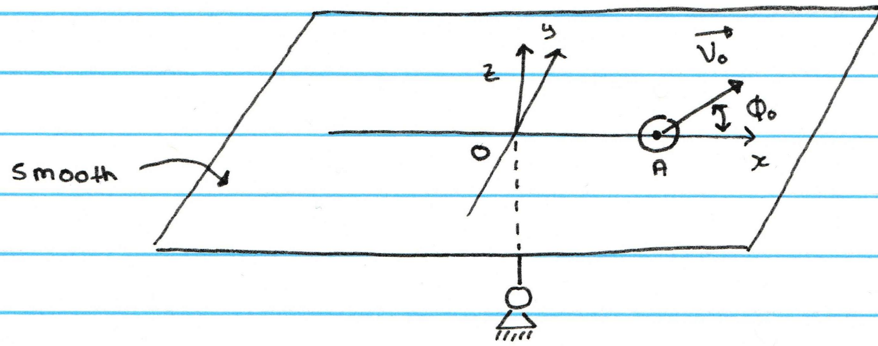
$$\vec{r}_0 \times m\vec{v}_0 = r_0 m v_0 \sin\phi\ \vec{k}$$

$$r \times mv = rmv \sin\phi\ \vec{k}$$



$$\Rightarrow m r_0 v_0 \sin\phi_0 = m r v \sin\phi$$

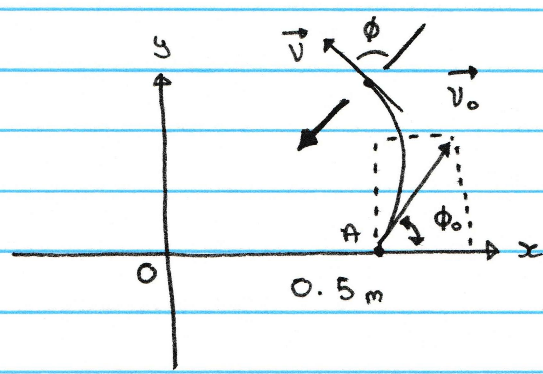
Example:



$OA = 0.5\,m$	$V_o = 20\ m/s$	$\Phi_o = 60°$
$k = 100\ N/m$	(spring unstretched when it is at point O.)	
$m = 0.6\ kg$		

Determine the min and max distance from the object to the origin O.

Solution:



Energy : $\frac{1}{2} m V_o^2 + \frac{1}{2} k r_o^2$

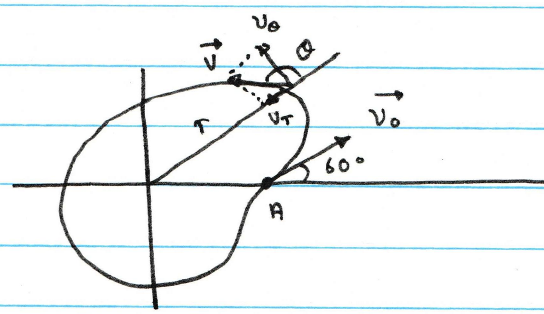
$= \frac{1}{2} m v^2 + \frac{1}{2} k r^2$

$\Rightarrow \frac{1}{2}(0.6)(20)^2 + \frac{1}{2}(100)(0.5)^2 = \frac{1}{2}(0.6) V^2 + \frac{1}{2}(100) r^2$ (1)

Angular Momentum : (not canceled out)

$\cancel{m} r_o V_o \sin\phi_o = \cancel{m}\, r V \sin\phi$

$\Rightarrow (0.5)(20) \sin 60° = r V \sin\phi$ (2)



r_{max} or min $\Leftrightarrow$ $\boxed{v_r = 0}$ $\Leftrightarrow$ $\phi = 90°$ (3)

$\phi = 90°$

(2) $\Rightarrow$ $v = \dfrac{(0.5)(20)\sin 60°}{r}$ (4)

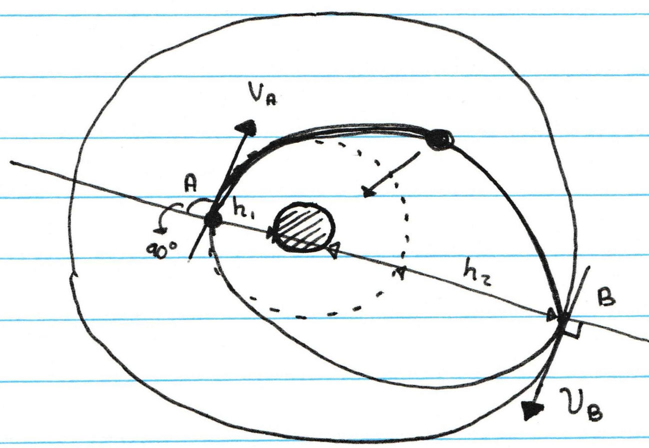
(4) $\Rightarrow$ (1)

$$\tfrac{1}{2}(0.6)(20)^2 + \tfrac{1}{2}(100)(0.5)^2$$
$$= \tfrac{1}{2}(0.6)\left[\frac{(0.5)(20)(\sin 60°)}{r}\right]^2 + \tfrac{1}{2}(100)r^2$$

$\Rightarrow$ $r^2 = 2.468$; 0.1824

$r = \underbrace{1.571}_{max}$; $\underbrace{0.427}_{min}$

Example:



$h_1 = 200$ miles
$h_2 = 500$ miles
$R = 6370$ KM

Determine a) the required increases in speed at A and B

b) the total energy per unit mass to execute the transfer