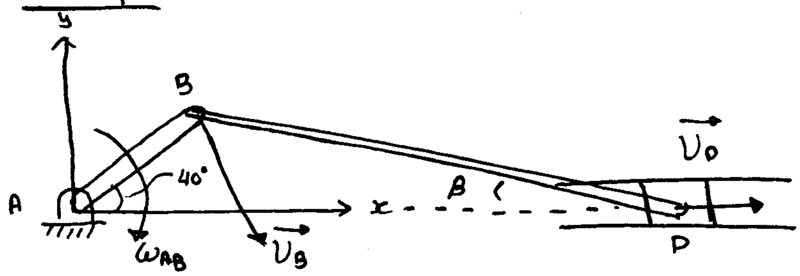
Nov. 7/17

Dynamics II

Example :

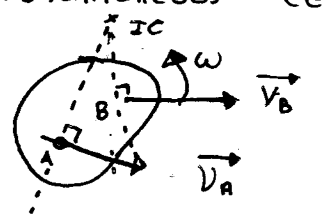


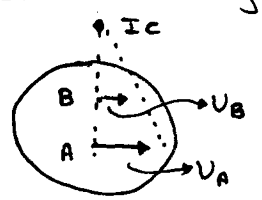
$\omega_{AB} = 2000$ rpm

$AB = 3$ in

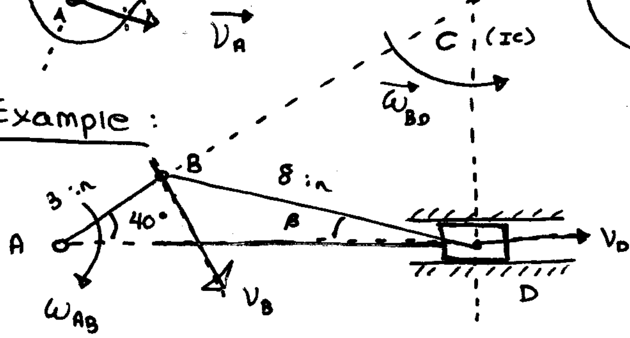
$BD = 8$ in

Instantaneous center of zero velocity :





Example :



$\beta = 13.95°$

$$V_B = AB \cdot \omega_{AB} = BC \cdot \omega_{BD}$$

$$\Rightarrow \omega_{BD} = \frac{AB}{BC}\omega_{AB}$$

$$\triangle BCD : \frac{BC}{\sin(90°+\beta)} = \frac{CD}{\sin(40°+\beta)} = \frac{BD}{\sin(50)}$$

$$\Rightarrow BC = \frac{(\sin(90°-\beta))}{\sin 50°} BD = 10.14$$

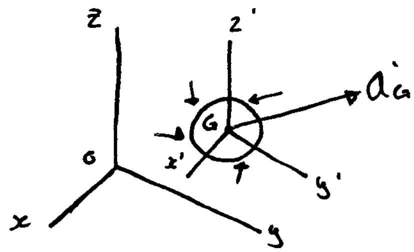
$$\Rightarrow CD = \frac{\sin(40°+\beta)}{\sin 50°} BD = 8.44$$

$$\therefore \omega_{BD} = \frac{AB}{BC}\omega_{AB} = \frac{3}{10.14}(209.4)$$

$$= 62.0 \text{ rad/s}$$

$$\therefore V_D = CD \cdot \omega_{BD} = 8.44(62.0) = 523.0 \text{ in/s}$$

Equations of motion for a rigid body:



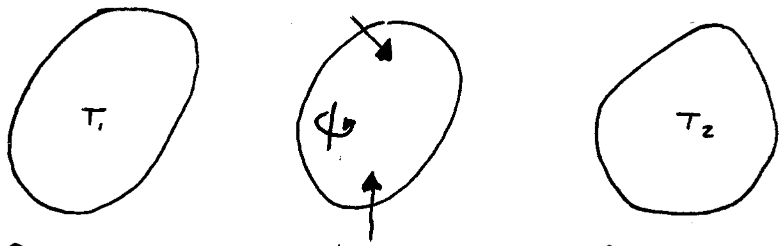
Translation:

$$\underline{\Sigma \vec{F} = m\vec{a}_G}$$

Rotation:

$$\underline{\Sigma \vec{M}_G = \dot{\vec{H}}_G}$$

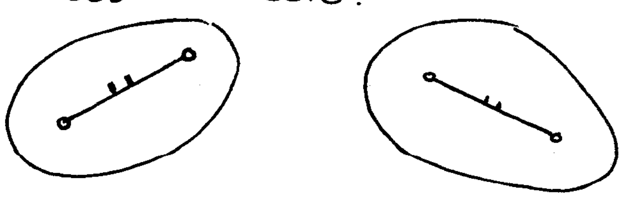
17.1 - Principle of Work and Energy

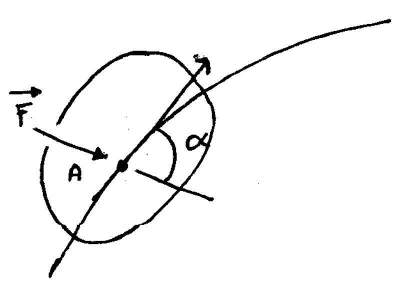


Position 1 $\quad U_{1\to 2} \quad$ Position 2

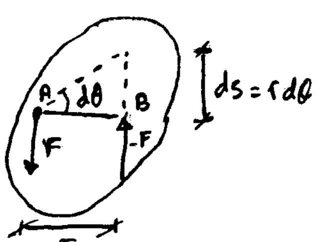
$$\boxed{T_1 + U_{1\to 2} = T_2}$$

For a rigid body, the net work done of internal forces is zero.





$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \Rightarrow \int_{s_1}^{s_2} F\cos(\alpha)\, ds$$



$$M = Fr$$

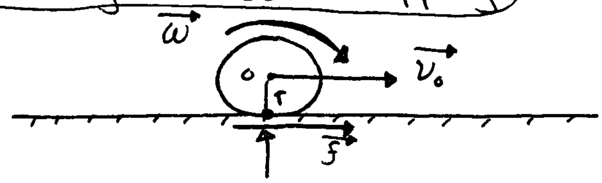
$$dU = F \cdot r\,d\theta = M\,d\theta$$

$$U_{1\to 2} = \int_{\theta_1}^{\theta_2} M\,d\theta$$

$\underline{M = \text{const}}$ $\qquad U_{1\to 2} = M(\theta_2 - \theta_1)$

* Friction force under the rotation of a disk:

rolling without slipping → no relative velocity at contact point. (For that moment)

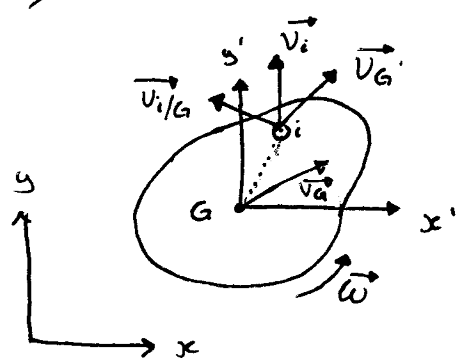


$$\boxed{v_o = r\omega}$$

$$dv = \vec{f} \cdot d\vec{r} = \vec{f} \cdot \cancel{\vec{v}\,dt}^{\,0} = 0$$

Kinetic energy:

$$\Delta T_i = \tfrac{1}{2}\Delta m_i \vec{v}_i \vec{v}_i$$



Total kinetic energy:

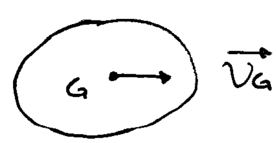
$$T = \sum_i \Delta T_i$$
$$= \sum_i \tfrac{1}{2}\Delta m_i \vec{v}_i \cdot \vec{v}_i$$
$$= \tfrac{1}{2}\left(\sum_i \Delta m_i\right) v_G^2 + \tfrac{1}{2}\left(\sum_i r_i^2 \Delta m_i\right)\omega^2$$

$$\begin{cases} m = \sum_i \Delta m_i \\ I_G = \sum r_i'^2 \Delta m_i \end{cases}$$

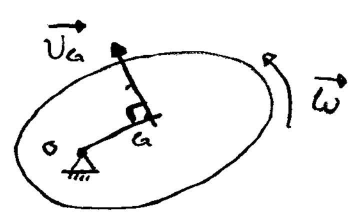
$$\boxed{T = \tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2}$$

Case 1: Translation

$$\omega = 0$$
$$T = \tfrac{1}{2} m v_G^2$$



Case 2: Rotation



$$v_G = OG \cdot \omega$$
$$T = \tfrac{1}{2} m (OG \cdot \omega)^2 + \tfrac{1}{2}(I_G)\omega^2$$
$$= \tfrac{1}{2}(J_G + m \cdot OG^2) \cdot \omega^2$$

$$I_O = I_G + m \cdot OG^2$$

$$\therefore \quad T = \tfrac{1}{2} I_O \omega^2$$

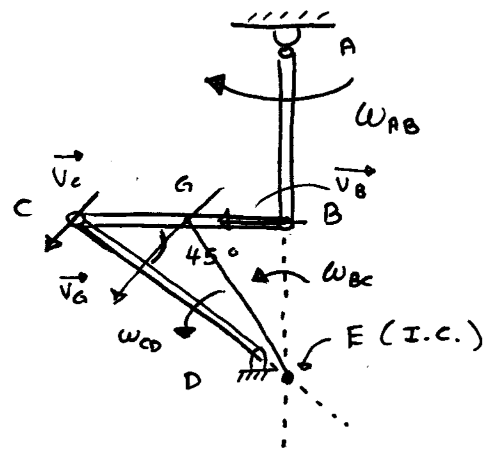
Example:

AB = 3 in $\qquad \omega_{AB} = 2$ rad/s

BC = 4 in $\qquad$ 0.5 lb/in

CD = 5 in

Determine the total kinetic energy of the system.



Example :

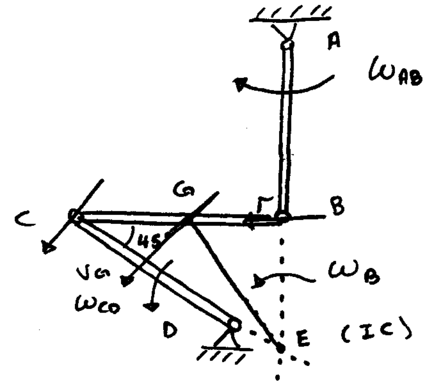
$\omega_{AB} = 2$ rad/s

0.5 lb/in

$AB = 3$ in

$BC = 4$ in

$CD = 5$ in



Solution: E is the IC of zero velocity of the bar BC

Bar AB : $V_B = AB\,\omega_{AB}$

Bar BC : $V_B = BE\,\omega_{BC}$

$\Rightarrow \omega_{BC} = \dfrac{AB}{BC}\omega_{AB}$

$AB = 3 \qquad BE = BC = 4$

$\omega_{BC} = 3/4 \times 2 = 1.5$

$V_C = CD\,\omega_{CD} = CE\,\omega_{BC}$ (*)

$\omega_{CD} = \dfrac{CE}{CD}\omega_{BC} = \dfrac{4\sqrt{2}}{5}\times 1.5 = \dfrac{6\sqrt{2}}{5}$

$= 1.697$

$V_G = GE\,\omega_{BC} = \sqrt{4^2+2^2}\times 1.5 = 6.708$

$\therefore \quad T_{AB} = \frac{1}{2} I_A \omega_{AB}^2$

$I_A = \frac{1}{3} m_{AB} AB^2$

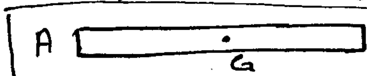
$= \frac{1}{3}\dfrac{(0.5)(3)}{(386.4)}(3)^2$

$I_{G,BC} = \frac{1}{12} m_{BC} BC^2$

$= \frac{1}{12}\dfrac{(4)(0.5)}{(386.4)}(4)$

$I_D = \frac{1}{3} m_{CD} CD^2 = \frac{1}{3}\dfrac{(5)(0.5)}{386.4}(5)^2$

m, L   By parallel axis theorem :



$I_G = \frac{1}{12} mL^2$

$I_A = I_G + md^2$

$= \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2$

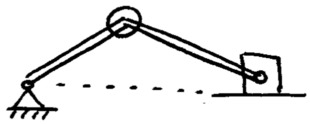
$= \frac{1}{3} mL^3$

$T_{BC} = \frac{1}{2} m_{BC} V_G^2 + \frac{1}{2} I_{G,BC}\,\omega_{BC}^2$

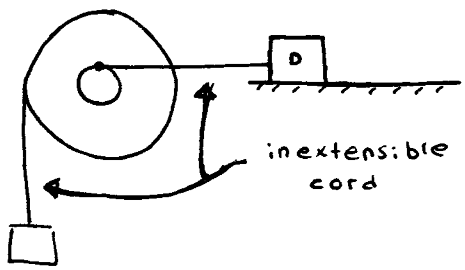
$T_{CD} = \frac{1}{2} I_D \omega_{CD}^2$

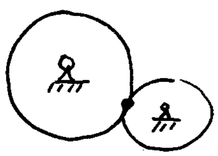
$\therefore \quad T = T_{AB} + T_{BC} + T_{CD} = \dfrac{87.00}{386.4} = 0.2252$ lb·in²

Systems of rigid bodies



(Pins)





meshed
gears

$$T_1 + U_{1\to 2} = T_2$$

Conservation of Energy

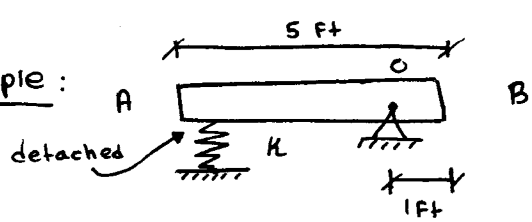
Conservative forces do the work

$$T_1 + V_1 = T_2 + V_2$$

$$\text{Power} = \frac{dv}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

$$\text{Power} = \frac{du}{dt} = \frac{M\,d\theta}{dt} = M\omega$$

Example:

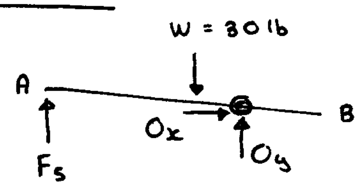


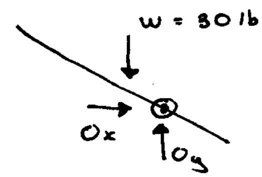
think newton's 2nd

Find the angular velocity and the reaction at the pivot as the rod passes through a vertical position.

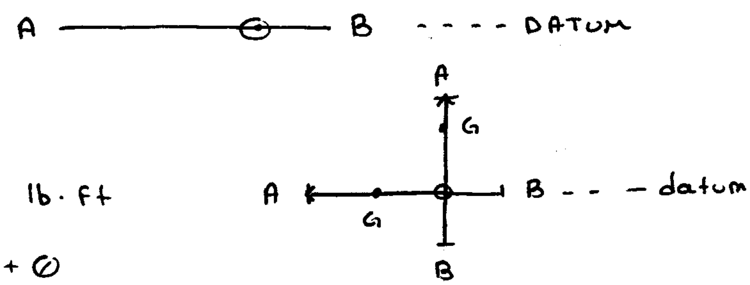
Given $k = 1800$ lb/in , $W_{AB} = 30$ lb

Solution





Conservation of Energy :



$$T_1 = 0$$

$$V_1 = V_{g1} + V_{e1}$$
$$= 0 + (1/2)(1800)(-1)^2$$
$$= 900 \text{ lb}\cdot\text{in} = 75 \text{ lb}\cdot\text{ft}$$

$$V_2 = V_{g2} + V_{e2} = (30)(1.5) + 0$$
$$= 45 \text{ lb}\cdot\text{ft}$$

$$I_O = I_G + md^2$$
$$= (1/12)(30/32.2)(5)^2 + (30/32.2)(1.5)^2$$
$$= 4.0373$$

$$T_2 = (1/2) I_O \omega^2 = 1/2(4.0373)\omega^2 = 2.019\,\omega^2$$

$$\Rightarrow \quad 0 + 75 = 45 + 2.019\,\omega^2$$

$$\omega = 3.86 \text{ rad/s}$$

Cont'd : Nov. 9/17

Dynamics

$\Sigma F_x = ma_x$:

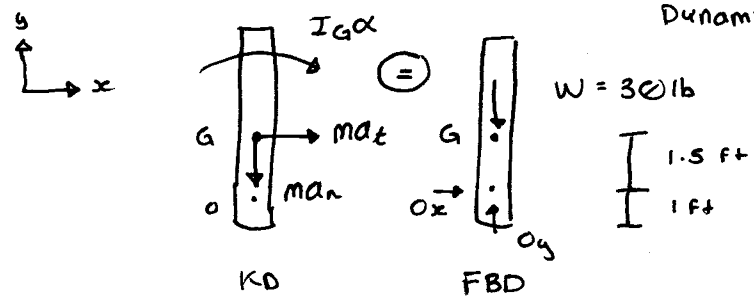
$O_x = ma_t$

$\Sigma F_y = ma_y$:

$O_y - W = -ma_n$

$\Sigma M_g = I_G\alpha$

$O_x(1.5) = -I_G\alpha$



$a_n = r\omega^2 = (1.5)(3.86)^2$

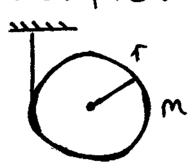
$a_t = r\alpha = (1.5)(\alpha)$

Since $m = \frac{30}{32.2}$; $I_G = \frac{1}{12}mL^2 \Rightarrow I_G = (\frac{1}{12})(\frac{30}{32.2})(5)^2$

$O_y = W - ma_n = 30 - 1.5(\frac{30}{32.2})(3.81)^2 = 9.22$ lb

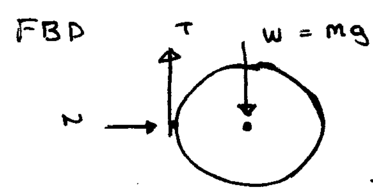
$O_x = 0$

Example:



Find the velocity of the mass center of the disk after it has moved downward a distance s.

Solution:

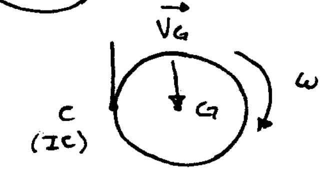


only weight does the work

At position 1, $T_1 = 0$

A position 2, $T_2 = \frac{1}{2}mV_G^2 + \frac{1}{2}I_G\omega^2$

$V_G = r\omega$



Since $I_G = \frac{1}{2}mr^2$

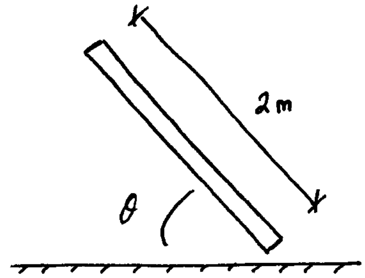
$\therefore T_2 = \frac{1}{2}m(r\omega)^2 + (\frac{1}{2})(\frac{1}{2})mr^2\omega^2 = \frac{3}{4}mr^2\omega^2$

$U_{1\to2} = W \cdot s = mgs$

$T_1 + U_{1\to2} = T_2 \Rightarrow 0 + mgs = \frac{3}{4}mr^2\omega^2$

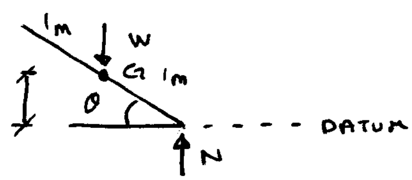
$\Rightarrow V_G = \sqrt{\frac{4}{3}gs}$

Example :



The 8-kg slender bar is released from rest with $\theta = 60°$. Find the angular velocity of the bar when $\theta = 30°$.

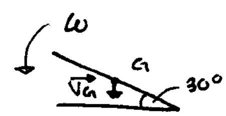
Solution : FBD



$T_1 + V_1 = T_2 + V_2$

$T_1 = 0$

$V_1 = mg(1)(\sin 60°)$



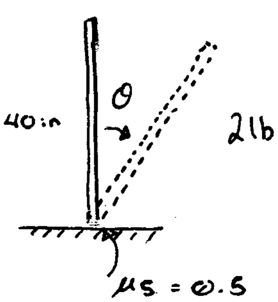
$V_2 = mg(1)(\sin 30°)$

$T_2 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$

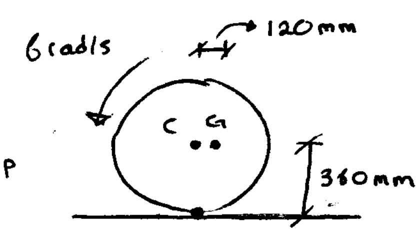
$\Rightarrow \quad 0 + mg \sin 60° = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2 + mg \sin 30°$

$V_G = r_{IC/G} \cdot \omega = 1 \cdot \cos(30°)\, \omega$ and $I_G = \frac{1}{12} m (2)^2$

$\omega = 2.57$ rad/s



Example :



rolling without slip

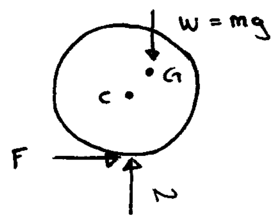
$m = 50$ kg

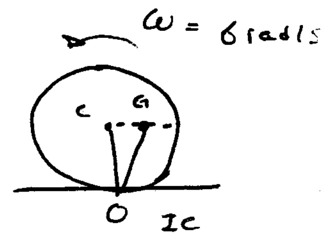
$k_G = 160$ mm

$(I_G = m \cdot k_G^2)$

Find the normal and friction forces exerted on the disk by the surface when the disk has rotated 210°

Solution : FBD





$T_1 + V_1 = T_2 + V_2$

$T_1 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

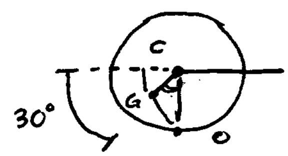
$V_G = OG \cdot \omega = \sqrt{OC^2 + CG^2}\,\omega$

$= \sqrt{0.36^2 + 0.12^2}\,(6)$

$I_G = m K_G^2 = 50(0.16)^2 = 1.28 (kg \cdot m^2)$

$\Rightarrow T_1 = \frac{1}{2}(50)(0.36^2 + 0.12^2)(6^2) + \frac{1}{2}(1.28)(6^2)$

$= 117\ J$



$V_1 = 0$

$V_2 = -mg \cdot CG \cdot \sin 30°$

$= -50(9.81)(0.12) \sin 30°$

$= -29.43\ J$

$T_2 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

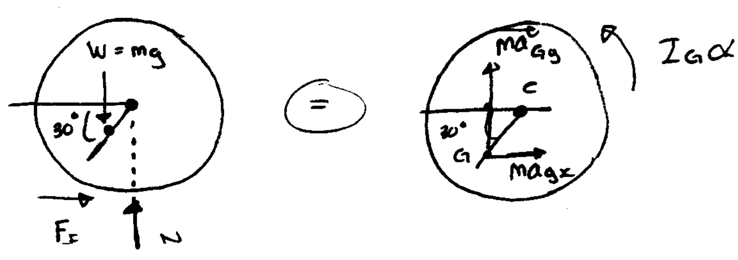
$\begin{cases} OG = \sqrt{OC^2 + CG^2 - 2\,OC\,CG \cos 60°} \\ OG = 0.2615 \end{cases}$

$\Rightarrow \frac{1}{2}(50)(0.2615)^2 \omega^2 + \frac{1}{2}(1.28)\omega^2 = 2.35\,\omega^2$

$117 + 0 = 2.35\omega^2 - 29.43$

$\omega^2 = 62.311 \qquad \omega = 7.8837$

FBD



$\Sigma F_x = m a_{Gx} \quad : \quad F_f = m a_{gx}$

$\Sigma F_y = m a_{Gy} \quad : \quad N - W = m a_{gy}$

$\Sigma M_G = I_G \alpha \quad : \quad F \cdot OD + N \cdot GD = I_G \alpha$

$\vec{a}_g = \vec{a}_c + \vec{a}_{g/c} \Rightarrow \vec{a}_G = \vec{a}_c + \alpha \vec{k} \times \vec{GC} + \omega \vec{k} \times (\omega \vec{k} \times \vec{GC})$