OCTOBER 17TH - MIDTERM

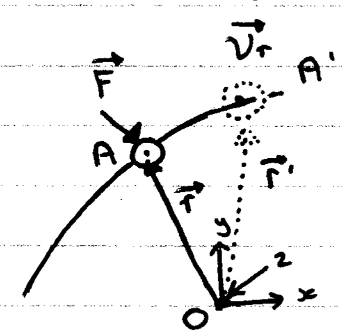
Office hours - to be determined later (TUES, WED, $10_A \rightarrow 2_P$) *
- can also email for appointment

Final exam is not cumulative, will not include material from before midterm.

Ch. 3 - Kinematics of a particle
Energy and Momentum Methods

13.1: Develop the principle of work and energy
develop the principle of impulse and momentum
apply the principals to solve problems
↳ that involve forces, velocity, disp., time

13.2: The work of a force
The work is the amount of energy transferred by a force acting through a distance in the direction of force.



- In this example:
studying the work done by the force on the particle.

A force $\vec{F}$ acting on a particle at A which moves along the path.

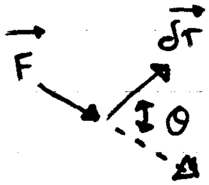
$\vec{r}$: the position vector

$$\boxed{\vec{d}_r = \vec{r} - \vec{r}'}$$

The differential displacement associated with an infinitesimal moment from A to A'

The work done by the force $\vec{F}$ during the displacement $\vec{dr}$ is defined as:

$$\boxed{dU = \vec{F} \cdot \vec{dr}}$$

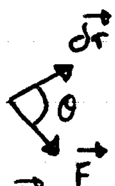


$$\boxed{dU = F \cdot ds \cdot \cos\theta}$$

where:

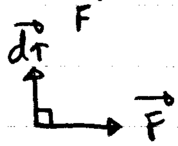
$F = |\vec{F}|$ $\quad ds = |\vec{dr}|$

(make both vectors start at the same location.)

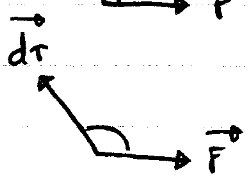


$0 < \theta < 90°$, $\cos\theta > 0$

$dU > 0$



$\theta = 90°$, $\cos\theta = 0$, $dU = 0$



$90° < \theta < 180°$, $\cos\theta < 0$

$dU < 0$

$\boxed{\text{Work is a scalar}}$

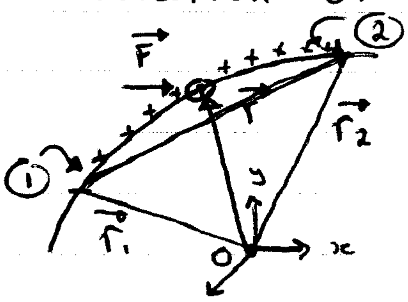
Unit in SI the Joule (J)

$1\,J = 1\,N\cdot m$

in FPS Foot-pounds (ft·lb)

$1\,ft\cdot lb = 1.3558\,J$

Calculation of Work



Does the following hold true?

$U_{1,2} \overset{?}{=} \vec{F} \cdot (\vec{r}_2 - \vec{r}_1)$ — No, the force is not constant

$$\boxed{U_{1,2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}}$$

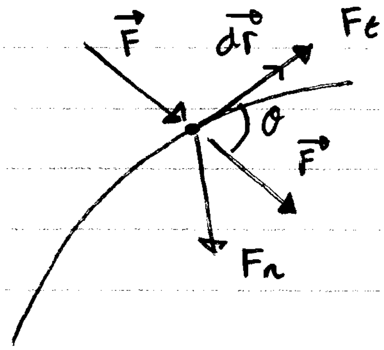
$$U_{1,2} = \int_{s_1}^{s_2} F \cdot \cos\theta \, ds$$

Given $\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$$

$$U_{1,2} = \int_{(x_1, y_1, z_1)}^{(x_2, y_2, z_2)} F_x dx + F_y dy + F_z dz$$

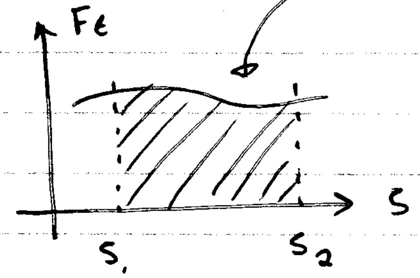


$$U_{1,2} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

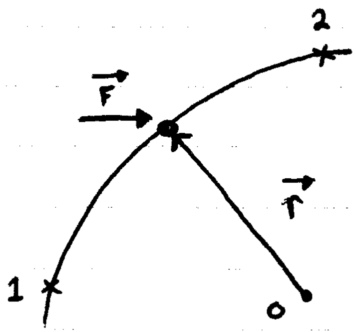
$$= \int_{s_1}^{s_2} F \cos\theta \, ds$$

$$\boxed{= \int_{s_1}^{s_2} F_t \, ds}$$

Given



Calculation of Work



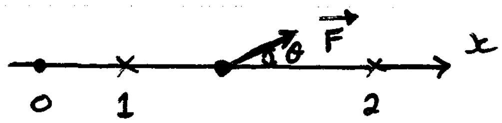
$$U_{1\to 2} = \int_{r_1 s_1}^{r_2 s_2} \vec{F} \cdot d\vec{r}$$

$$U_{1\to 2} = \int_{s_1}^{s_2} F \cos\theta \, ds$$

$$U_{1\to 2} \int_{(x_1, y_1, z_1)}^{(x_2, y_2, z_2)} F_x dx + F_y dy + F_z dz$$

Case 1 : Rectilinear motion

$\vec{F}$ = const.

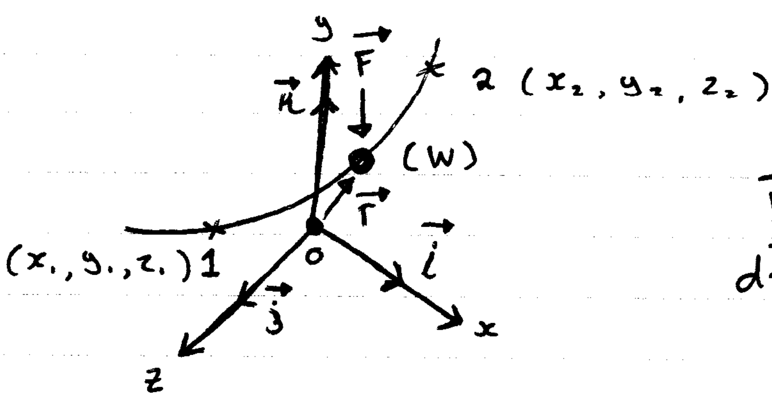


$$\vec{F} \cdot d\vec{r}$$
$$= \underbrace{F \cos\theta}_{\text{constant}} ds$$

$$U_{1\to 2} = \int_{s_1}^{s_2} F \cos\theta \, ds = F\cos\theta \int_{s_1}^{s_2} ds$$

$$\boxed{= F\cos\theta \, (s_2 - s_1)}$$

Case 2 : Work of a Weight



$$\vec{F} = W(-\vec{j})$$
$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

Since $\vec{i} \cdot \vec{i} = 1$ $\quad \vec{j} \cdot \vec{j} = 1$ $\quad \vec{k} \cdot \vec{k} = 1$

$\vec{i} \cdot \vec{j} = 0$ $\quad \vec{j} \cdot \vec{k} = 0$ $\quad \vec{k} \cdot \vec{i} = 0$

$$\vec{F}\cdot d\vec{r} = -w\vec{j}\cdot(dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k})$$
$$= -w\,dy$$

$$U_{1\to2} = \int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} \vec{F}\cdot d\vec{r} = \int_{y_1}^{y_2} -w\,dy$$

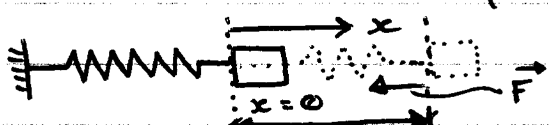
$$= -w(y_2 - y_1) = -w\Delta y$$

Here, $\Delta y = y_2 - y_1$

The particle moves upward, $U_{1\to2} < 0$

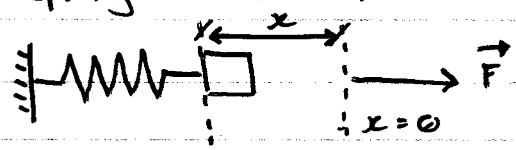
The particle moves downward, $U_{1\to2} > 0$

Case 3 : Work of a (linear) spring Force

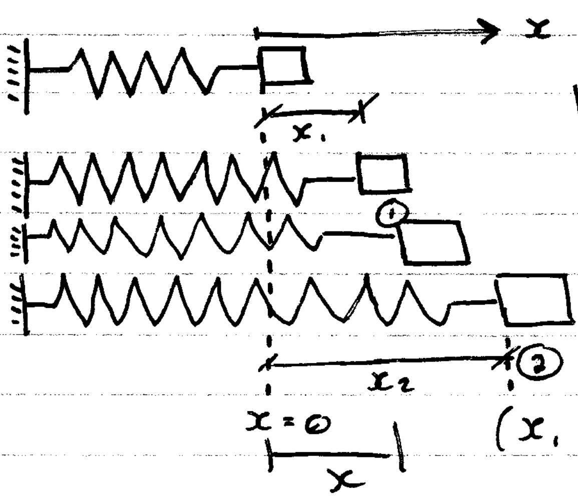


$F = kx$

spring const ?



Note :
The spring Force is always trying to move the particle back to the original position.



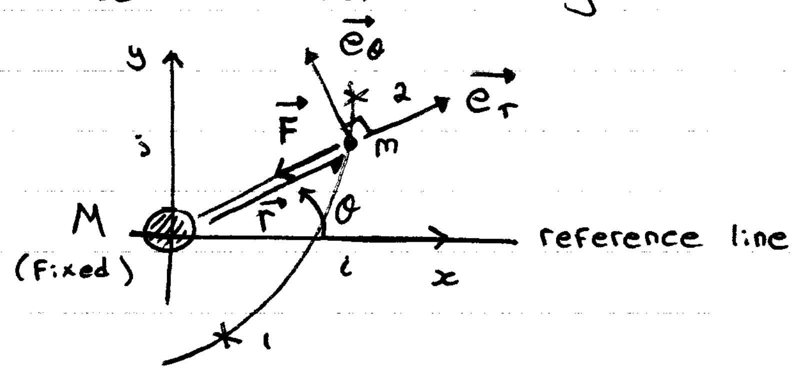
$(x_1 \le x \le x_2)$

Spring Force
$\vec{F} = k\vec{x}(-\vec{i})$

The disp.
$d\vec{r} = dx(\vec{i})$

$$U_{1\to 2} = \int \vec{F} \cdot d\vec{r}$$
$$= \int_{x_1}^{x_2} kx(-\vec{i})\, dx\,\vec{i}$$
$$= -k \int_{x_1}^{x_2} x\,dx$$
$$= -k\left(\tfrac{1}{2}x_2^2 - \tfrac{1}{2}x_1^2\right)$$
$$\boxed{U_{1\to 2} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2}$$

Case 4 : Work of a gravitational Force



$$\begin{cases} \vec{e_r} = \cos\theta\,\vec{i} + \sin\theta\,\vec{j} \\ \vec{e_\theta} = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j} \end{cases}$$

$$\vec{F} = \frac{GMm}{r^2}(-\vec{e_r})$$

$$\vec{r} = r\vec{e_r}$$

$$\Rightarrow d\vec{r} = d(r\vec{e_r}) = dr\,\vec{e_r} + r\,d\vec{e_r}$$

Since $d\vec{e_r} = (-\sin\theta\,\vec{i} + \cos\theta\,\vec{j})\,d\theta$

$$= \vec{e_\theta}\,d\theta$$

$$\boxed{\Rightarrow d\vec{r} = dr\,\vec{e_r} + r\vec{e_\theta}\,d\theta}$$

$$\Rightarrow \vec{F}\cdot d\vec{r} = -\frac{GMm}{r^2}\cdot\vec{e_r}\left[dr\,\vec{e_r} + r\vec{e_\theta}\,d\theta\right]$$
$$= -\frac{GMm}{r^2}\,dr$$

$$\Rightarrow U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

$$= \int_{r_1}^{r_2} -\frac{GMm}{r^2}\, dr \quad \boxed{\Rightarrow \frac{GMm}{r_2} - \frac{GMm}{r_1}}$$

$$W = mg = \frac{GMm}{R^2}$$

$$\frac{GM}{R^2} = g \qquad GM = R^2 g$$

$$U_{1\to 2} = \frac{R^2 mg}{r_2} - \frac{R^2 mg}{r_1}$$

$$= \frac{WR^2}{r_2} - \frac{WR^2}{r_1}$$

R : The radius of the Earth

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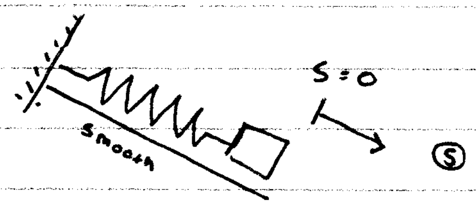
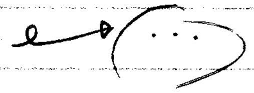
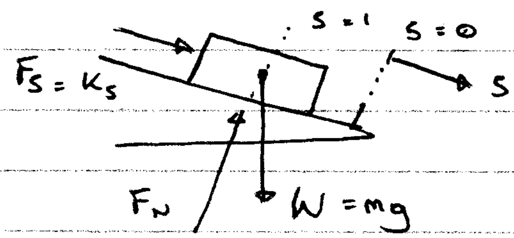
DYNAMICS II



$k = 20\ N \cdot m$
$m = 5kg$



Draw FBD of block @ $S(S_1 \le S \le S_2)$

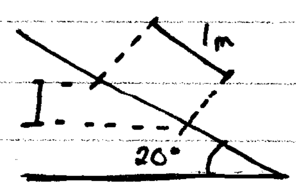


Spring Form:

$U_{1\to 2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$
$= \frac{1}{2}(20)(-1)^2 - \frac{1}{2}(20)(0)^2$
$= 10\ J$

Weight:

$U_{1\to 2} = -W\Delta y$
$U_{1\to 2} = -mg(-\sin 20°)$
$= (-5)(9.81)(\sin 20°)$
$= 16.8\ J$



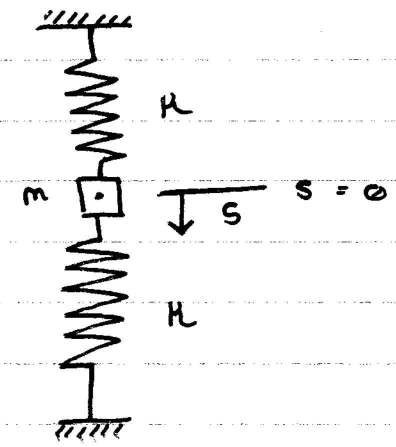
Normal Force:

$U_{1\to 2} = 0$

Example :

$k = 25$ lb/ft

$W = 50$ lb



When the block has fallen 1ft, how much work is done by the spring?

Solution :

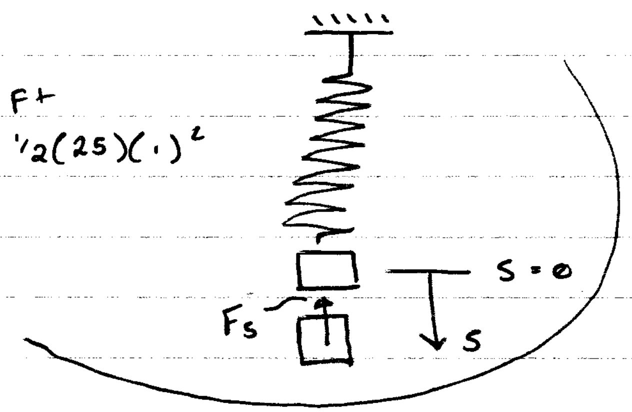
$$U_{1\to2} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

Top Spring :

$x_1 = 0$, $x_2 = 1$ ft

$$U_{1\to2} = \tfrac{1}{2}(25)(0)^2 - \tfrac{1}{2}(25)(1)^2$$

$$= 12.5 \text{ ft}\cdot\text{lb}$$

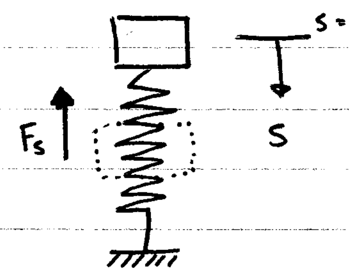


Bottom Spring :

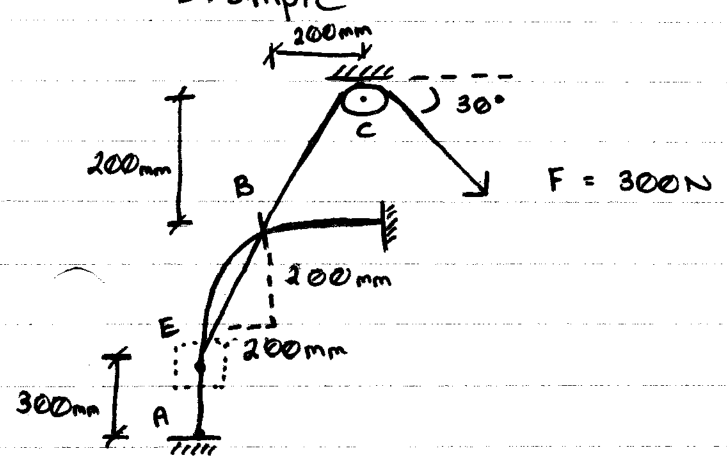
$x_1 = 0$, $x_2 = 1$ ft

$$U_{1\to2} = \tfrac{1}{2}(25)(0) - \tfrac{1}{2}(25)(1)^2$$

$$= 12.5 \text{ ft}\cdot\text{lb}$$



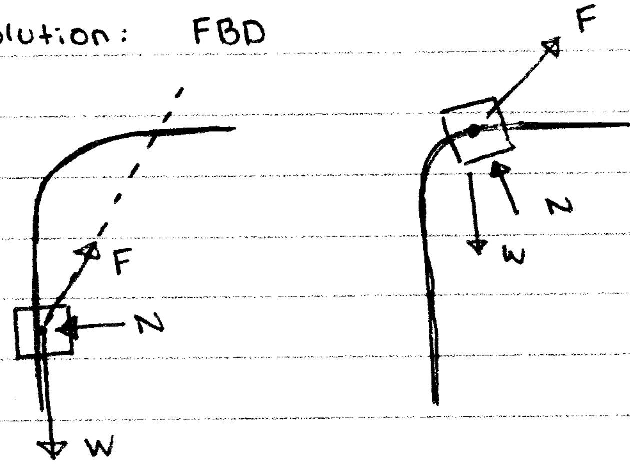
Example



Find the work done by forces applied to the block.

$m = 15$ kg

Solution: FBD



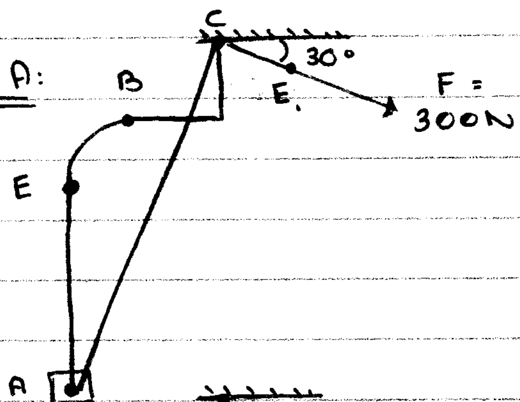
$$U_{1\to2}^{N} = 0$$

$$U_{1\to2}^{W} = W\Delta y$$

$$= -15\,(9.81)$$

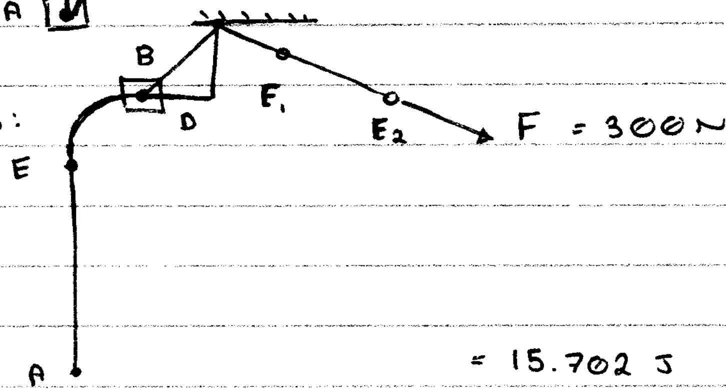
$$= -73.58 \text{ J}$$

At A:



AC + CE

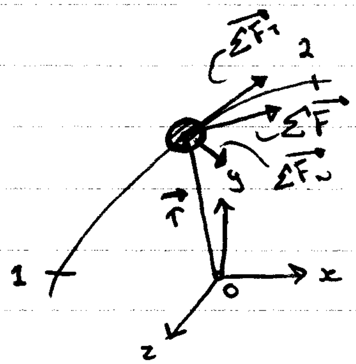
At B:



= 15.702 J

13.3 Kinetic Energy of a particle

Definition of Work and Energy



$$\sum \vec{F} = m\vec{a}$$

The tangential component

$$\sum F_t = m a_t = m \, dv/dt$$

$$v = ds/dt$$

$$\Rightarrow \sum F_t = m \, dv/ds \; ds/dt = m v \, dv/ds$$

$$\Rightarrow \sum F_t \, ds = m v \, dv$$

$$\Rightarrow \int_{s_1}^{s_2} \sum F_t \, ds = \int_{v_1}^{v_2} m v \, dv$$

$$\boxed{\Rightarrow U_{1\to 2} = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2}$$