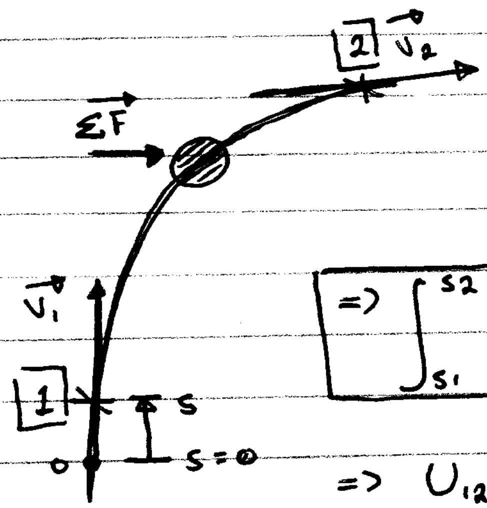
SEPT. 12/17
DYNAMICS 2



$T = \frac{1}{2} m v^2 = \frac{1}{2} m \vec{v} \cdot \vec{v}$

$\Sigma \vec{F} = m \vec{a}$

$$\boxed{\Rightarrow \int_{s_1}^{s_2} \Sigma F_t \, ds = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2}$$

$\Rightarrow U_{12} = T_2 - T_1$

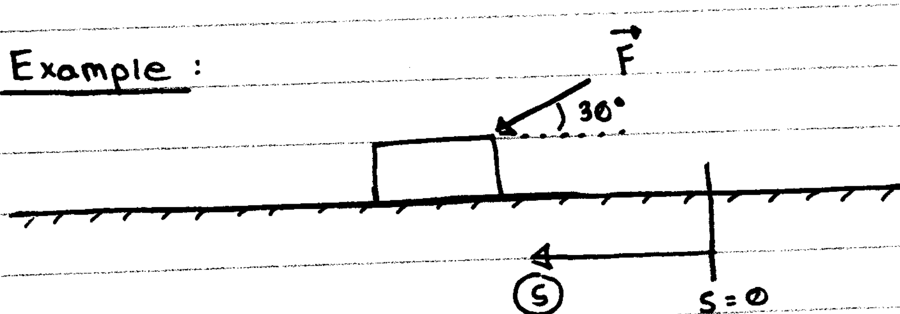
Work-energy Principle :

$$\boxed{\Rightarrow \underbrace{T_1}_{\text{initial kinetic energy}} + \underbrace{U_{12}}_{\text{Total work done}} = \underbrace{T_2}_{\text{final kinetic energy}}}$$

* Force, Displacement, and Velocity
↳ instances when question asks about work-energy principle

* Cannot find acceleration directly
↳ Cannot find normal force
→ go back to Newton's 2nd

<u>Example</u> :



→ mass = 2 kg

→ $F(s) = \frac{300}{1+s}$ (N)

[s is in meters]

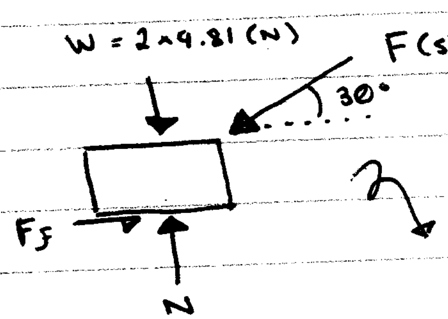
→ $\mu_k = 0.25$

→ The speed of the block is 8 m/s to the left when S = 4 m

Find the speed of the block when S = 12 m.

Solution : (use work-energy principle)

<u>FBD</u>



Moves From $S_1 = 4m$ to $S_2 = 12m$

$U_{12}(\omega) = -\omega \Delta y = 0$

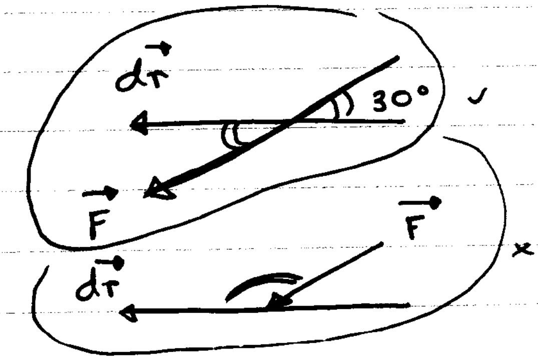
$U_{12}(N) = 0$

$$U_{12}(F) = \int_{s_1}^{s_2} F \cos\theta \, ds$$

$$= \int_4^{12} \frac{300}{1+s} \cos 30° \, ds$$

$$= 300 \cos 30° \ln(1+s) \Big|_4^{12}$$

$$= 248.25 \text{ J}$$



vectors have to start @ same location to det. θ

Friction Force :

$F_f = \mu_k N$

$\boxed{\Sigma F_y = 0}$: $N - W - F\sin 30° = 0$

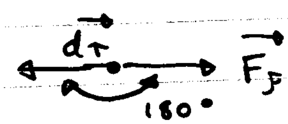
$N = W + F\sin 30°$

$= mg + F\sin 30°$

$F_f = \mu_k (mg + F\sin 30°)$

$= 0.25\left(2 \times 9.81 + \frac{300}{1+s} \sin 30°\right)$

$$U_{12}(F_f) = \int_{s_1}^{s_2} F \cos\theta \, ds$$



$$= \int_4^{12} 0.25\left(2 \times 9.81 + \frac{300}{1+s} \sin 30°\right) \cos 180° \, ds$$

$$= -75.07 \text{ J}$$

Since $T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(2)(8)^2 = 64 \text{ J}$

$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(2)(v_2)^2 = v_2^2$

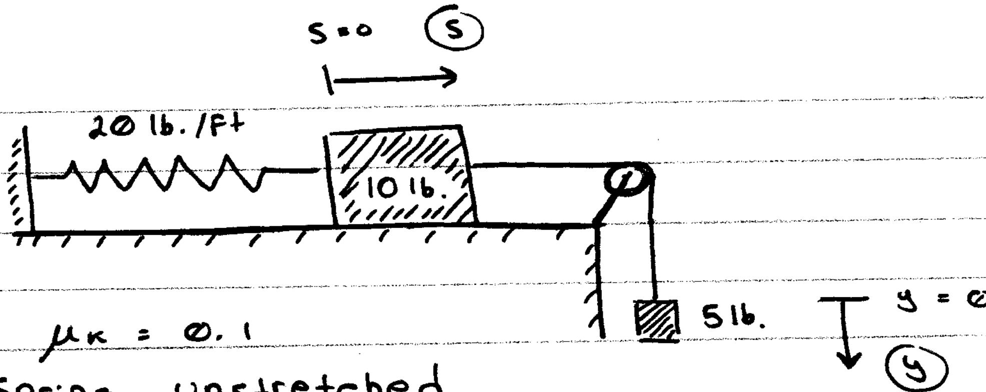
Work - Energy :

$T_1 + U_{12} = T_2$

$64 + 0 + 0 + 248.25 - 75.07 = v_2^2$

$\Rightarrow v_2 = 15.4 \text{ m/s}$

Example :



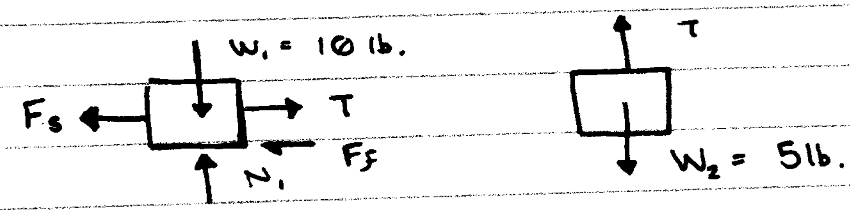
$\mu_s = 0.2$, $\mu_K = 0.1$

At rest, spring unstretched

Find a) the max velocity of the blocks and the stretch in the spring at that position and b) the max amount that the 5 lb. block will drop.

Solution :

FBD



At position 1: $S_1 = 0$, $V_1 = 0$

At position 2: S, $V = ?$

$$T_1 = \sum \tfrac{1}{2} m v^2 = 0$$

$$T_2 = \sum \tfrac{1}{2} m v^2 = \tfrac{1}{2}\left(\frac{10}{32.2}\right) v^2 + \tfrac{1}{2}\left(\tfrac{5}{32.2}\right) v^2$$

$$U_{12}\text{ (block 1)} = \int_0^s T\,ds + \tfrac{1}{2} K x_1^2 - \tfrac{1}{2} K x_2^2 - \mu_K W_1 S$$

$$= \int_0^s T\,ds + 0 - \tfrac{1}{2}(20) S^2 - 0.1\,(10)\,S$$

$$= \int_0^s T\,ds - 10S^2 - S$$

$$U_{12}\text{ (block 2)} = W_2 \cdot S - \int_0^s T\,ds$$

$$= 5S - \int_0^s T\,ds$$

$$\therefore U_{12} = U_{12}\,(\text{block } 1) + U_{12}\,(\text{block } 2)$$
$$= 4s - 10s^2$$

Sept. 13/17

DYNAMICS I

Work-energy Principle

$T_1 + U_{12} = T_2$

$\emptyset + 4s - 10s^2 = (1/2)(10/32.2)v^2 + (1/2)(5/32.2)v^2$

$\Rightarrow \quad 4s - 10s^2 = \frac{15}{64.4}v^2$

a) at max. velocity $v = v_{max}$

$\frac{dv}{ds} = \emptyset$

$\Leftrightarrow \quad \frac{dv^2}{ds} = \emptyset$

$\Leftrightarrow \quad \frac{d}{ds}(4s - 10s^2) = \emptyset$

$4 - 20s = \emptyset \quad ; \quad s = 1/5$

At $s = 1/5 = \emptyset.2$ ft, $v = v_{max}$

$4(\emptyset.2) - 10(\emptyset.2)^2 = (15/64.4)v_{max}^2$

$v_{max} = 1.311$ ft/s

$$\left(\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v . \frac{dv}{dt} \right)$$

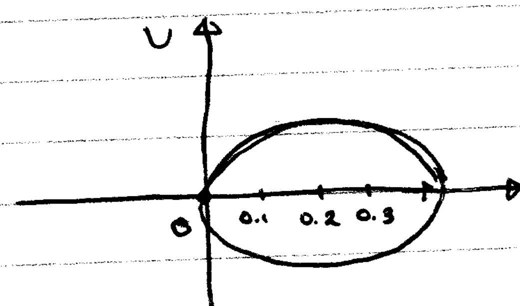
b) the max amount occurs when $v = \emptyset$

$4s - 10s^2 = \emptyset$

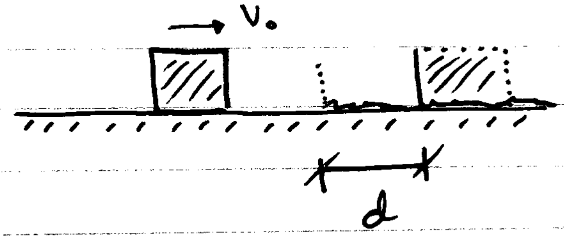
$s = \emptyset.4$ ft $= s_{max}$

$4s - 10s^2 = \frac{15}{64.4}v^2$

$\frac{15}{64.4}v^2 + 10(s - \emptyset.2)^2 = \emptyset.4$



Example :



If the block were traveling twice as fast, that is, at speed $2V_0$, how far will it travel on the rough surface ?

1) $d/2$ 2) d 3) $\sqrt{2}d$ 4) $2d$ 5) $4d$

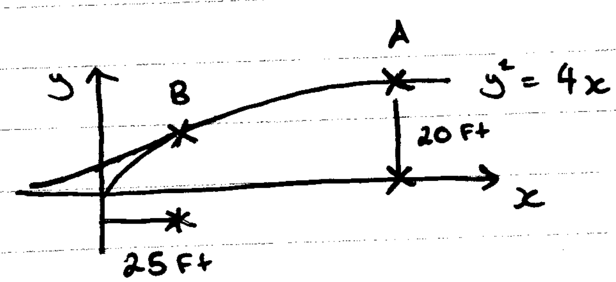
Example: A 400 kg satelite in a circle orbit 1500 km above the surface of the Earth. $g = 6.43\ m/s^2$. Determine the kinetic energy of the satelite knowing that its speed is 25.6×10^3 km/h

Solution $m = 400$ kg

$v = 25.6 \times 10^3$ km/h $\rightarrow$

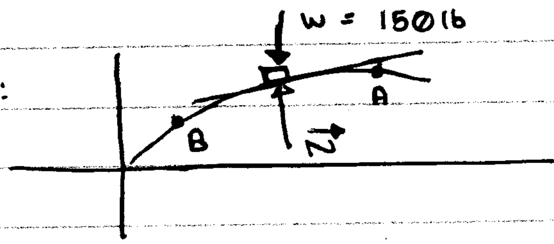
$$\therefore\ T = \tfrac{1}{2} m v^2$$
$$= \tfrac{1}{2}(400)\left(25.6 \times 10^3 \times \frac{1000}{3600}\right)^2$$
$$= 1.0113 \times 10^{10} \ldots$$

Example :



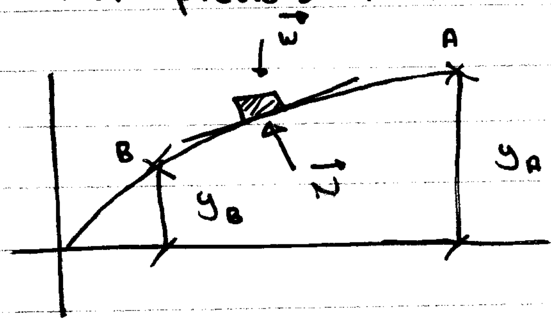
At A, $v_A = 6$ ft/s, Find the velocity when he reaches point B and the normal force exerted on him by the track at this point (B).

Solution :



TBC . .

→ From previous lecture



$y_A = 20\text{ ft}$

$x_B = 25\text{ ft} \qquad \therefore \quad y_B^2 = 4x_B = 4(25) = 100$

$y_B = 10\text{ ft}$

$\Delta y = y_B - y_A = 10 - 20 = -10$

$U_{12}(w) = -w\Delta y$

$= -150(-10) = 1500$

$T_A = \frac{1}{2} m_A V_A^2 = \frac{1}{2}(150/32.2)(6)^2$

$T_B = \frac{1}{2} m_b V_b^2 = \frac{1}{2}(150/32.2)(v_B)^2$

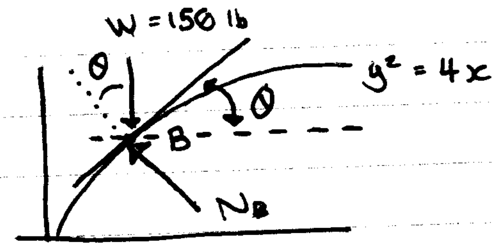
Work - energy principle

$T_1 + U_{12} = T_2$

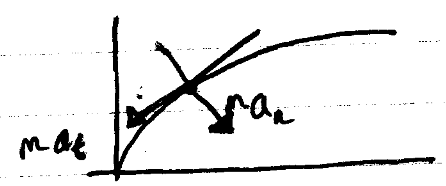
$\Rightarrow \frac{1}{2}(150/32.2)(6)^2 + 1500 = \frac{1}{2}(150/32.2)v_B^2$

$\Rightarrow V_B = 26.08 \text{ ft/s}$

FBD @ B :



KD @ B :



Newton's 2nd in the normal :

$\Sigma F_n = ma_n$

$\boxed{W\cos\theta - N_B = ma_n}$

? ?

Since $y^2 = 4x \Rightarrow y = 2\sqrt{x}$

$y' = dy/dx = 1/\sqrt{x} = x^{-1/2}$

$y'' = -1/2\, x^{-3/2}$

$\Rightarrow \tan\theta = y' = x^{-1/2}$

$$\rho = \frac{[1+(y')^2]^{3/2}}{|y''|} \Rightarrow \frac{[1+x^{-1}]^{3/2}}{|-\frac{1}{2}x^{-3/2}|}$$

At B., $x_B = 25$

$\tan\theta = 25^{-1/2} = 1/5 \Rightarrow \theta = 11.31°$

$$\rho = \frac{(1+1/25)^{3/2}}{\frac{1}{2}(25^{-3/2})} = 265.15 \text{ ft}$$

and $a_n = \dfrac{v^2}{\rho}$

$\Rightarrow N_B = W\cos\theta - m a_n$

$\Rightarrow W\cos\theta - m\dfrac{v_B^2}{\rho}$

$$\Rightarrow 150\cos 11.31° - \left(\frac{150}{32.2}\right)\left(\frac{26.08^2}{265.15}\right)$$

$= 135$ lb

13.5 Power and Efficiency

Power: the time rate at which work is performed or energy is converted.

$\boxed{P = \dfrac{dU}{dt}}$ Power is a scalar.

Units: SI: Watt

$1\,W = 1\,J/s = 1\,N\cdot m/s$

FPS Horsepower

$1\,hp = 550\,ft\cdot lb/s$

$\boxed{1\,hp = 746\text{ watt}}$

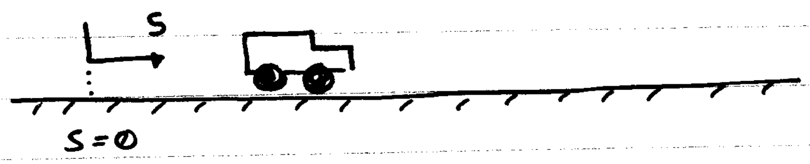
Formula: $dU = \vec{F}\cdot d\vec{r}$

$\Rightarrow P = \dfrac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$

Efficiency: Mechanical efficiency of a machine is defined as the ratio of the output of useful power produced by the machine to the input of the power supplied to the machine.

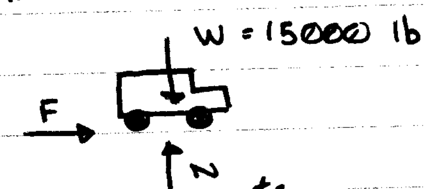
$$\eta = \frac{\text{power output}}{\text{Power input}} < 1$$

Example: $W = 1500\ \text{lb}$, $P = 100\ \text{hp}$



Determine how far it must travel to reach a speed of 40 ft/s ?

Solution:



At position 1: $S_1 = 0$, $v_1 = 0$

At position 2: $S_2 = ?$, $v_2 = 40\ \text{ft/s}$

$$U_{12} = \int_{S1}^{S2} F ds = \int_{t_1}^{t_2} F \frac{ds}{dt} \cdot dt$$

$$= \int_0^{t_2} (F v) dt$$

$$= F v \cdot t_2 = P t_2$$

Work-energy Principle

$$T_1 + U_{12} = T_2$$

$$\frac{1}{2}\left(\frac{15000}{32.2}\right)(0)^2 + P t_2 = \frac{1}{2}\left(\frac{15000}{32.2}\right)(40)^2$$

$$v = v(t)$$

Position 1, $S_1 = 0$, $v_1 = 0$

Position 2, S, and speed of car: v

$$T_1 + U_{12} = T_2$$

$$\boxed{0 + Pt = \tfrac{1}{2}\left(\frac{15000}{32.2}\right)v^2}$$

$$100 \times 550 \times t = \tfrac{1}{2}\left(\frac{15000}{32.2}\right)v^2$$

$$\boxed{v = 15.367\sqrt{t}}$$

Kinematic : $v = \dfrac{ds}{dt}$; $ds = v\,dt$

$$\Rightarrow \int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt$$

$$\Rightarrow S_2 = \int_0^{t_2} 15.367\sqrt{t}\,dt$$

$$= 10.244\,t_2^{3/2}$$

At $v_2 = 40$ ft/s

$40 = 15.367\sqrt{t_2} \Rightarrow t_2 = 6.7755$

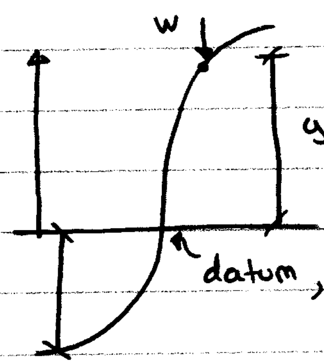
$S_2 = 10.244(6.7755)^{3/2}$

$= 180.7$ ft

$$a = \frac{dv}{dt} = \frac{15.367}{2} \cdot \frac{1}{\sqrt{t}}$$

13.6 Potential Energy

Gravitational Potential Energy



Work done by weight

$U_{12} = -w\Delta y$

$= -w(y_2 - y_1)$

$= wy_1 - wy_2$

$V_{g1} = wy_1$, $V_{g2} = wy_2$

$\therefore \boxed{U_{12} = V_{g1} - V_{g2}}$

Potential Energy

$V_g = wy$