$$\begin{cases} \epsilon_x = 1/E\,[\sigma_x - \nu(\sigma_y + \sigma_z)] \\ \epsilon_y = 1/E\,[\sigma_y - \nu(\sigma_z + \sigma_x)] \\ \epsilon_z = 1/E\,[\sigma_z - \nu(\sigma_x + \sigma_y)] \end{cases}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G} \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\boxed{G = \frac{E}{2(1+\nu)}}$$

Principal stress directions

σ_1, σ_2, σ_3

$$\Rightarrow \begin{cases} \epsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)] \\ \vdots \qquad \qquad \vdots \end{cases}$$

Plane stress :

$\sigma_z = 0$, $\tau_{xz} = 0$, $\tau_{yz} = 0$

$$\Rightarrow \begin{cases} \epsilon_x = \frac{1}{E}[\sigma_x - \nu\sigma_y] \\ \epsilon_y = 1/E\,[\sigma_y - \nu\sigma_x] \\ \gamma_{xy} = \frac{\tau_{xy}}{G} \end{cases}$$

$$\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) \neq 0$$

Plane strain :

$\epsilon_z = 0$, $\gamma_{xz} = \gamma_{yz} = 0$

Since $\epsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) = 0$

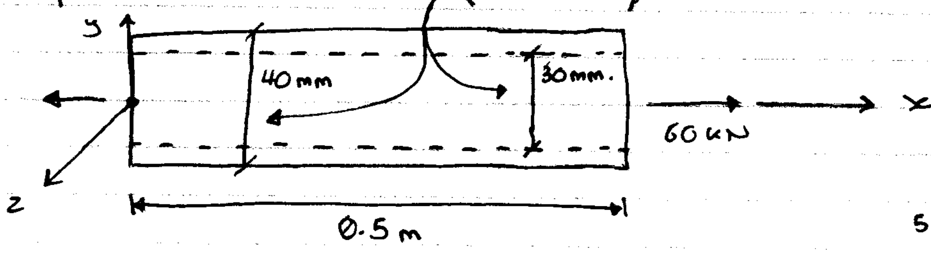
$$\Rightarrow \sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu\sigma_y - \nu \cdot \nu(\sigma_x + \sigma_y)]$$

$$\epsilon_x = \frac{1+\nu}{E}[(1-\nu)\sigma_x - \nu\sigma_y] \quad \ldots$$

and
$$\begin{cases} \cdots \\ \epsilon_y = \dfrac{1+\nu}{E}\left[(1-\nu)\,\sigma_y - \nu\,\sigma_x\right] \\ \gamma_{xy} = \dfrac{\tau_{xy}}{G} \end{cases}$$

Examples :

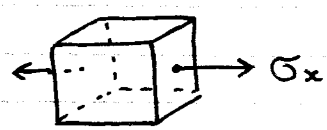


5-01 to 5-19 - CALC 2
5-23 to 6-12 - DYNA 2

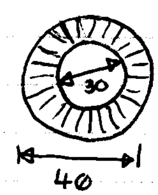
$E = 200\ GPa \qquad \nu = 0.32$

Determine the change in volume of the material after the load is applied.

Solution : uniaxial state of stress



where $\sigma_x = \dfrac{P}{A}$



$$e = \frac{\Delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z$$

$$\hookrightarrow \frac{1-2\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\therefore e = \frac{\Delta V}{V} = \frac{1-2\nu}{E}\,\sigma_x$$

$$\therefore \Delta V = \frac{1-2\nu}{E}\cdot\sigma_x\cdot V$$

$$= \frac{1-2\nu}{E}\cdot\frac{P}{A}\cdot(A\cdot L)$$

$$= \frac{1-2\nu}{E}\cdot P\cdot L \Rightarrow \frac{1-2(0.32)}{200\times10^9}\times 60\times10^3\times 0.5$$

$$\Rightarrow 54.0\times10^{-9}\ m^3$$

Hooke's Law

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{\sigma_x}{E}$$

$$= \frac{P}{AE}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$= \frac{60 \times 10^3}{\pi/4 (d_o^2 - d_i^2) \cdot E}$$

$$= 545.674 \times 10^{-6}$$

$$\Delta L = L\epsilon_x = 0.5 \times 545.674 (10^{-6})$$
$$= 272.837 (10^{-6}) \text{ m}$$
$$= 0.0728 \text{ mm}$$

$$\epsilon_{lat} = -\nu \epsilon_{long} = -0.32 \times (545.674)(10^{-6})$$
$$= -174.616 (10^{-6})$$

$$\Delta d_o = d_o \epsilon_{lat}$$
$$= 40 (-174.614)(10^{-6})$$
$$= 6.985 (10^{-3}) \text{ mm}$$

$$\Delta d_i = d_i \epsilon_{lat}$$
$$= -5.238 (10^{-3}) \text{mm}$$

New dimensions

$$L' = L + \Delta L = 500 + 0.2728 \Rightarrow 500.2728 \text{ mm}$$

$$d_o' = d_o + \Delta d_o = 40 - 6.885 \times 10^{-3}$$
$$= 39.9930 \text{ mm}$$

$$d_i' = d_i + \Delta d_i = 30 - 5.238 \times 10^{-3}$$
$$= 29.9948 \text{mm}$$

$$V' = \frac{\pi}{4}(d_o' - d_i') \cdot L'$$

$$V = \pi/4 (d_o - d_i) \cdot L$$

$$\therefore \Delta V = V' - V \Rightarrow 52.54 \text{ mm}^3$$

Example:

The principal plane stress and the associated strains in a member at a point are

$\sigma_1 = 36$ ksi $\qquad \sigma_2 = 16$ ksi

$\epsilon_1 = 1.02 \times 10^{-3} \qquad \epsilon_2 = 0.180 \times 10^{-3}$

Determine the modulus of elasticity and poisson's ratio.

Solution: Plane stress $\Rightarrow \sigma_3 = 0$

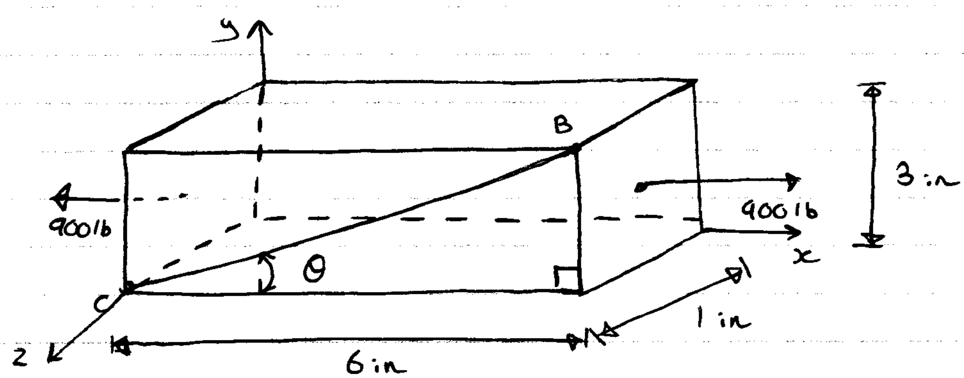
$$\begin{cases} \epsilon_1 = 1/E\,[\sigma_1 - \nu(\sigma_2 + \sigma_3)] \\ \epsilon_2 = 1/E\,(\sigma_2 - \nu\sigma_1) \\ \epsilon_3 = -\nu/E\,(\sigma_1 + \sigma_2) \end{cases}$$

$$\Rightarrow \quad 1.02 \times 10^{-3} = 1/E\,(36 - 16\nu)$$
$$0.180 \times 10^{-3} = 1/E\,(16 - 36\nu)$$

$$\Rightarrow \begin{cases} 1.02 \times 10^{-3} E + 16\nu = 36 \\ 0.180 \times 10^{-3} E + 36\nu = 16 \end{cases}$$

$$\Rightarrow \quad E = 30.7 \times 10^3 \text{ ksi}$$
$\nu = 0.291$ (no units, its a ratio)

Example:



The angle θ decreases by $\Delta\theta$ 0.01° after the load is applied. Find the poisson's ratio, Given E = 800 ksi

Solution ↓