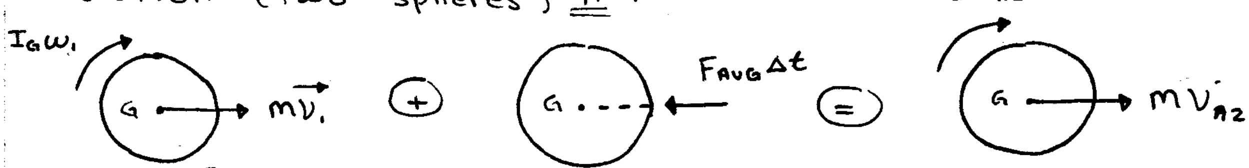
NOV. 28/17

DYNAMICS

Solution (two spheres) $\underline{A}$:

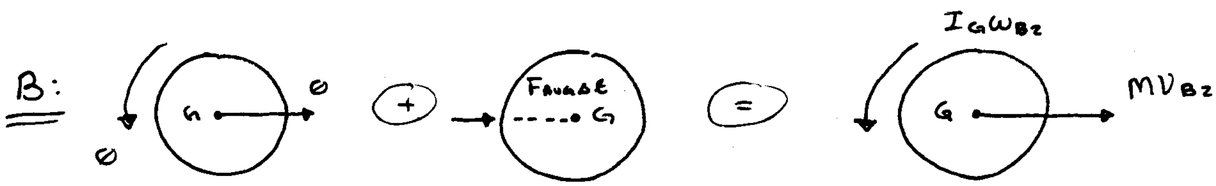


PRINCIPLE OF LINEAR IMPULSE:

$$\underline{x}: \quad mv_1 - F_{AVG}\Delta t = mv_{A2} \qquad (1)$$

ANGULAR MOMENTUM (About mass centre G):

$$\underline{+G\circlearrowright}: \quad -I_G\omega_1 = -I_G\omega_{A2} \Rightarrow \omega_{A2} = \omega_1 \qquad (2)$$

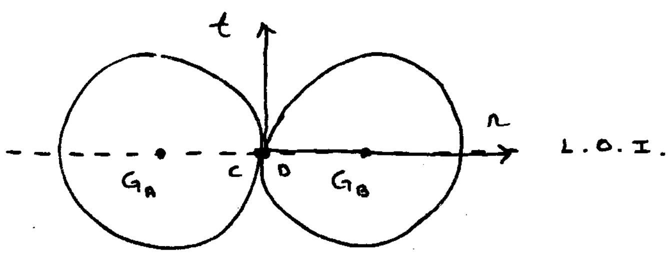


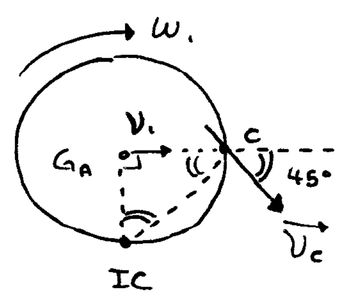
$$\underline{x}: \quad 0 + F_{AVG}\Delta t = mv_{B2} \qquad (3)$$

$$\underline{+G\circlearrowright}: \quad 0 + 0 = -I_G\omega_{B2} \Rightarrow \omega_{B2} = 0 \qquad (4)$$

$$\underline{(1)+(3)}: \quad mv_1 = mv_{A2} + mv_{A2}$$

$$v_{A2} + v_{B2} = v_1 \qquad (5)$$





$$e = -\left(\frac{v_{cn}' - v_{Dn}'}{v_{cn} - v_{Dn}}\right)$$

Before: $v_{Dn} = 0$

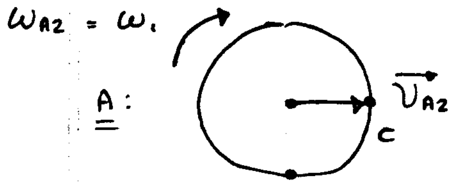
$$v_{cn} = v_c\cos 45° = \sqrt{2}\,r\omega_1 \cdot \cos 45°$$

$$= v_1$$

$$\vec{v}_c = \underbrace{\vec{v}_1}_{\text{normal}} + \underbrace{\vec{v}_{c/G_A}}_{\text{tangential}}$$

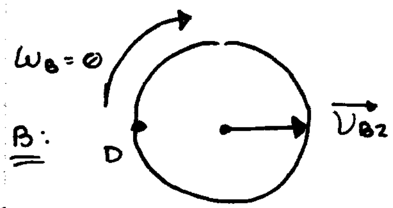
$\therefore v_{cn} = v_1$

After impact:



$$\vec{v}_c = \vec{v}_{A2} + \vec{v}_{c/GA}$$

$$\Rightarrow v_{cn}' = v_{A2}$$

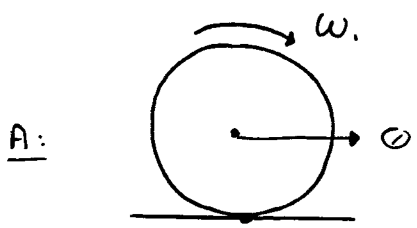


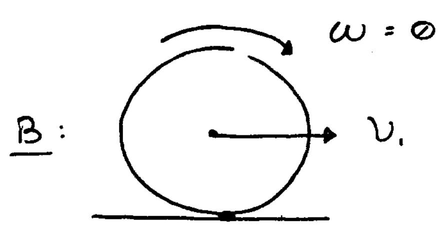
$$v_{Dn}' = v_{B2}$$

$$\therefore \quad e = 1 = -\left(\frac{v_{cn}' - v_{Dn}'}{v_{cn} - v_{Dn}}\right)$$

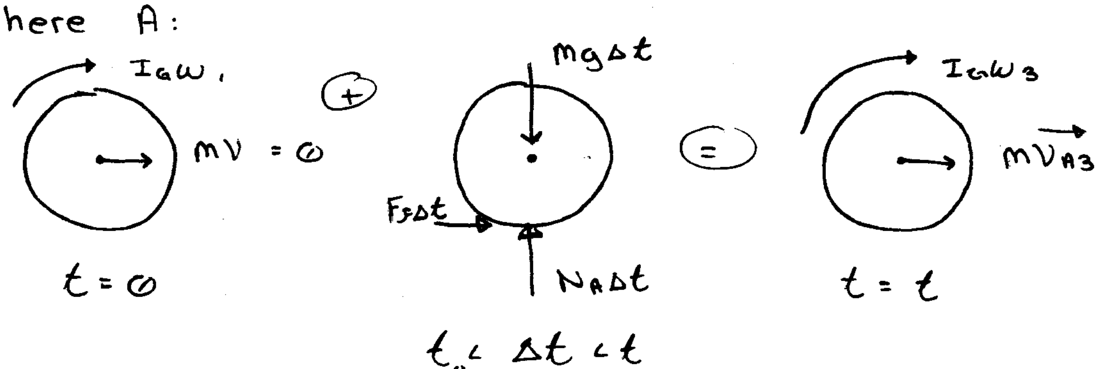
$$\rightarrow \quad 1 = -\frac{v_{A2} - v_{B2}}{v_1 - 0} \quad \ldots \quad (6)$$

$$\Rightarrow v_{A2} = 0, \quad v_{Bn} = v_1$$





Sphere A:



$\underline{y}$: $\emptyset + N_A t - mg t = \emptyset \Rightarrow N_A = mg$ (7)

$\underline{x}$: $\emptyset + F_f t = m v_{A3}$ (8)

$\underline{+G\circlearrowleft}$: $-I_G \omega_1 + F_f r t = -I_G \omega_3$ (9)

Rolling without slipping : $v_{A3} = r\omega_3$ (10)

$F_f = \mu_k N_A$ (11)

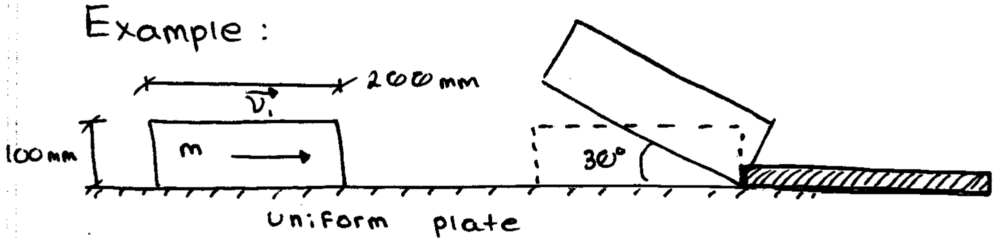
Since $I_G = \frac{2}{5} m r^2$

(7) ~ (11) : $\omega_{A3} = \frac{2}{7} \cdot \frac{v_1}{r}$; $v_{A3} = \frac{2}{7} v_1$

Sphere B :

$\omega_{B3} = \frac{5}{7} \cdot \frac{v_1}{r}$; $v_{B3} = \frac{5}{7} v_1$

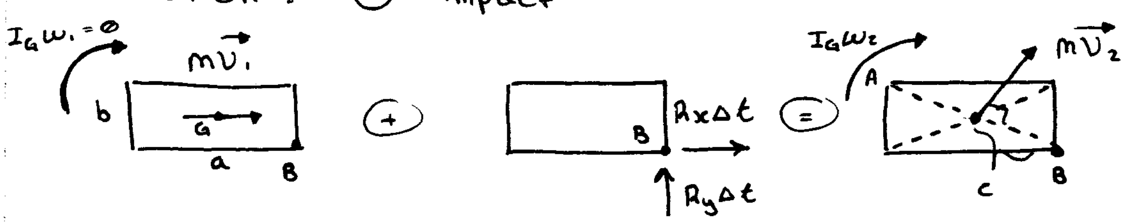
Example :



Impact : Perfectly plastic ($e = \emptyset$)

Find $\underline{\vec{v}_1}$.

Solution : (1) impact



$\underline{+B\circlearrowleft}$: $-m v_1 (b/2) + \emptyset = \underbrace{-I_G\omega_2 - m v_2 \cdot c}_{\text{OR USE } -I_B\omega_2}$

$AB = d = \sqrt{a^2+b^2}$

$GB = 1/2\, d = 1/2 \sqrt{a^2+b^2}$

$I_B = I_G + m GB^2 = 1/12\, m(a^2+b^2) + m\left(\frac{\sqrt{a^2+b^2}}{}\right)$

$I_B = \frac{1}{3} m(a^2 + b^2)$

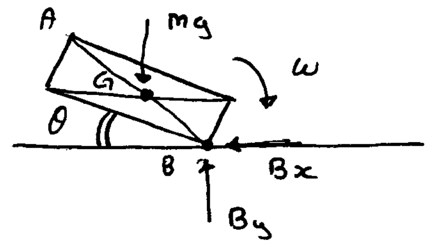
$\Rightarrow \quad \dfrac{m v_1 b}{2} = \dfrac{1}{3} m(a^2 + b^2)\omega_2$

$\Rightarrow \quad v_1 = \frac{2}{3} \cdot \left(\dfrac{a^2 + b^2}{b}\right)\omega_2$

(2) Rotation :

$\theta = 0, \ \omega_2 :$

$\theta = 30°, \ \omega = 0$



Conservation of energy :

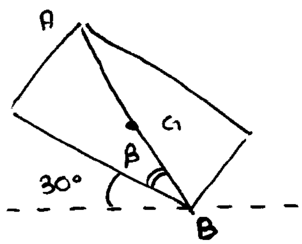
$T_1 + V_1 = T_2 + V_2$

$T_1 = \frac{1}{2} I_B \omega_2^2 = \frac{1}{2} \cdot \frac{1}{3} m(a^2 + b^2)\omega_2^2$

$V_1 = \ldots$

Nov. 29/17
DYNAMICS

$V_{1A} = mg\frac{b}{2} = mg \times \frac{0.1}{2} = 0.05mg$



$T_2 = 0$

$V_2 = mg \cdot \frac{AB}{2} \cdot \sin(30° + \beta)$

$\tan\beta = \frac{b}{a} = \frac{100}{200}$; $\beta = 26.565°$

$V_2 = mg \cdot \frac{\sqrt{0.2^2 + 0.1^2}}{2} \sin(30° + 26.565°)$

$\Rightarrow \frac{1}{6} m(0.2^2 + 0.1^2)\omega_2^2 + 0.05mg$

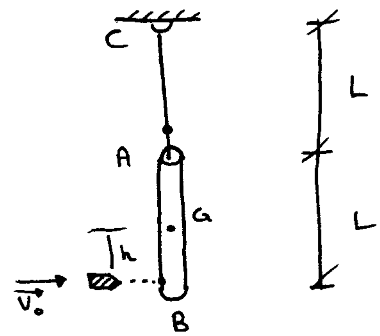
$= 0 + mg \cdot \frac{\sqrt{0.2^2 + 0.1^2}}{2} \sin(56.565°)$

$\Rightarrow \omega_2 = 7.1396$ rad/s

$\Rightarrow V_1 = \frac{2}{3}\frac{a^2 + b^2}{b}\omega_2 = \frac{2}{3}\frac{(0.2)^2 + 0.1^2}{0.1} \times 7.1396$

$= 2.38$ m/s

Example :



$L = 30$ in

$W_{AB} = 15$ lb

$W_b = 0.08$ lb

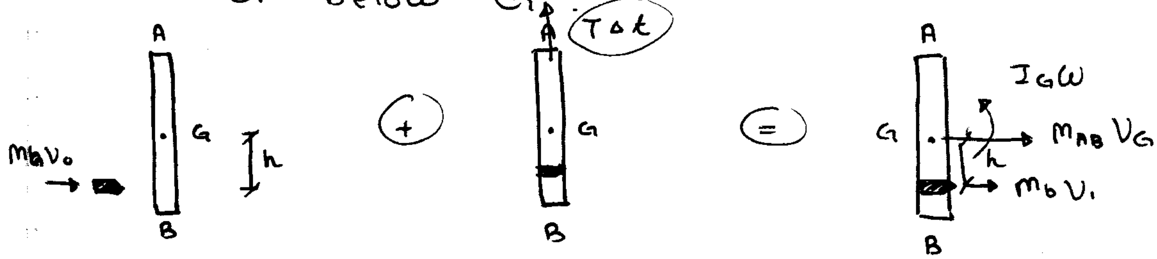
$v_0 = 1800$ ft/s

C is the IC of zero velocity of AB

Find h

Impact problem (don't worry about weight)

- Consider support force
- how do you know if the distance h is above or below G?



x: $m_b v_0 + 0 = m_{AB} v_G + m_b v_1$

Angular momentum $+G\circlearrowright$: $m_b v_0 h + 0 = I_G \omega + m_b v_1 h$

[y: $0 + T\Delta t = 0$ then $T = 0$ during impact]

$v_G = (1.5L)\,\omega$

$v_1 = (1.5L + h)\,\omega$ $\Rightarrow$ $h = L/18$

$\hookrightarrow$ between $0 \rightarrow \frac{1}{2}L$