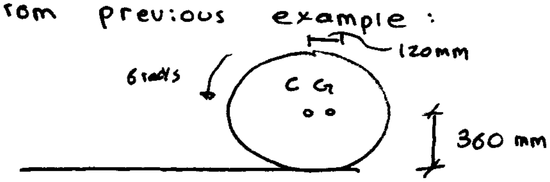
Nov. 14/17

Dynamics II

From previous example:



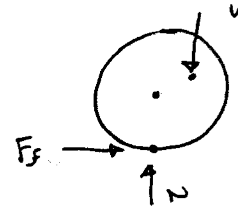
$m = 50$ kg

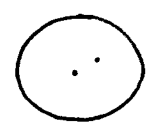
$k_G = 160$ mm

(rolling without slipping)

- Find the normal and friction forces exerted on the disk by the surface when the disc has rotated 210°

FBD:



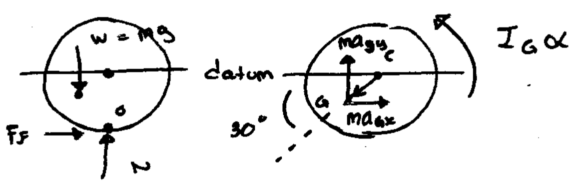


(repeated info)

$T_1 + V_1 = T_2 + V_2$

$T_1 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

...



$\Sigma F_x = ma_{Gx}$: $F_f = ma_{Gx}$

$\Sigma F_y = ma_{Gy}$: $N - W = ma_{Gy}$

$\Sigma M_G = I_G \alpha$:

$\vec{a}_G = \vec{a}_C + \vec{a}_{G/C}$

$\vec{a}_G = \vec{a}_C + \alpha \vec{k} \times \vec{GC} + \omega \vec{k} \times (\omega \vec{k} \times \vec{GC})$

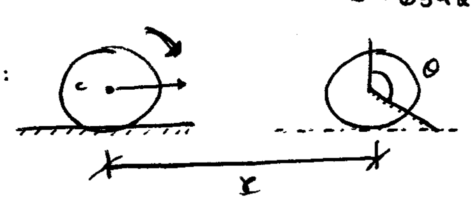
$\Rightarrow \vec{a}_G = \vec{a}_C + \alpha \vec{k} \times \vec{GC} - \omega^2 \cdot \vec{GC}$

Here $\vec{GC} = -GC \cos(30°)\, i - GC \sin(30°)\, j$

$= -0.12 \cos(30°)\, i - (0.12) \sin(30°)\, j$

$\vec{GC} = -0.10392\, i - 0.0600\, j$

Consider:



$$\boxed{\begin{aligned} x &= r\theta \\ v &= r\omega \\ a &= r\alpha \end{aligned}}$$

$\vec{a}_C = -r\alpha i = (-0.36)\alpha \vec{i}$

$\vec{a}_G = -0.36 \alpha \vec{i} + \alpha \vec{k} \times (-0.10392 \vec{i} - 0.06 \vec{j})$

$- 62.311 (-0.10392 \vec{i} - 0.06 \vec{j})$

$$a_{gx}\vec{i} + a_{gy}\vec{j} = -0.36\,\alpha\,\vec{i} - 0.10392\,\alpha\,\vec{j} \cdots$$

$$\cdots + 0.06\,\alpha\,\vec{i} + 6.4754\,\vec{i} + 3.7387\,\vec{j}$$

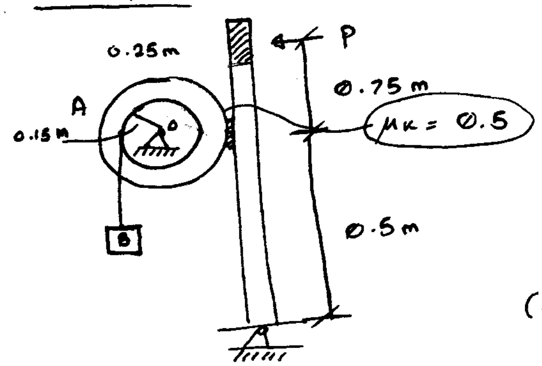
$$\Rightarrow \begin{cases} a_{Gx} = 6.4754 - 0.24\,\alpha \\ a_{Gy} = 3.7387 - 0.10392\,\alpha \end{cases}$$

$$\Rightarrow \quad \alpha = 31.513 \ \text{rad/s}^2$$

$$F = -54.4 \ \text{N}$$

$$N = 514.0 \ \text{N}$$

Example:



Drum: $m = 50$ kg

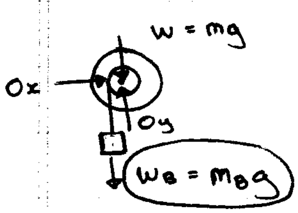
$k_o = 0.23$ m

Block: $m_B = 15$ kg

(1) Determine the speed of the block after it falls down 3m from rest.

(2) Determine the force P that must be applied at the brake handle which will then stop the block after it decends another 3m.

Solution 1° – FBD



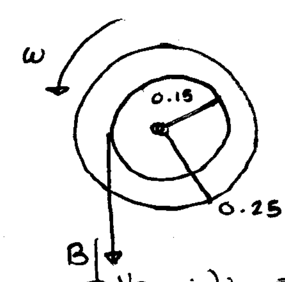
$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = 0 \ ; \ V_1 = 0$$

$$V_2 = -m_B g h$$

$$T_2 = \ ?$$

$$\boxed{I_O = m k_o^2}$$



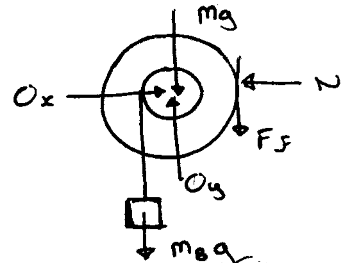
$$\therefore V_B = 2.58 \ \text{m/s}$$

$$T_2 = \tfrac{1}{2} m_B V_B^2 + \tfrac{1}{2} I_o \omega^2$$

$$= (\tfrac{1}{2})(15) V_B^2 + (\tfrac{1}{2})(50)(0.23)^2 \left(\frac{V_B}{0.15}\right)^2$$

$$\Rightarrow 0 + 0 = \tfrac{1}{2}(15) V_B^2 + (\tfrac{1}{2})(50)(0.23)^2 \left(\frac{V_B}{0.15}\right)^2 - 15(9.81)(3)$$

(2)



Position 1 : $T_1 = 0$

Position 2 : $T_2 = 0$

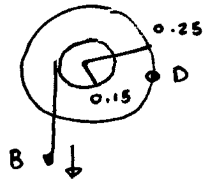
$U_{1\to 2}\,(W_B) = m_B g \times 6$

$U_{1\to 2}\,(F_f) =$

$\theta = \frac{S_B}{r} = \frac{6}{0.15}$

3?

$S_D = (0.25)\,\theta = 0.25\left(\frac{6}{0.15}\right)$

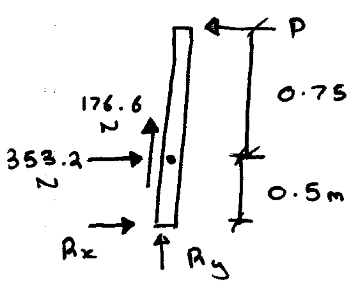


$T_1 + U_{1\to 2} = T_2$

$0 + (6)(15)(9.81) - F(0.25)\left(\frac{6}{0.15}\right) = 0$

$\Rightarrow \quad F = 176.6\ N$

The normal $N = F/\mu_k = \frac{176.6}{0.5} = 353.2$



$\Sigma M_A = 0$

$P(0.7 + 0.5) - 353.2(0.5) = 0$

$P = 141\ N$

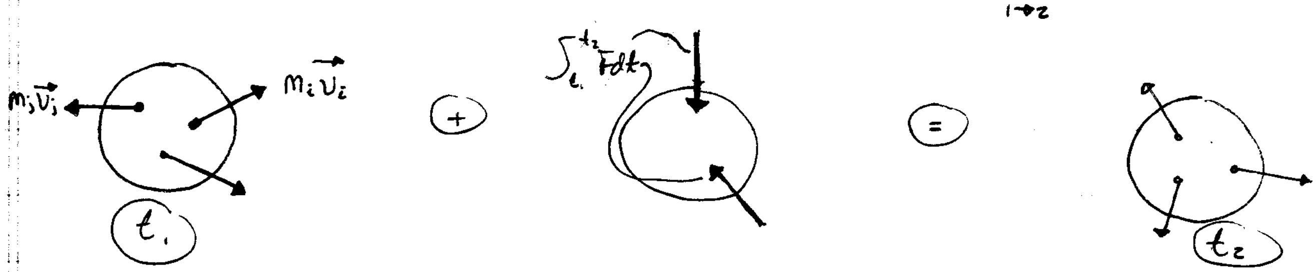
Nov. 15/17

Dynamics II

17.8 - Principle of Impulse and Momentum

* time and velocity
* impact

System momentum$_1$ + System external impulse$_{1\to2}$ = Sys. momentum$_2$

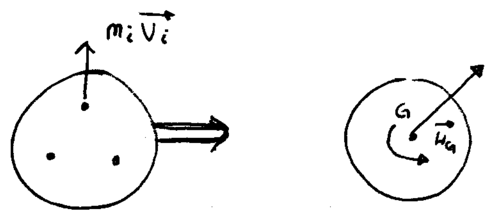


linear momentum

$$\vec{L} = m\vec{v}_G$$

Angular momentum about mass centre G :

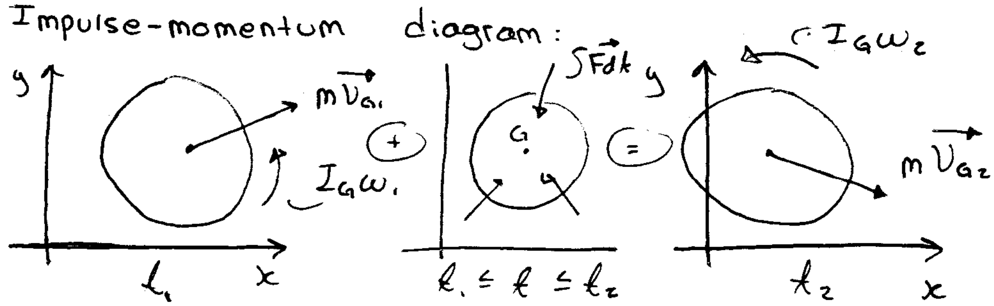
$$\vec{H}_G = I_G\vec{\omega}$$



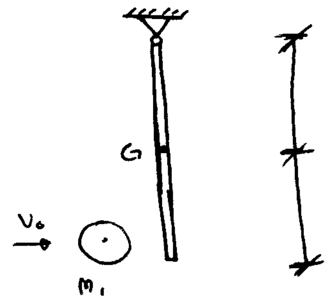
$$\vec{L} = m\vec{v}_G$$

$$= I_G\vec{\omega}$$

Impulse-momentum diagram:



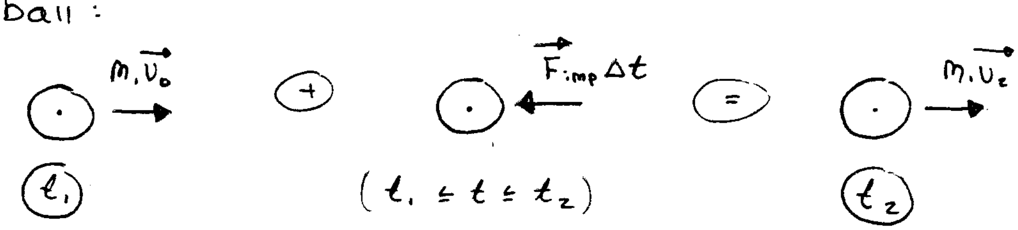
Example :



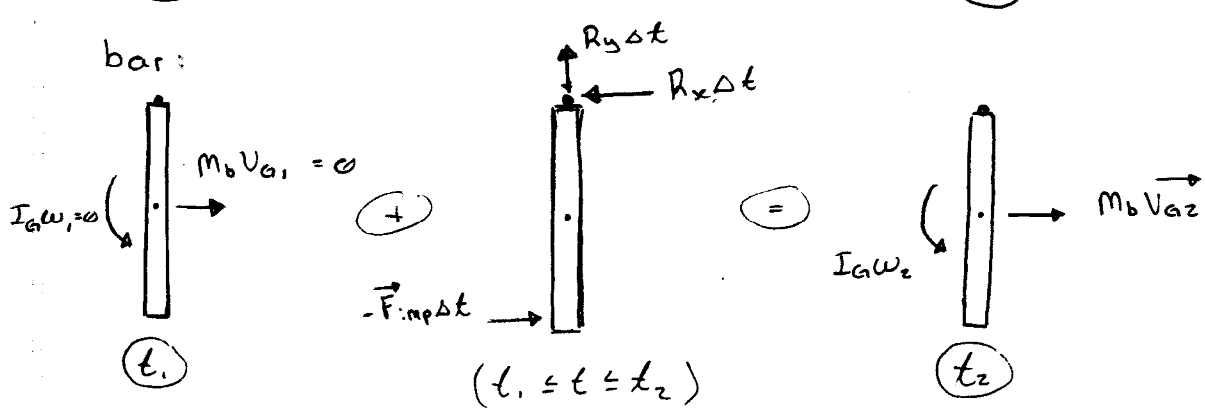
Draw the impulse - momentum diagram for the ball and the bar separately. Time 1 is immediately before the impact and time 2 is immediately after.

Solution :

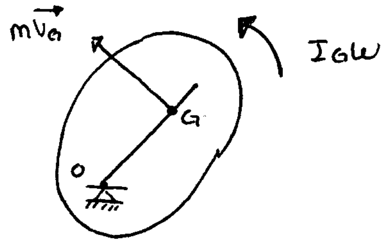
ball :



bar :



Fixed-axis Rotation:



The angular momentum about O:

$$I_o \omega = I_G\omega + m V_G(\overline{OG})$$

$$\Rightarrow I_G\omega + m\,OG^2\cdot\omega$$

$$\Rightarrow \omega(I_G + m\,OG^2)$$

$V_G = OG\cdot\omega$

Principle of angular impulse and momentum:

$$I_o\omega_1 + \Sigma\int_{t_1}^{t_2} M_o\,dt = I_o\omega_2$$

17.9 - Systems of Rigid Bodies

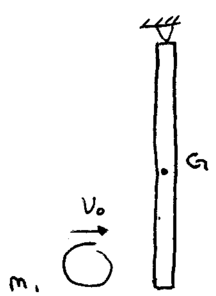
17.10 - Conservation of Angular Momentum

↳ 1° no external force acts on a rigid body or a system of rigid bodies

$\vec{L_1} = \vec{L_2}$ and $(H_o)_1 = (H_o)_2$

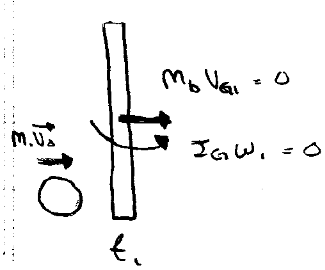
2° The sum of the angular impulse about O is zero:

$(H_o)_1 = (H_o)_2$



Angular momentum about G, conserved? - NO

Angular momentum about O, conserved? YES!



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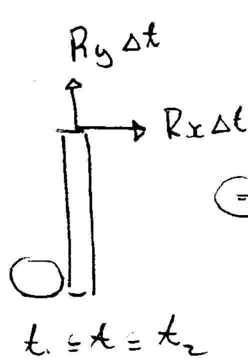

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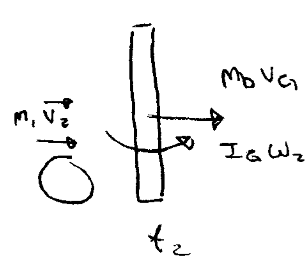
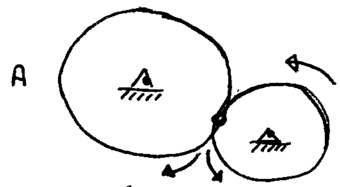


Example :

$M = 6\ N \cdot m$



$\omega_A = \omega_B$ (no slip)

$r_A = 250$ mm

$r_B = 100$ mm

$m_A = 10$ kg $m_B = 3$ kg

$K_A = 200$ mm $K_B = 80$ mm

1) Determine the time required for gear B to reach an angular velocity of 600 rpm.

2) The tangential force exerted by gear B on gear A.

Solution :

$$\omega_B = 600\ \text{rpm} \times \frac{2\pi}{60} = 2\pi\ \text{rad/s}$$

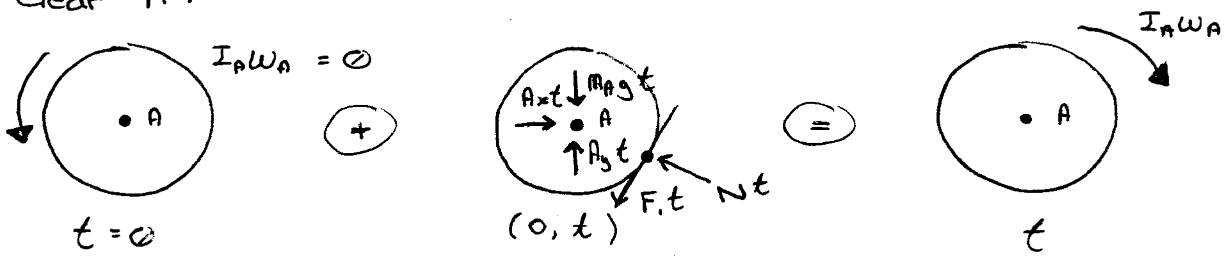
$$r_A/\omega_A = r_B/\omega_B$$

$$\omega_A = \frac{r_A}{r_B}\omega_B = \frac{100}{250} \times 20\pi = 8\pi\ \text{rad/s}$$

$$I_A = m_A K_A^2 = 10\left(\frac{200}{1000}\right)^2 = 0.4$$

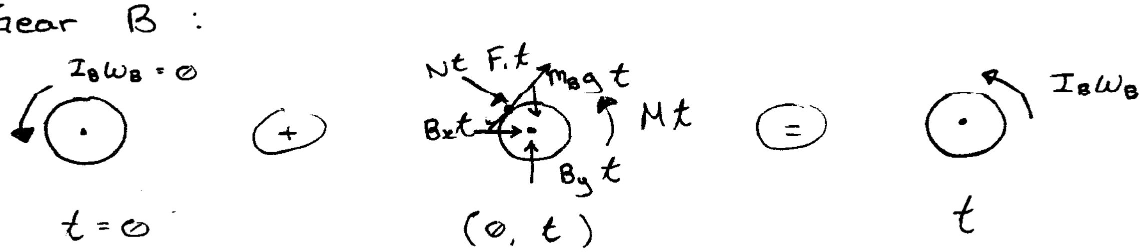
$$I_B = m_B K_B^2 = 3\left(\frac{80}{1000}\right)^2 = 0.0192$$

Gear A :



$$0 + (-Ft \cdot r_A) = -I_A\omega_A \quad (1)$$

Gear B :



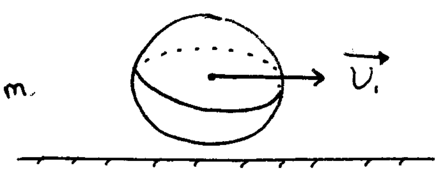
$$0 + (Mt - Ft \cdot r_B) = I_B\omega_B \quad (2)$$

① : $F \cdot t \cdot (0.25) = 0.4(8\pi)$

② : $6t - Ft(0.1) = 0.0192(20\pi)$

$$\Rightarrow \begin{cases} t = 0.871\,s \\ F = 46.2\,N \end{cases}$$

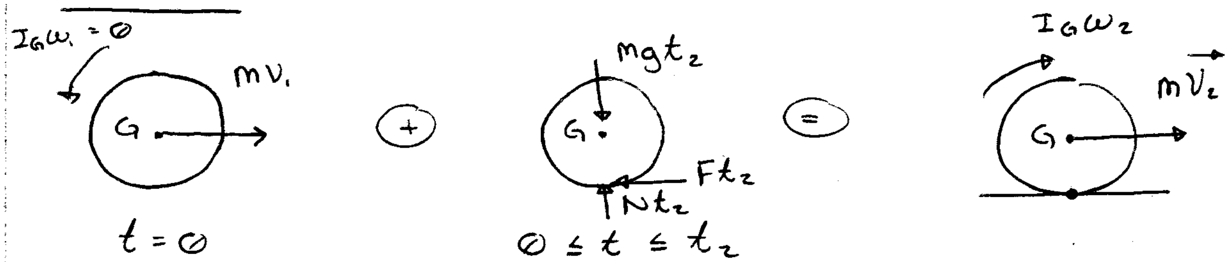
Example :



Determine a) The time at which the sphere will start rolling without slipping

b) The velocity of the sphere at time t_2

Solution :



y component : $0 + Nt_2 - mgt_2 = 0$

$N = mg$

x component : $mv_1 - Ft_2 = mv_2$

Angular moment about G : $0 + Fr\, t_2 = I_G\omega_2$

$\Rightarrow 2/5\, mr^2\omega_2$

Unknowns : F_1, t_2, v_2, ω_2

$F = \mu_k N = \mu_k mg$

Rolls w/o slipping : $v_2 = r\omega_2$

$\Rightarrow t_2 = (2/7)(v_1/\mu_k g)$ $\Rightarrow v_2 = (5/7)v_1$ $\Rightarrow \omega_2 = (5/7)(v_1/r)$

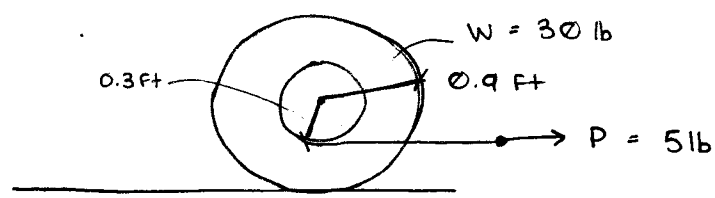
For a bowling ball:

$m = 7\ kg$ $\qquad r = 10.86\ cm$ $\qquad \mu_k = 0.1$

$v_1 = 8\ m/s$ $\qquad t_2 = 2.33\ s$

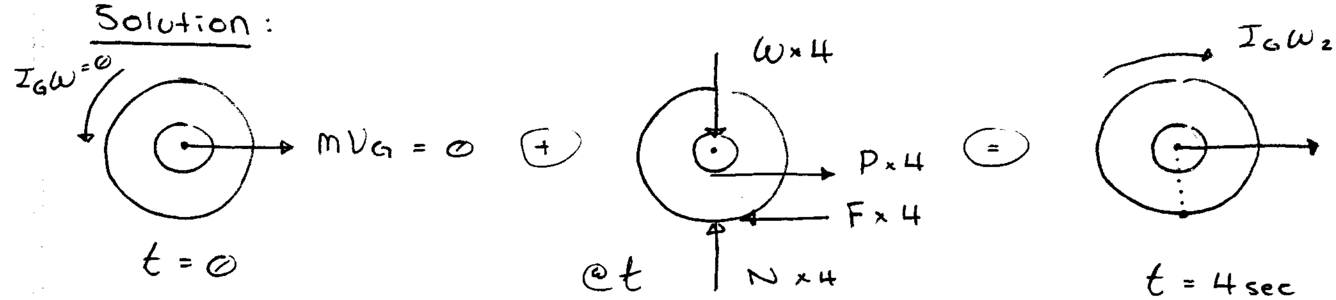
Example:

$K_o = 0.45\ ft$



Find the angular velocity of the disk at $t = 4$ sec. Rolling without slipping.

Solution:



$\underline{y}$: $\quad 0 + N \times 4 - W \times 4 = 0 \quad \Rightarrow \quad N = W = 30\ lb$

x: $\quad 0 + P \times 4 - F \times 4 = mV_{G2}$

$$5 \times 4 - 4F = \frac{30}{32.2} V_{G2} \qquad (1)$$

Angular moment about O:

$$0 + P \times 4 \times 0.3 - F \times 4 \times 0.9 = -I_G \omega_2$$

$$\rightarrow 5 \times 4 \times 0.3 - F \times 4 \times 0.9 = -\left(\frac{30}{32.2}\right)(0.45)^2 \omega_2 \qquad (2)$$

Kinematics: $\quad V_{G2} = r\omega$

$$V_{G2} = 0.9\,\omega_2 \qquad (3)$$

$\Rightarrow \quad \omega_2 = 12.72\ rad/s$

$F = 2.333\ lb$