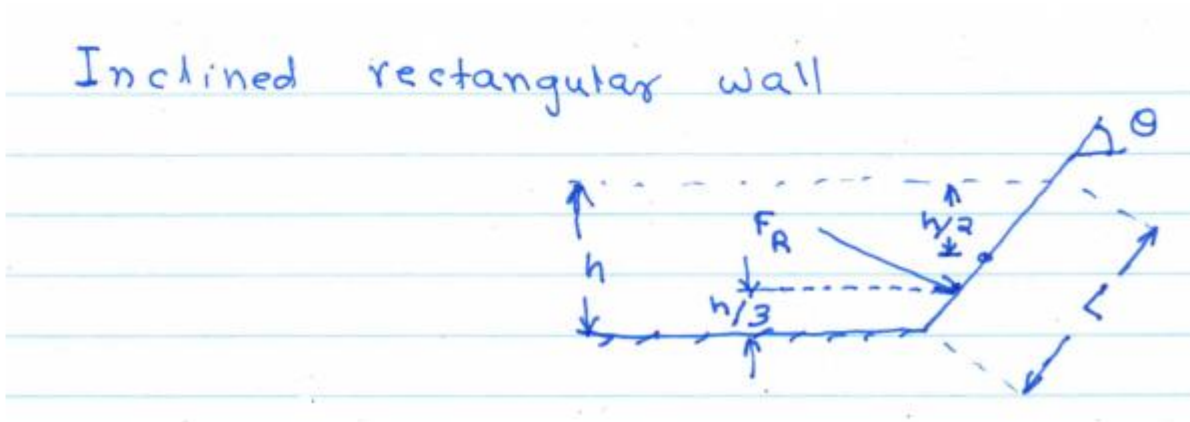
Jan 23rd /17



$$L = \frac{h}{\sin\theta}$$

$F_R = \gamma\left(\frac{h}{2}\right)A$ where: $\gamma = pg$

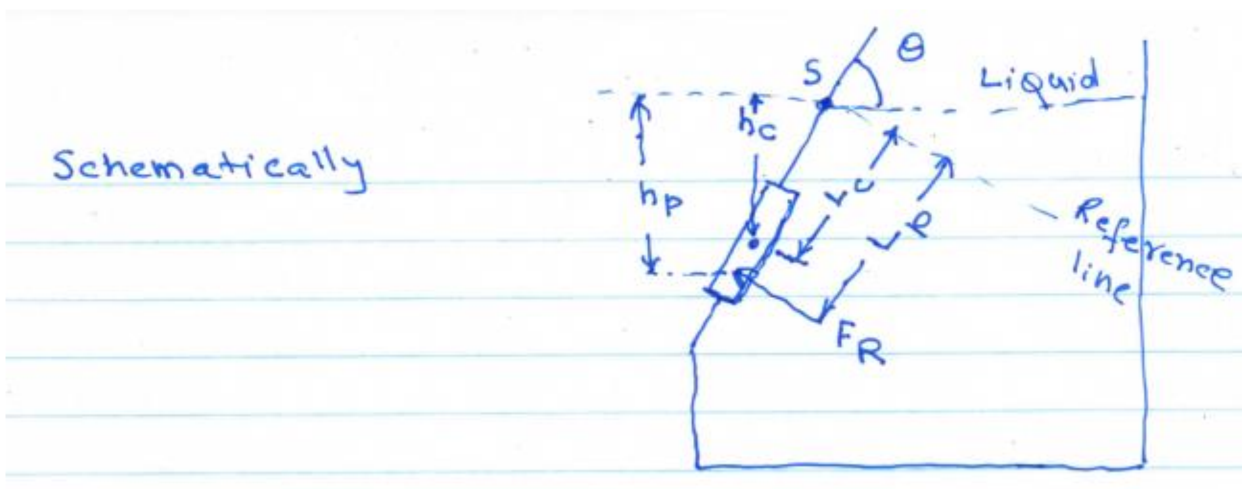
If the width of the wall is w

Then $A = L \cdot W$

$$F_R = \gamma\left(\frac{h}{2}\right)\left(\frac{h}{\sin\theta}\right)W$$

(You should remember $\sin\theta$, $\cos\theta$, and $\tan\theta$ $at\ least$)

Many times, engineers have to do calculations for a general plane area that is submerged. Aquarium is one such example.



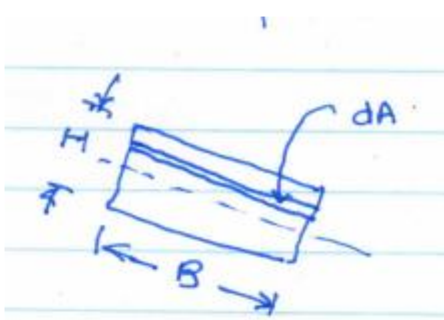
To analyze the given window

1. Locates point S, angle θ
2. Locate the centroid of the area from geometry

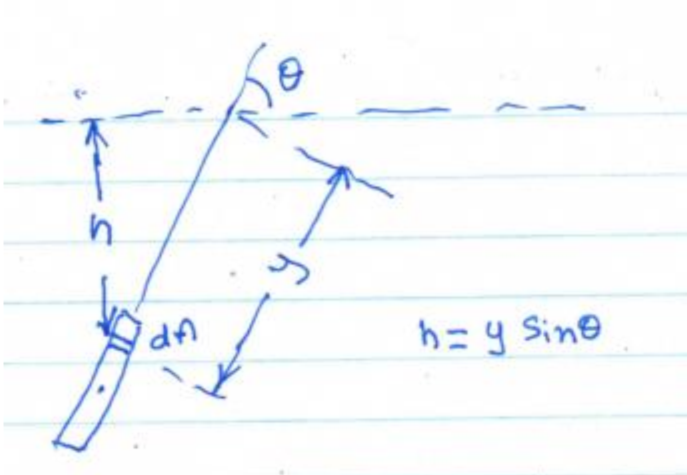
3. h_C = vertical distance from the centroid to the liquid surface.
4. L_C = inclined distance = $\frac{hc}{\sin\theta}$
5. Calculate the total area of which force is to calculated (use geometry)
6. Calculate resultant force $F_R = \gamma \cdot h_c \cdot R$
7. Calculate I_c, the moment of inertia of area about its centroidal axis.
   [3 – example of Ic]
8. Calculate the location of the centre of pressure from:
   $L_D = L_C + \frac{I_c}{L_c \cdot A}$ (need not remember)
9. Sketch the resultant force F_R acting at the centre of pressure, perpendicular to the area.
10. Show the dimension L_P
11. L_P and L_c are drawn from reference line
12. If it is desired to compute vertical depth
    $h_p = L_p \sin\theta$ etc.

Step 8 could be modified $h_p = h_c + \frac{I_c \sin^2\theta}{h_c \cdot A}$

Resultant force is defined as the summation of forces on a small element of interest



dA is not d times A, it is elemental area.



Because the area is inclined at an angle θ, it is convenient using y to denote the position of area at any depth, h.

So, $dF = \gamma(\sin\theta)dA$

F_R is the integrated value of dF

F_R is the integrated value of dF

$$F_R = \int_A dF = \int_A \gamma\,(y \sin\theta)\,dA$$

$$= \gamma \sin\theta \int_A y\,dA$$

$$\int_A y\,dA = L_c\,A$$

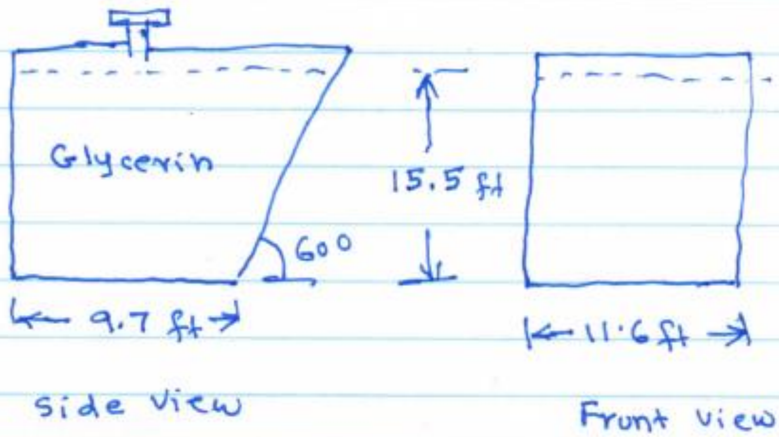
L_c = distance of the centroid of the area from reference line.

So, $F_R = \gamma \sin\theta\,(L_c A) = \gamma\,h_c A$

The resultant force acts perpendicular to the area.

Centre of pressure – The centre of pressure is that point on an area where the resultant forces can be assumed to act so as to have the same effect as the distributed force over the entire area due to fluid pressure.

Problem 4.15

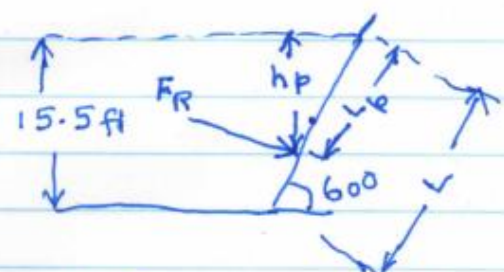


Compute the resultant force on the sloped side. Also compute the location of centre of pressure

Length of sloped side

$L = \frac{15.5}{\sin 60} = 17.90$ ft

$A = 17.90 \times 11.6 = 207.6$ ft^2



$F_R = \gamma \frac{h}{2} A = \underset{\text{given}}{78.5} \frac{\text{lb}}{\text{ft}^3} \times \frac{15.5}{2} \text{ ft} \times 207.6 \text{ ft}^2$

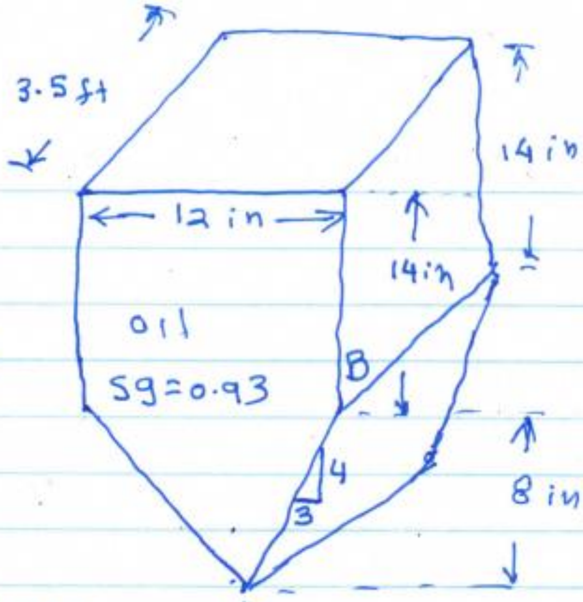
$= 126298.65$ lb

$h_p = \frac{2}{3} h = \frac{2}{3}(15.5) = 10.33$ ft

$L_p = \frac{2}{3} L = \frac{2}{3}(17.90) = 11.93$ ft

(Final answer has to have correct units).

Problem 4.18



Compute force on side AB

centroid is at midpoint of AB

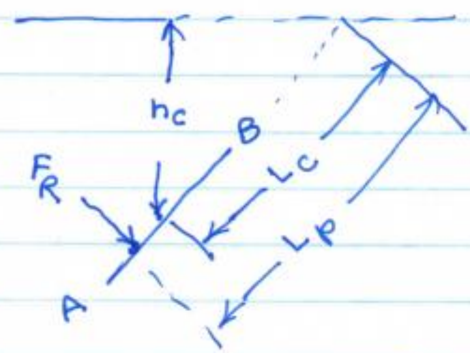
$h_c = 14 + 4 = 18$ in

$= 1.5$ ft

$\overline{AB} = 10.0$ in

$L_c = h_c \frac{5}{4}$

$= 22.5$ in



$\text{Area} = \overline{AB} \times 3.5 = \frac{10}{12} \times 3.5 = 2.92 \text{ ft}^2$

$F_R = \gamma h_c A = (0.93)(62.4)(1.50)(2.92)$

$= 254$ lb

$I_c = \frac{BH^3}{12} = \frac{(42)(10)^3}{12} \text{ in}^4 = 3500 \text{ in}^4$

Formula given

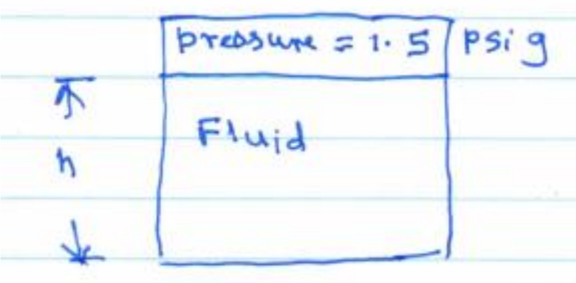
$$L_p - L_c = \frac{I_c}{L_c A} = \frac{3500}{(22.5)(420)} = 0.37 \text{ in}$$

Given in some form

$$L_p = L_c + 0.37 = 22.5 + 0.37 = 22.87 \text{ in}$$

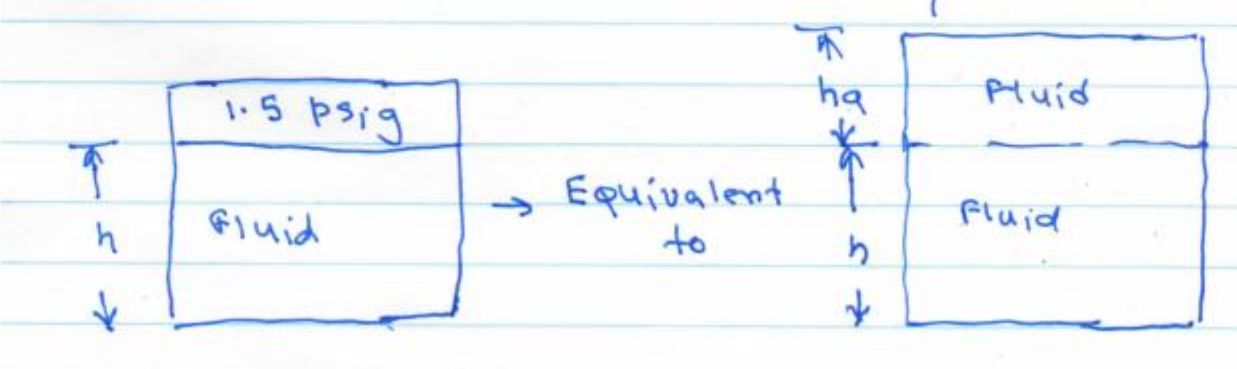
Piezometric head: in the problems considered so far, the liquid surface was exposed to atmosphere ($p = 0$ $gage\ pressure\ at\ the\ liquid\ surface$)

Sometimes the pressure above this free liquid surface is different from the ambient pressure outside the area.



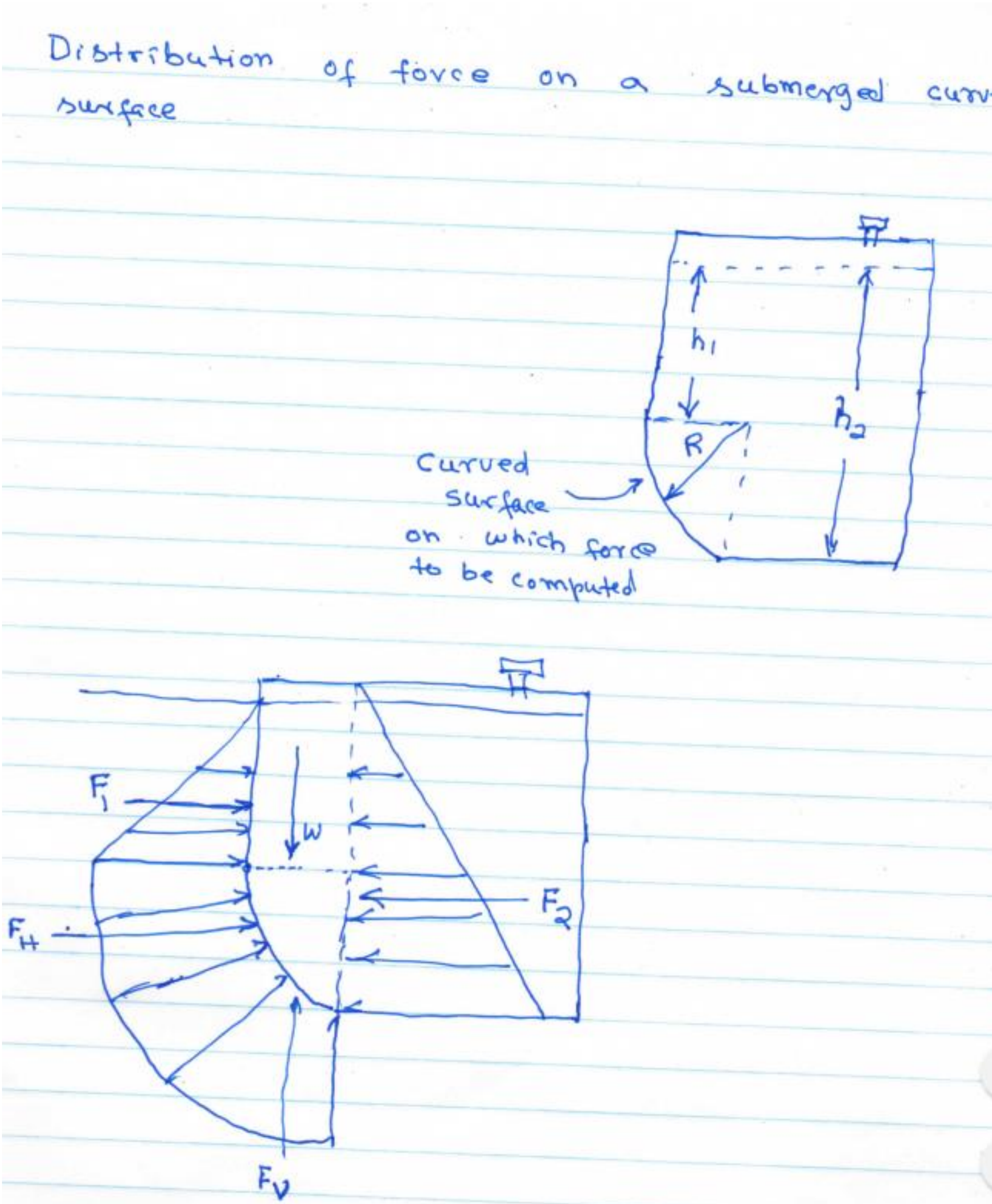
A convenient method to deal with such situations is to use the concept of piezometric head, in which the actual pressure above the fluid, P_A, is converted into an equivalent depth of the fluid, h_a, that would create the same pressure.

piezometric head $h_a = \frac{p_a}{\gamma}$



Jan. 25th /17

Distribution of force on a submerged curved surface



Vertical force F_v is balanced by weight of the fluid above it, W

Horizontal force on left F_1 and F_2 is balanced with F_2 on the right.

One way to visualize the total force system involved is to isolate the volume of fluid directly above the surface of interest as a free body and show all forces acting on it.

The objective is to determine the horizontal force F_h and the vertical force F_v exerted on the fluid by the curved surface and their resultant force F_R.

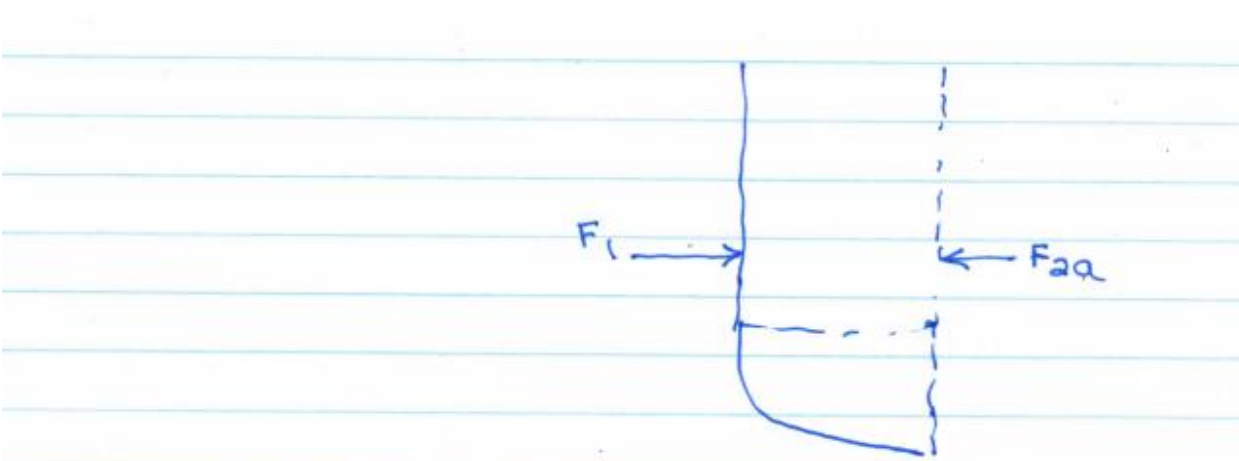
The line of action of the resulting force acts through the centre of curvature of the curved surface (Given in appendix, and if needed given in exam).

This is because each of the individual force vectors due to fluid pressure acts perpendicular to the boundary surface, which is along then along the radius of curvature.

Horizontal component – The vertical solid wall at the left exerts horizontal force on the fluid in contact with it in reaction to the force due to fluid pressure. This part of the system is the same manner as the vertical walls studied earlier.

The resultant force F_1 acts at a distance $\frac{h}{3}$ from the bottom of the wall.

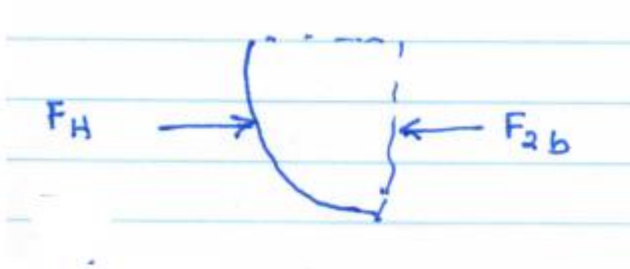
If we look at the right side of the plane wall in a free body diagram:



The force F_{2a} on the right (upper part) is equal to F_1

This way, they have no effect on the curved surface.

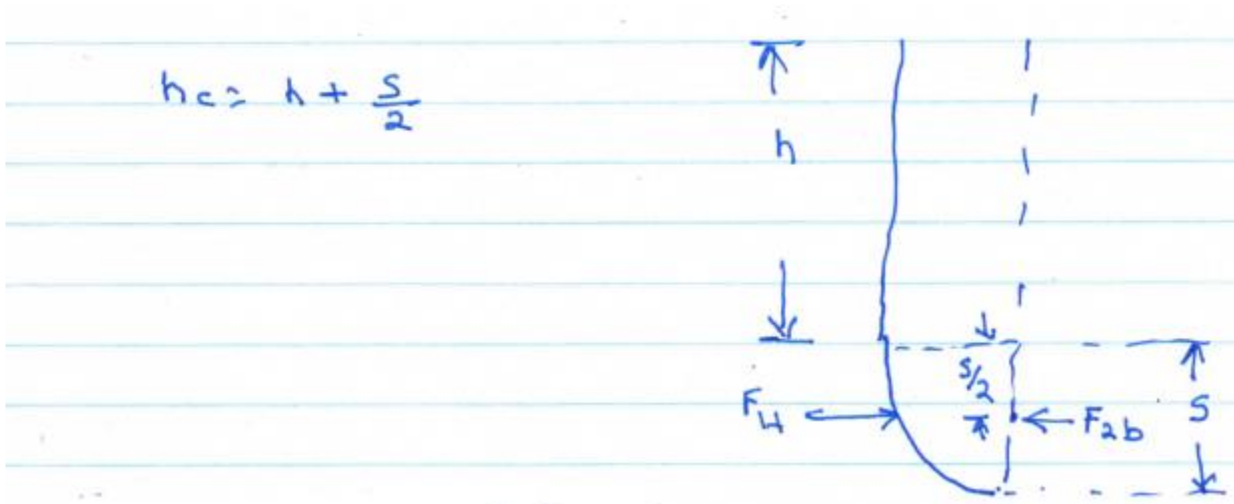
For the curved surface F_h is balanced by F_{2b}:



The area on which F_{2b} acts is the projection of the curved surface on the vertical plane.

$F_{2b} = \gamma h_c A$

$h_c A$ is the depth of the centroid of the projected area.



$h_c = h + \frac{5}{2}$

If width (perpendicular to paper) is w

The relevant area $A = S \cdot W$

Then $F_{2b} = F_H = \gamma \cdot S \cdot W \left(h + \frac{5}{2}\right)$

The location of F_{2b} is the centre of pressure of the projected area.

$h_p - h_c = \frac{I_c}{h_c A}$ (no need to remember)

$I_c = \frac{WS^3}{12}$ $\qquad A = SW$

$$h_p - h_c = \frac{WS^3}{12(h_c)(\mathrm{SW})} = \frac{S^2}{12h_c}$$

Vertical component – The weight of the fluid acts downwards, and so the vertical component F_V equals in magnitude of the weight

$$F_v = \gamma\ (volume) = \gamma A w$$

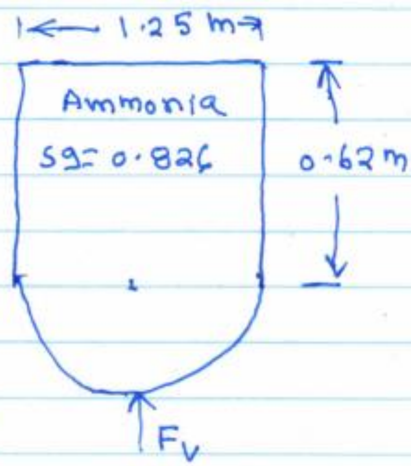
The resultant force F_R is

$$F_R = \sqrt{F_H^2 + F_V^2}$$

The resultant force acts at an angle Ø relative to the horizontal found from

$$\tan \text{Ø} = \frac{F_V}{F_H}$$

Problem 4.48



$$F_V = \gamma V = \gamma A w$$

$$= (0.826)(9.81)\left[(0.62 \times 1.25) + \frac{\pi (1.25)^2}{8}\right]$$

$$= 28.1 \text{ KN}$$

$F_H = 0$ because horizontal forces are balanced

$$F_R = F_V = 28.1 \text{ KN}$$

Practice problems from chapter 4

4.20 4.25 4.40 4.44

Practise problems from chapter 4

4.20, 4.25, 4.40, 4.44

Chapter 6 – Flow of fluids and Bernoulli's Equation (we skipped chapter 5)

Chapter 6-10 in this textbook are usually just one chapter in other textbooks.

The quantity of fluid flowing in a system per unit time can be expressed by these different terms:

Q The volume flow rate – is the volume of fluid flowing past a section per unit time.
W The weight flow rate – the weight of fluid flowing past a section per unit time.
m The mass flow rate – the mass of fluid flowing past a section per unit time.

The most fundamental of these three terms is the volume flow rate Q, which is calculated from
$Q = AV$
$A - the\ area\ of\ the\ section$
$V -$ the average velocity of flow

Units of $Q = AV\ m^2 \cdot \frac{m}{s} = \frac{m^3}{s}$

The weight flow rate W is related to Q

$W = \gamma Q$ where γ is the specific weight of the fluid

Units of $W = \gamma P\ \frac{N}{m^3} \cdot \frac{m^3}{s} = \frac{N}{s}$

The mass flow rate M

Units of $M = eQ$ where e is density

$M = eQ\ \frac{kg}{m^3} \cdot \frac{m^3}{s} = \frac{kg}{s}$

Useful conversions (need not memorize)

$$1.0\frac{L}{min} = 0.06\frac{m^3}{hr}$$
$$2.0\frac{m^3}{s} = 60000\frac{L}{min}$$
$$1.0\frac{gal}{min} = 3.785\frac{L}{min}$$
$$1.0\frac{gal}{min} = 0.2271\frac{m^3}{hr}$$
$$1.0\frac{ft^3}{s} = 499\frac{gal}{min}$$

(gal for gallon – US gallon is slightly smaller quantity of fluid than the imperial gallon)

Typical volume flow ratio

Typical volume flow rates

m^3/h	$\frac{L}{min}$		gal/min
0.9 - 7.5	15-125	Reciprocating pumps handling heavy fluids and slurries	4-33
2.4-276	40-4500	Centrifugal pumps in chemical processes	16-1200
12 - 240	2000-4000	Flood control and drainage pumps	50-1000
108-750	1800-9500	centrifugal firefighting pumps	500-2500