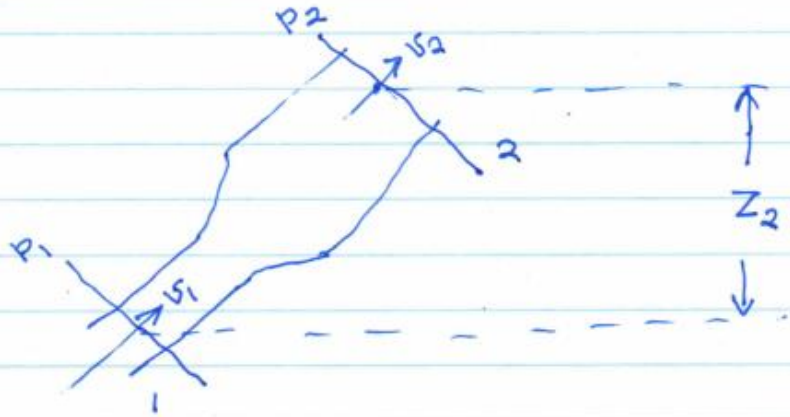
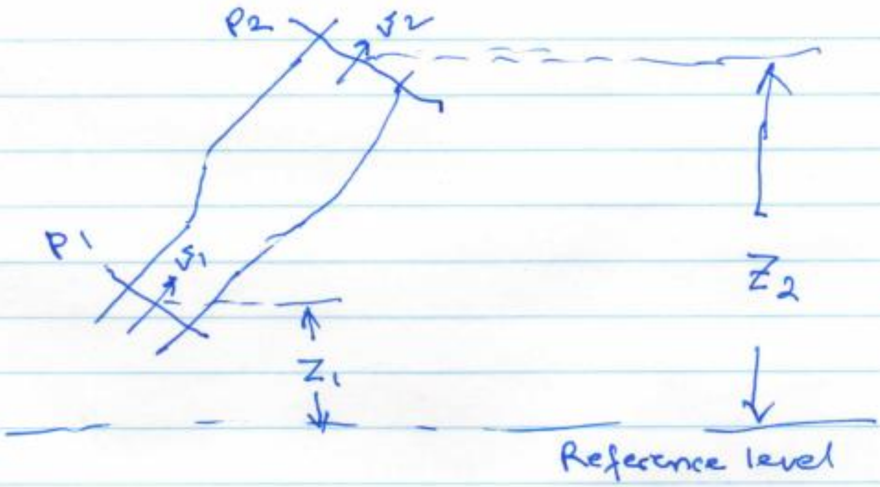
The method of calculating the velocity of flow of a fluid in a closed pipe system depends on the principle of continuity.



we can take a difference level and have z_1 and z_2 with respect to that level



According to continuity principle

$M_1 = M_2$ mass flow rates

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

In this course we will only deal with incompressible liquids $\Rightarrow$ $\rho_1 = \rho_2$

So, $A_1V_1 = A_2V_2$

Or $\emptyset_1 = \emptyset_2$

Commercially available pipe and tubing:

The normal sizes for commercially available pipe still refer to an "inch" size even though the transition to the SI system is an international trend.

For many applications, codes and standards must be followed as established by government agencies. There are many organizations that advise on these standards.

American Water Works Association
American Fire Sprinkler Association
International Standards Organization

Steel pipe – General purpose pipelines are often constructed of steel pipe.

Standard Pipe sizes are designated by the nominal size and schedule number.

Schedule numbers are related to the permissible operating pressure of the pipe and to the allowable stress of the pipe.

Tube and pipe – Although there is no fixed demarcation, but tube means a smaller diameter, and pipe means a larger diameter.

Copper tubing – mostly used in homes.

Plastic pipes and tubing are used increasingly these days.

Conservation of Energy – Bernoulli's Equation (you need to know this)

In Physics you learned that energy can neither be created nor destroyed, but it can be transformed from one form to another.

This is a statement of the law of conservation of energy.

There are three forms of energy that are always considered when analyzing a pipe flow problem.

1. Potential Energy - Due to its elevation the potential energy of a fluid element relative to some reference level is: $PE = WZ$ (PE is potential energy)
   Where W is the weight of the fluid element under consideration, and Z is its elevation with respect to some reference level.

2. Kinetic energy – Due to the velocity, the kinetic energy of the fluid element is
   $KE = W\frac{v^2}{2g}$
3. Flow energy – Or sometimes called pressure energy or flow work: This represents the amount of work necessary to move the element across a certain section against the pressure P. Flow Energy FE is
   $FE = W\frac{P}{\gamma}$

The work done in moving the fluid element through distance L

Work = $pAL = pV - V$ for volume (not velocity, in this instance)

Work = Force x Distance = Pressure x Area x Distance

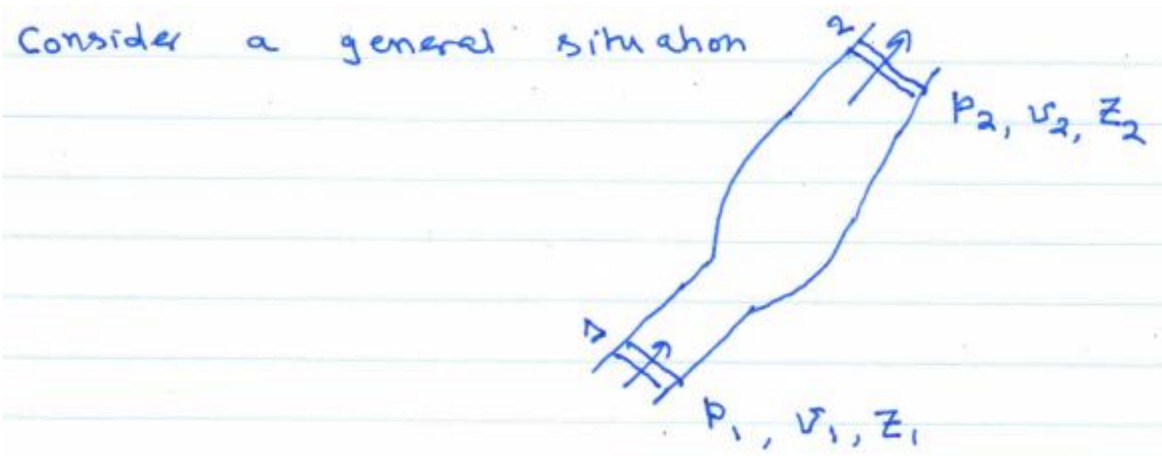
Conservation of Energy:

The total amount of energy of these three forms possessed by the element of fluid is the sum E

$$E = FE + PE + KE$$
$$E = W\frac{P}{\gamma} + WZ + W\frac{v^2}{2g}$$

Each of these terms are expressed in units of energy which are $Newton\ \cdot metres\ (N \cdot m)$ in SI units and $ft \cdot pound$ in US units.

Consider a general situation:



When the fluid element moves from neutral to section 2

$$Energy\ at\ 1\ E_1 = W\frac{P_1}{\gamma} + WZ + W\frac{{v_1}^2}{2g}$$

$$Energy\ at\ 2\ E_2 = W\frac{P_2}{\gamma} + WZ + W\frac{{v_2}^2}{2g}$$

From conservation of energy principle

$$E_1 = E_2$$

So equation E_1 and E_2 and canceling out W

$$\frac{P_1}{\gamma} + Z_1 + \frac{{v_1}^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{{v_2}^2}{2g}$$

(This is Bernoulli's equation, you should remember it).

Each term in the Bernoulli's equation is in the form of energy possessed by the fluid per unit weight.

The units for each term $\frac{N \cdot m}{N} = m\ (metres)$

Each term has dimension of length in Bernoulli's Equation.

Practicing Engineers refer to this unit as "head"

$$\frac{P}{\gamma}\ is\ called\ pressre\ head$$

$$Z\ is\ called\ elevation\ head$$

$$\frac{v^2}{2g}\ is\ called\ velocity\ head$$

The sum of these is called $total\ head$.

So, when diameter is increased in a horizontal pipe, the pressure is increased and velocity is decreased.

The sum of these three terms remains constant according to Bernoulli's equation.

(obviously at this stage we have not included friction in the pipe)

Restriction on Bernoulli's Equation:

1. Is it valid only for incompressible fluids ($\gamma = constant$)
2. No mechanical devices between two sections under consideration
3. No heat transfer
4. No energy losses due to friction (in later chapters, we will include friction losses and modify the equation)

In reality, no practised situation that meets these conditions, but engineers make reasonable assumptions to solve practice problems.

Procedures for applying Bernoulli's Equation:

1. Device which items are known and what is to be found.
2. Decide which two sections in the system will be used.
3. Write the Bernoulli's equation for two sections.
4. Be explicit when labeling the subscripts for the pressure head, elevation head, and velocity head
5. Simplify the equation, if possible by cancelling terms that are zero or that are equal on both sides.
6. Solve the equation for the desired term.
7. Substitute known quantities and calculate the result.

Simplifications in using Bernoulli's Equation for special cases:

The velocity at the free surface of a large reservoir can be taken as zero, because it is very small.

Sometimes we apply Bernoulli's equation at two points in a same pipe (constant diameter), so the velocity term cancels out on both sides.

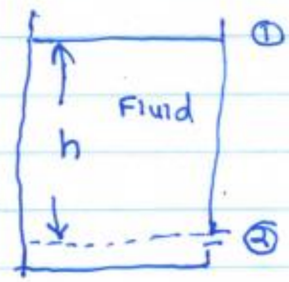
When a pipe is horizontal $Z_1 = Z_2$ so the elevation terms cancels out.

Feb 1st /17

Application of Bernoulli's Theorem:

Fluid draining out of a tank

Consider a tank with free surface at section (1) and drain at section (2)



Apply Bernoulli's Equation between sections (1) and (2)

$$\frac{P_1}{\gamma} + Z_1 + \frac{{v_1}^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{{v_2}^2}{2g}$$

Both sections (1) and (2) are exposed to atmosphere

$$P_1 = P_2 = P_{atm} = 0 \; gage$$

If the tank diameter is much larger than the drain diameter, we can assume $v_1 = 0$

So we are left with the following:

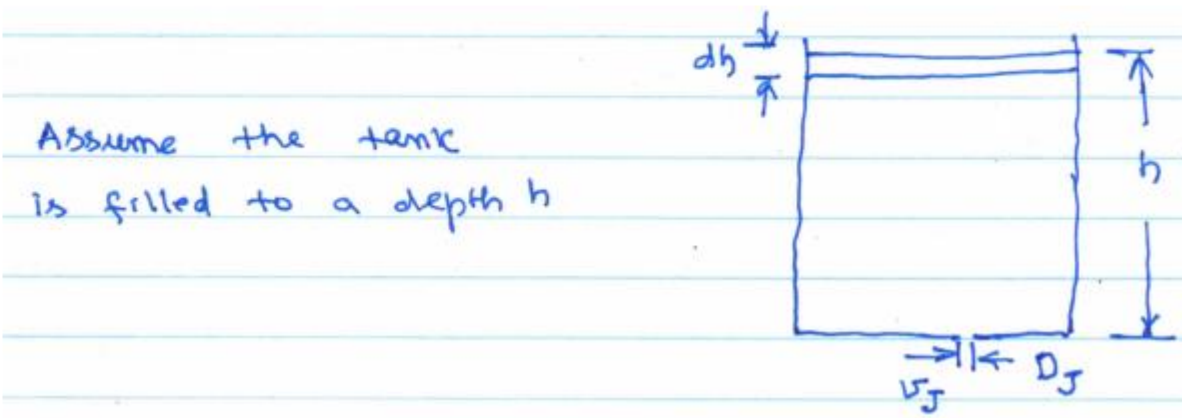
$$Z_1 = Z_2 + \frac{{v_2}^2}{2g}$$

So, $v_2 = \sqrt{2g\,(Z_1 - Z_2)} = \sqrt{2gh}$

This is called Torricelli's theorem

When a tank is being emptied, as the tank level falls, the exit velocity v_2 will also fall

How to figure out the time required to empty a tank:



When we let it drain out with drain diameter D_J, and assume V_J is the velocity of the jet

Volume flow rate through the nozzle (drain) at the bottom y

$$Q = A_J v_J$$

Where A_J is the area of the exit jet

In a time dt, the volume flow is:

$$Q\ dt = A_J v_J\ dt$$

During time dt, let the fluid level decrease by dh

So, the volume of fluid removed from the tank in time dt can also be expressed as:

$$-A_t\ dh$$

Where A_t is the area of the tank

The minus sign is there because the level is decreasing.

$$A_J v_J\ dt = -A_t\ dh$$

$$dt = -\left(\frac{A_t}{A_J}\right)\frac{dh}{v_J}$$

$$= -\left(\frac{A_t}{A_J}\right)\frac{dh}{\sqrt{2gh}}$$

$$(This\ is\ Terricelli's\ Theorem)$$

The time required for the fluid level to change from a depth h_1 to h_2

$$\int_{t_1}^{t_2} dt = -\left(\frac{A_t}{A_J}\right) \frac{1}{\sqrt{2g}} \int_{h_1}^{h_2} h^{-\frac{1}{2}} \, dh$$

$$t_2 - t_1 = 2 \frac{A_t}{A_J} \frac{\left(h_1^{1/2} - h_2^{1/2}\right)}{\sqrt{2g}}$$

$$= 2 \frac{A_t}{A_J} \frac{\sqrt{h_1} - \sqrt{h_2}}{\sqrt{2g}}$$

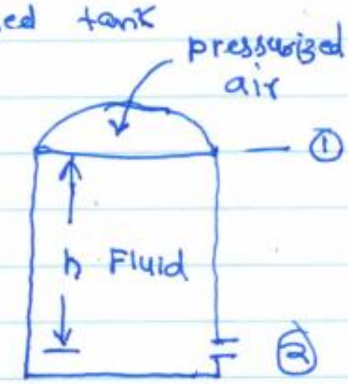
This equation can be used to find the time required.

If the tank is pressurized, the piezometric head $\frac{P_1}{\gamma}$ should be added to the actual liquid depth before completing calculation.

Problem 6.67 Emptying a pressurized tank

Given h = 8 ft

Air pressure at the top is 20 psig



(psig means pounds per square inch gage)

Take liquid surface as section ①
The centerline of exit drain as section ②

Write Bernoulli's equation.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$v_1 \simeq 0$ (assuming large diameter)

$p_2 = 0$ (exposed to atmosphere)

p_1 is given as 20 psig

$$v_2 = \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - z_2\right)}$$

$$= \sqrt{(2)(32.2)\left[\left(\frac{20 \times 144}{62.4}\right) + 8.0\right]}$$

$$= 59.06 \text{ ft/s}$$

So, $Q = A_2 v_2 = \pi \frac{3^2}{4} \frac{59.06}{144} = 2.90 \text{ ft}^3/\text{s}$

Chapter 7 – General Energy Equation

In Bernoulli's equation application, there were no mechanical devices between the two sections under consideration.

In reality there are various devices and components in fluid systems.

You either add energy to the fluid or remove energy from the fluid or cause undesirable loss of energy from the fluid.

Pumps – A mechanical device that adds energy to a fluid, an electric motor or some other prime mover drives as rotating shaft in a pump.

The pump takes this kinetic energy and delivers it to the fluid resulting in a fluid flow and increased fluid pressure.

Fluid friction – A fluid in motion offers frictional resistance to flow. Part of this energy lost is converted to heat and dissipated through the walls of the pipe.

(In this course we ignore heat transfer)

The magnitude of energy lost depends on properties of the fluid, the flow velocity, the pipe size, the smoothness of the pipe wall and the length of the pipe.

Valves and fitting – control the direction of flow or flow rate of a fluid in a given system. They set up turbulence in the fluid causing energy dissipation as heat.

In a large system, usually the frictional loss is the largest loss.

The loss in valves and fittings is usually less than function losses. These losses are called "minor losses"

Friction losses are called "major losses."

So, the words "minor" and "major" used here are only terminology.

Nomenclature of Energy losses and additions

$h_A =$ Energy added to the fluid with a mechanical device such as a pump this is often referred to as the total head on the pump.

$h_R =$ Energy removed from the fluid with a mechanical device such as a fluid motor or a turbine.

The major difference between a pump and a fluid motor is that, when acting as a motor the fluid drives the rotating elements of the device; the reverse is true for a pump.

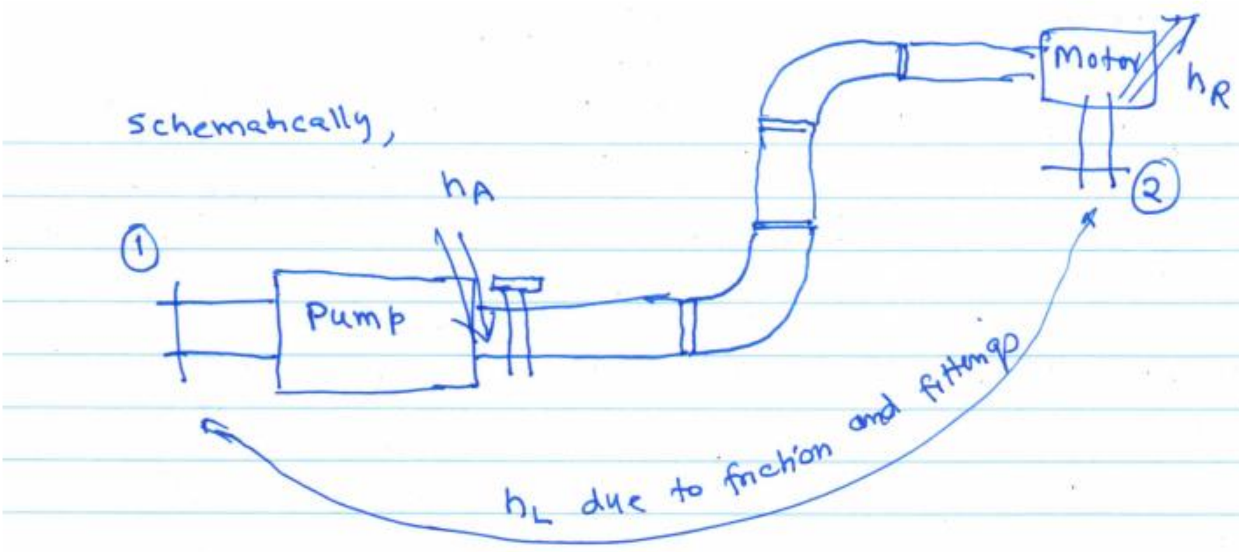
h_L = Energy losses from the system due to friction in pipes or minor losses due to valves and fittings

We ignore heat transfer losses in this course.

Generally, $h_L = k\frac{v^2}{2g}$ for valves and fittings

k is called the resistance coefficient.

The numerical values of k are found from tables in handbooks or appendices in textbooks.



Look at section (1)

Energy per unit weight of the fluid, E_1', can be expressed as

$$E_1' = \frac{P_1}{\gamma} + Z_1 + \frac{{v_1}^2}{2g}$$

Similarly, at section (2)

$$E_2' = \frac{P_2}{\gamma} + Z_2 + \frac{{v_2}^2}{2g}$$

To relate $E_1{}'$ and $E_2{}'$, we need to account for h_A, h_R, and h_L

$$\frac{P_1}{\gamma} + Z_1 + \frac{{v_1}^2}{2g} + h_A + h_R + h_L = \frac{P_2}{\gamma} + Z_2 + \frac{{v_2}^2}{2g}$$

This is called “General Energy Equation” you should remember this.

Each term represents energy per unit weight.