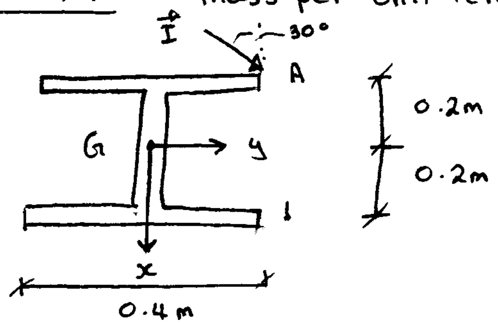
Nov. 21/17

Dynamics II

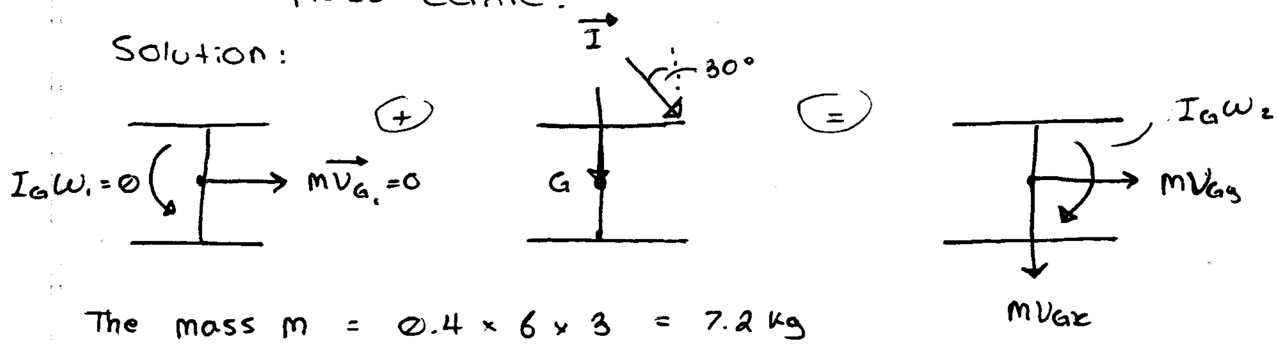
Example Mass per unit length : 6 kg/m

$\vec{I}$ = 10 N·s



Find the angular velocity and the magnitude of its mass centre.

Solution:



The mass $m = 0.4 \times 6 \times 3 = 7.2$ kg

The moment of inertia

$$I_G = \frac{1}{12} m_1 L^2 + \left[\frac{1}{12} m_1 L^2 + m_1 \left(\frac{L}{2}\right)^2\right] \times 2$$

$$= (1/12)(6)(0.4)(0.4)^2 + \left[1/12(6)(0.4)(0.4)^2 + (6)(0.4)(0.2)^2\right] \times 2$$

$$= 0.288 \ \text{kg} \cdot \text{m}^2$$

x: $0 + I\cos(30°) = mV_{Gx}$

$(10)\cos 30° = 7.2 V_{Gx} \rightarrow V_{Gx} = 1.203$ m/s

y: $0 + I\sin(30°) = mV_{Gy}$

$(10)\sin 30° = 7.2 V_{Gy} \rightarrow V_{Gy} = 0.6944$ m/s

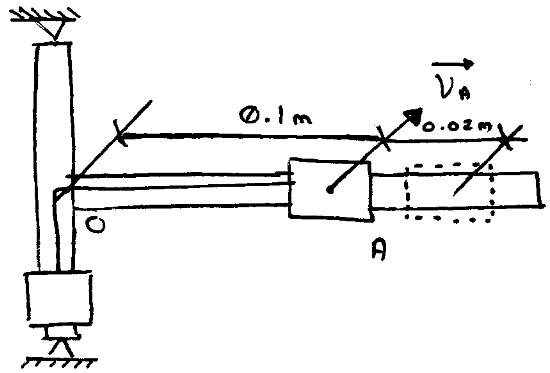
Angular momentum about G:

$$0 + \left[-(10)\cos 30°(0.2) - (10)\sin 30°(0.2)\right] = -I_G\omega_2$$

$$\Rightarrow (-10)\cos 30°(0.2) - (10)\sin 30°(0.2) = -0.288\,\omega_2$$

$\omega_2 = 9.49$ rad/s

Example :



$m_A = 1.8$ kg

$m_B = 0.7$ kg

(Ø.1m): $V_A = 2.1$ m/s

(Ø.12m): $V_A = 2.5$ m/s

Find the angular velocity of the Frame at that instant and the moment of inertia of the Frame.

Solution: FBD

Conservation of energy

Conservation of angular momentum about the rotating axis

→ At position 1, $V_A = 2.1$ m/s, $V_B = 0$

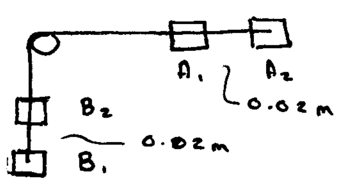
$V = r\omega = OA \cdot \omega \Rightarrow \omega = 21$ rad/s

$\therefore T_1 = \frac{1}{2} I\omega^2 + \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$

$\Rightarrow \frac{1}{2} I (21)^2 + (\frac{1}{2})(1.8)(2.1)^2 + 0$

$V_1 = 0$

→ At position 2



$\vec{V}_{A2} = \vec{V}_{Ar2} + \vec{V}_{A\theta 2}$

$V_{A2} = OA \cdot \omega_2 = 0.12\,\omega_2$

Since $V_{A2} = \sqrt{V_{Ar2}^2 + V_{A\theta 2}^2}$

$2.5 = \sqrt{(V_{Ar2})^2 + (0.12\omega_2)^2}$

$V_{Ar2} = \sqrt{(2.5)^2 - (0.12\omega_2)^2}$

$\therefore V_{B2} = V_{Ar2} = \sqrt{(2.5)^2 - (0.12\omega_2)^2}$

$\therefore T_2 = (\frac{1}{2}) I\omega_2^2 + (\frac{1}{2}) m_A V_{A2}^2 + (\frac{1}{2}) m_B V_{B2}^2$

$= (\frac{1}{2}) I\omega_2^2 + (\frac{1}{2})(1.8)(2.5)^2 + (\frac{1}{2})(0.7)(2.5^2 - (0.12\omega_2)^2)$

$V_2 = m_B g h = (0.7)(9.81)(0.02)$

$$\Rightarrow (1/2)\, I\, (21)^2 + (1/2)(1.8)(2.1)^2 + \emptyset + \emptyset \cdots$$

$$(1)\quad \cdots = (1/2)\, I\, \omega_2^{\,2} + (1/2)(1.8)(2.5)^2 + (1/2)(0.7)(2.5^2 - (0.12\,\omega_2)^2) + (0.7)(9.81)(0.02)$$

At position 1: (time 1)

$$H_{G1} = I\omega_1 + OA \cdot m_A v_{A1}$$
$$= (21)\, I + \emptyset.1 \times (1.8) \times (2.1)$$

At position 2: (time 2)

$$H_{G2} = I\omega_2 + OA \cdot m_A v_{A02}$$
$$= I\omega_2 + \emptyset.12 \times 1.8 \times \emptyset.12\, \omega_2$$

$$(2)\quad \Rightarrow (21) I + [0.01 \times 1.8 \times 2.1] = I\, \omega_2 + [\emptyset.12 \times 1.8 \times 0.12\,\omega_2]$$

$\hookrightarrow$ $I = \emptyset.\emptyset 5\emptyset 8$ kg·m^2 $\qquad \omega_2 = 18.83$ rad/s

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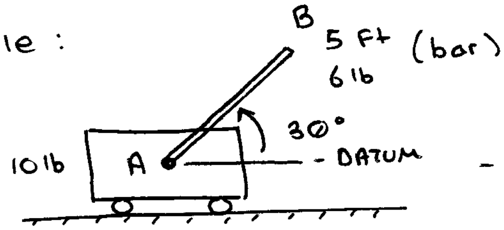
DYNAMICS II

Example :



- neglect friction

Find a) the velocity of the point B as a bar passes through a vertical position

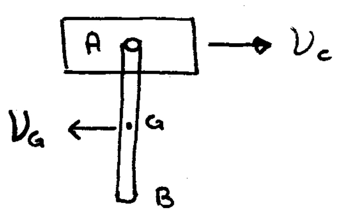
b) the corresponding velocity of the cart

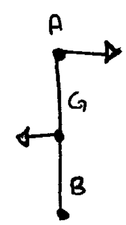
Given $g = 32.2 \text{ ft/s}^2$

Solution : FBD

No horizontal external forces

- Conservation of linear momentum in the horizontal direction
- Only work done is by the weight (conservation of energy)



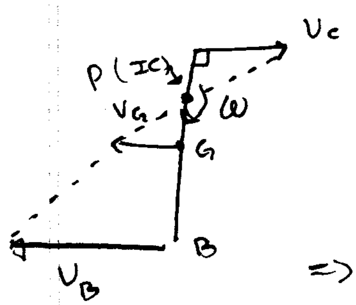


$\vec{v}_G = \vec{v}_{G/A} + \vec{v}_A$

$\underline{x}$:

$$\emptyset = \frac{W_c}{g} v_c - \frac{W_b}{g} v_G$$

$$\emptyset = 10/g\, v_c - 6/g\, v_G \quad \Rightarrow \quad v_G = 5/3\, v_c$$



P : The I.C. of AB

$$\begin{cases} \dfrac{GP}{PA} = \dfrac{v_G}{v_c} = \dfrac{5}{3} \\ GP + PA = 1/2\, AB = 1/2(5) = 2.5 \end{cases}$$

$\Rightarrow GP = 1.5625$

$\Rightarrow v_G = GP \cdot \omega$

$$\omega = \frac{v_G}{GP} = \frac{(5/3)\, v_c}{(1.5625)} = \frac{16}{15} v_c$$

<u>POSITION 1</u> : $T_1 = \emptyset$

$V_1 = mgh$

$= (6)(5/2 \sin 30°)$

$= 7.5$

<u>Position 2</u> : $T_2 = \frac{1}{2} m_C v_C^2 + \frac{1}{2} m_G v_G^2 + \frac{1}{2} \bar{I}_G \omega^2$

$= (1/2)(\frac{10}{32.2}) v_C^2 + (1/2)(6/32.2)(5/3\, v_C)^2 \ldots$

$\ldots + (1/2)(1/12)(6/32.2)(5^2)(16/15\, v_C)^2$

$= 0.63492\, v_C^2$

$V_2 = mgh = 6(-5/2) = -15$

$T_1 + V_1 = T_2 + V_2$

$\emptyset + 7.5 = 0.63492\, v_C^2 - 15$

$v_C = 5.9529$ ft/s

$v_B = PB \cdot \omega$

$= (1.5625 + 2.5) \times (\frac{16}{15}) \times (5.9529)$

$= 25.80$ ft/s

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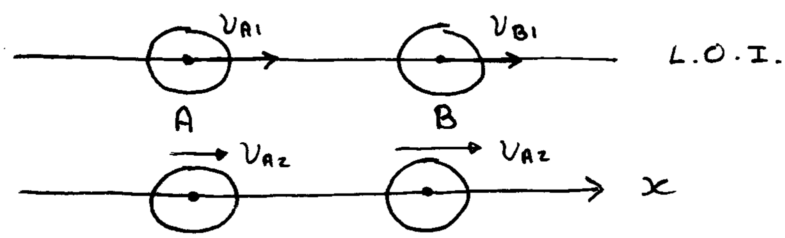
DYNAMICS

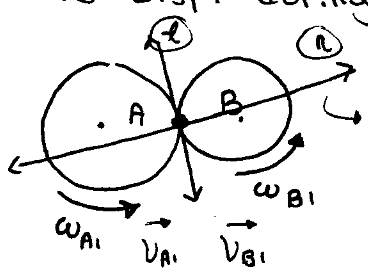
17.12 Eccentric Impact



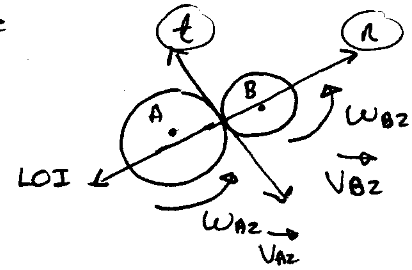
Collision of particles
$$\begin{cases} m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2} \\ e = -\left(\dfrac{v_{A2} - v_{B2}}{v_{A1} - v_{B1}}\right) \end{cases}$$

* Principle of impulse and momentum

* No disp. during the impact



↳ not necessarily through mass centre



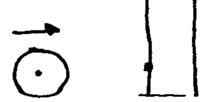
before

$$\vec{v}_{A1} = \vec{v}_{A1n}\,\vec{e}_n + \vec{v}_{A1t}\,\vec{e}_t$$
$$\vec{v}_{B1} = \vec{v}_{B1n}\,\vec{e}_n + \vec{v}_{B1t}\,\vec{e}_t$$

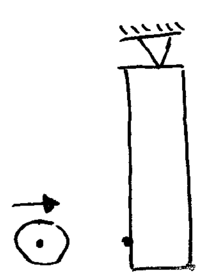
after

$$\vec{v}_{A2} = \vec{v}_{A2n}\,\vec{e}_n + \vec{v}_{A2t}\,\vec{e}_t$$
$$\vec{v}_{B2} = \vec{v}_{B2n}\,\vec{e}_n + \vec{v}_{B2t}\,\vec{e}_t$$

$$e = -\left(\frac{v_{A2n} - v_{B2n}}{v_{A1n} - v_{B1n}}\right)$$

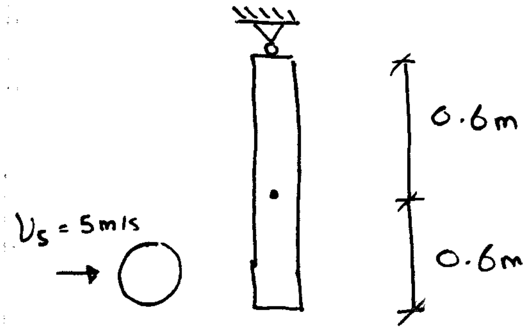


Free impact



Constraint impact

Example



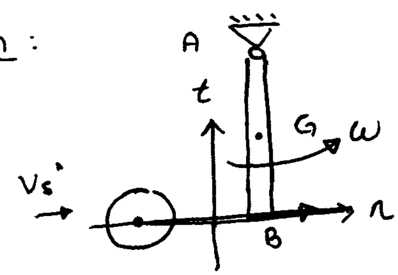
$m_{AB} = 8$ kg

$m_S = 2$ kg

$e = 0.8$

Determine the angular velocity of the rod and the velocity of the sphere immediately after impact.

Solution:



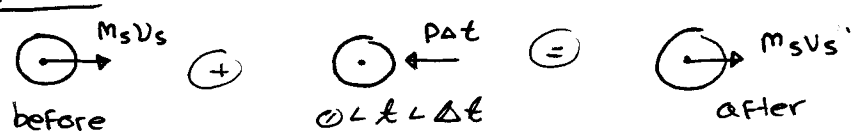
$$V_B' = AB \cdot \omega = 1.2\omega$$

$$e = -\left(\frac{V_s' - V_B'}{V_s - V_B}\right)$$

$$0.8 = -\left(\frac{V_s' - 1.2\omega}{5 - 0}\right)$$

$$\Rightarrow 1.2\omega - V_s' = 4 \quad \cdots \text{(1)}$$

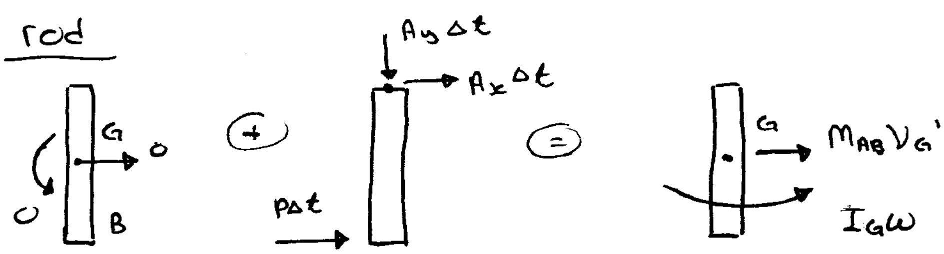
Sphere:



$$m_s V_s - P\Delta t = m_s V_s'$$

$$2(5) - P\Delta t = 2V_s \quad \cdots \text{(2)}$$

rod



$x:\quad \emptyset + P\Delta t + A_x\Delta t = m_{AB}V_G' = 8(0.6)\omega$

$P\Delta t + A_x\Delta t = 4.8\,\omega$

$y:\quad \emptyset + A_y\Delta t = \emptyset \Rightarrow A_y = \emptyset$

$+\circlearrowright :\quad \emptyset + P\Delta t \times \emptyset.6 - A_x\Delta t \times \emptyset.6 = I_G\omega$

$(P\Delta t - A_x\Delta t(\cdots))$

$P\Delta t - A_x\Delta t = \frac{1}{0.6} \times \frac{1}{12} \times 8 \times 1.2^2\omega = 1.6\,\omega \quad (4)$

$P\Delta t,\ A_x\Delta t,\ V_s',\ \omega$: unknown

(3)(4) $2P\Delta t : 6.4\,\omega$

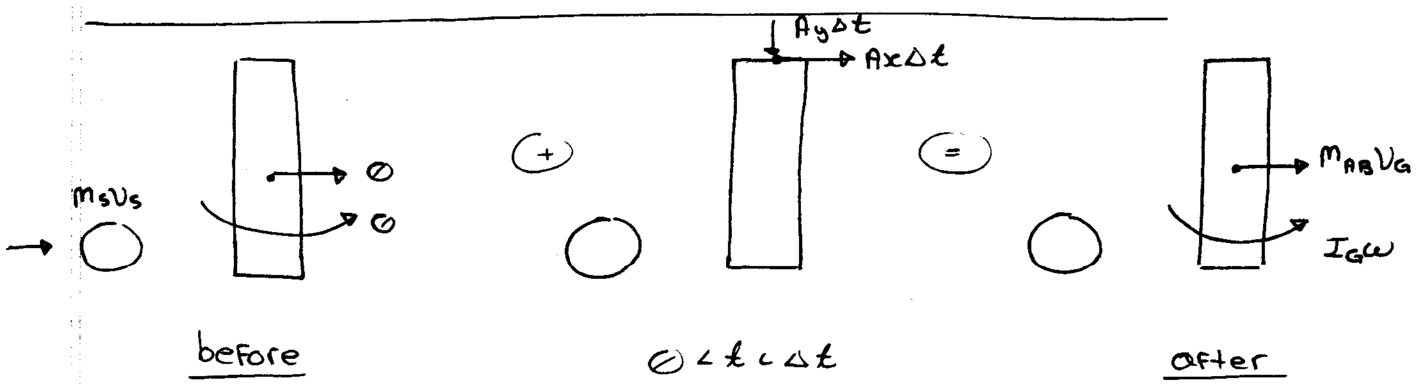
$P\Delta t : 3.2\omega \quad \cdots \quad (5)$

(5) into (2) $\quad 2(5) - 3.2\omega = 2V_s'$

$\Rightarrow \quad V_s' + 1.6\omega = 5 \quad \cdots \quad (6)$

(1)(6) :

$$\begin{cases} \omega = 9/2.8 = 3.2143 \text{ rad/s} \\ V_s' = -\emptyset.14286 \text{ m/s} \end{cases}$$

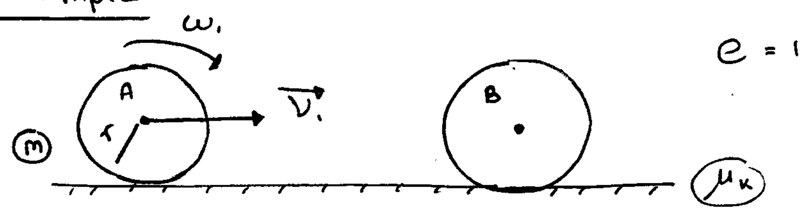


$A\circlearrowright :\quad m_sV_s \times AB + \emptyset + \emptyset = m_sV_s' \times AB + I_A\omega$

$2 \times 5 \times 1.2 = 2 \times V_s' \times 1.2 + \frac{1}{3} \times 8 \times 1.2^2\omega$

$\Rightarrow \quad 5 = V_s' + 1.6\omega$

Example

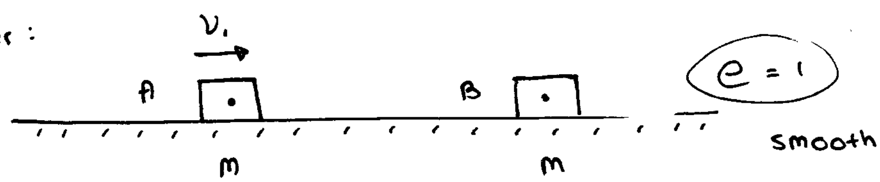


rolling w/o slipping

Find 1) the linear and angular velocities of each sphere immediately after impact

2) the velocity of each sphere after it has started rolling w/o slipping

Another :



After : $v_A = \emptyset$, $v_B = v_1$