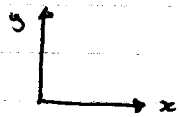
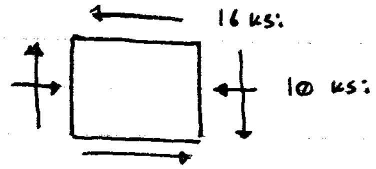
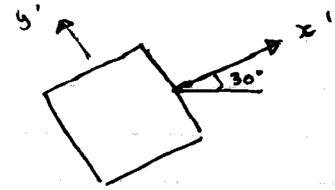
Example:







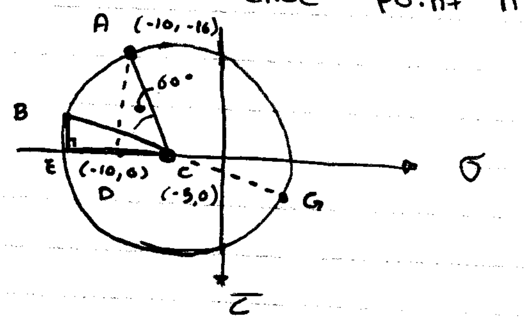
Determine the equivilent state of stress of an element if it is oriented 30° ccw from the element shown

Sol: $\sigma_x = -10 \quad \sigma_y = 0 \quad \tau_{xy} = -16$

$$\sigma_{AVG.} = \frac{\sigma_x + \sigma_y}{2} = \frac{-10+0}{2} = -5$$

$\therefore$ Centre $= C(-5, 0)$

Reference point A (-10, -16)



$\sigma_x = -10$
$\sigma_y = 0$
$\tau_{xy} = -16$
$\sigma_{AVG} = -5$
$R =$

$\Delta ACD \longrightarrow \quad CD = 5 \qquad AD = 16$

$$\tan \angle ACD \quad \frac{AD}{CD} = \frac{16}{5}$$

$\angle ACD = 72.646°$

$\angle BCE = \angle ACD - \angle ACB = 72.646° - 60°$
$= 12.646°$

Since $BC = AC = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \sqrt{\left(\frac{-10-0}{2}\right)^2 + (-16)^2} = 16.763$$

$\Delta BCE \longrightarrow \quad CE = BC \cos(\angle BCE) = 16.763 \cos(12.646°)$

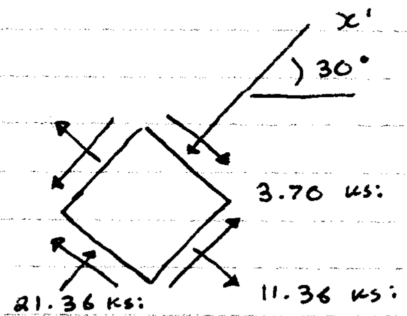
$\therefore \sigma_{x'} = -(CE + OC) \Rightarrow -(16.763 \cos(12.646°) + 5)$
$=$
$= -21.36$ ksi

$\tau_{x'y'} = -BE = -16.763 \sin(12.646°)$
$= -3.70$ ksi

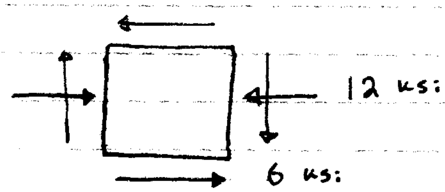
$$\frac{\sigma_B + \sigma_G}{2} = \sigma_c$$

$$\frac{\sigma_{x'} + \sigma_{y'}}{2} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'}$$
$$= 10 + 0 - (-21.36)$$
$$= 11.36 \text{ ksi}$$



Example :

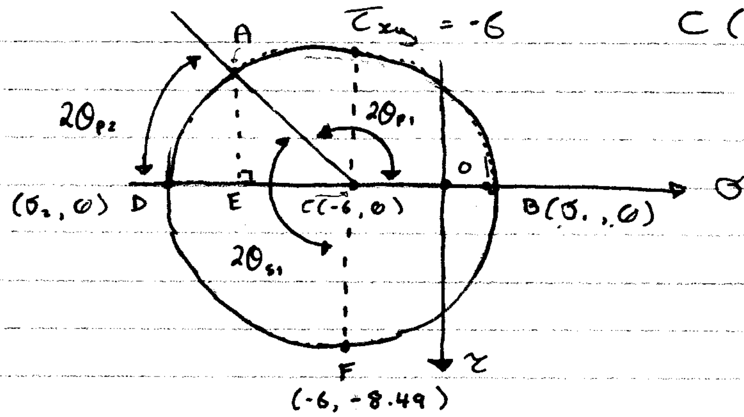


Determine the principal stresses and orientations.

Solution : $\sigma_x = -12$ $\qquad \sigma_{AVG} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-12 + 0}{2} = -6$

$\sigma_y = 0$

$\tau_{xy} = -6$ $\qquad C(-6, 0) \qquad A(-12, -6)$



Since $CA = R = \sqrt{(-12-(-6))^2 + (-6-0)^2}$
$= 8.49$

$\therefore \sigma_1 = R - |OC| = 8.49 - 6 = 2.49 \text{ ksi}$

$\sigma_2 = -(R + |OC|) = -(8.49 - 6) = -14.49 \text{ ksi}$

$\triangle ACE$: $AE = 6$ $\quad CE = 6$

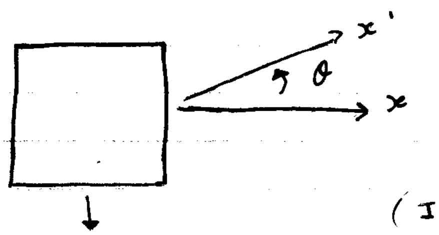
$$\therefore \tan \angle ACE = \frac{AE}{CE} = \frac{6}{6} = 1$$

$$\angle ACE = 45°$$

$$\therefore \theta_{P2} = +(1/2)\angle ACE = 22.5°$$

and

$$\theta_{P1} = -(1/2)(180° - 45°) = -67.5°$$

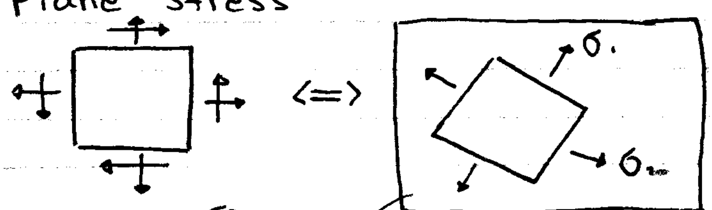


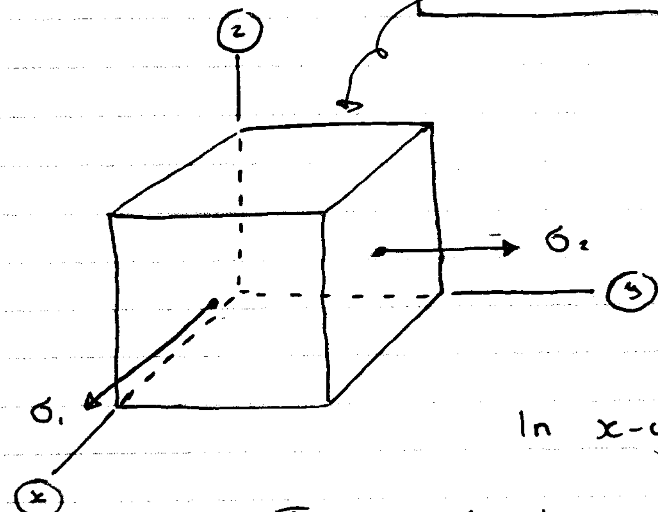
$\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$

(IGNORE THIS)

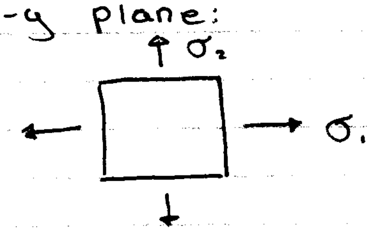
9.5 - Absolute Maximum Shear Stress

Plane Stress

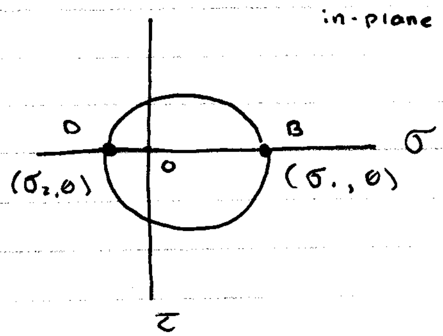




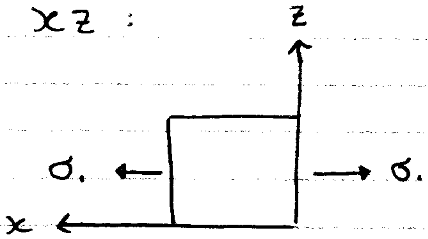
In x-y plane:

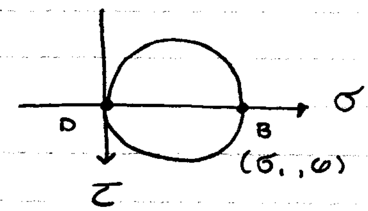


$$\tau_{\substack{max \\ \text{in-plane}}} = \frac{\sigma_1 - \sigma_2}{2}$$



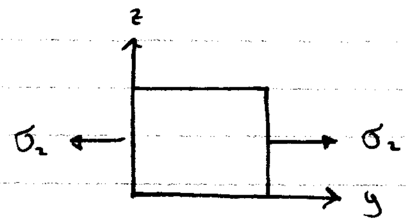
In xz :





$$\tau_{\substack{max \\ in\text{-}plane}} = \frac{\sigma_1}{2}$$

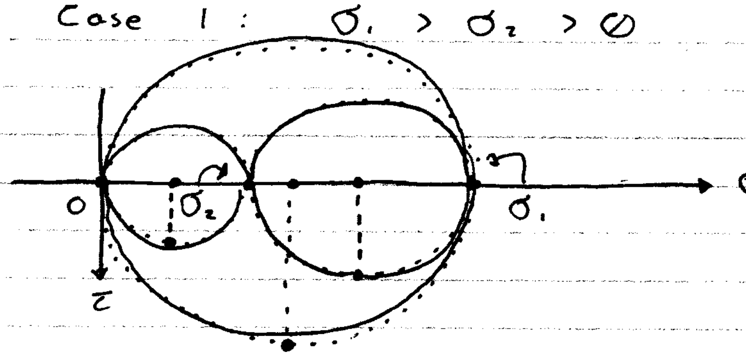
In yz :



$$\tau_{\substack{max \\ in\text{-}plane}} = \frac{\sigma_2}{2}$$

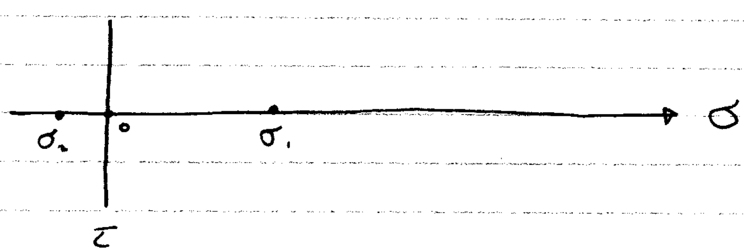
$$\tau_{\substack{abs \\ max}} = \max\left(\left|\frac{\sigma_1}{2}\right| , \left|\frac{\sigma_2}{2}\right| , \left|\frac{\sigma_1 - \sigma_2}{2}\right| \right)$$

Case 1 : $\sigma_1 > \sigma_2 > \emptyset$



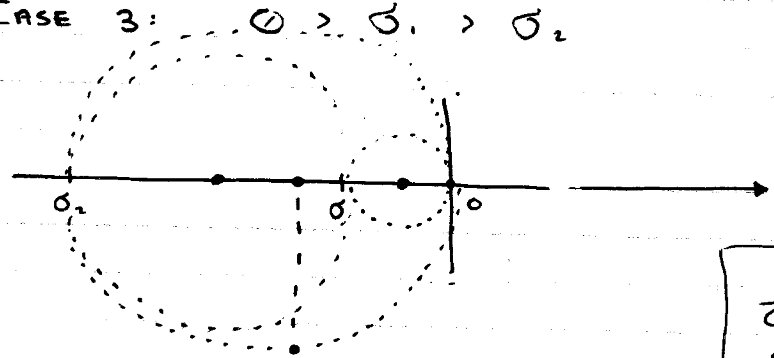
$$\boxed{\tau_{\substack{abs \\ max}} = \frac{\sigma_1}{2}}$$

Case 2 : $\sigma_1 > \emptyset > \sigma_2$



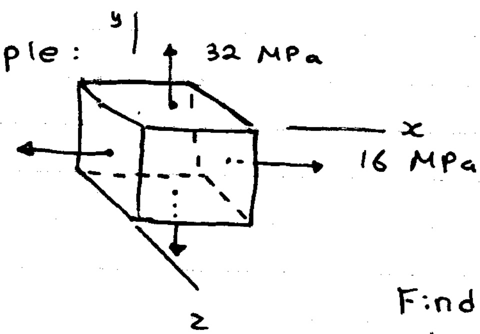
$$\boxed{\tau_{\substack{abs \\ max}} = \frac{\sigma_1 - \sigma_2}{2}}$$

Case 3: $0 > \sigma_1 > \sigma_2$



$$\boxed{\tau_{\substack{abs\\max}} = -\frac{\sigma_2}{2}}$$

Example:



Find the absolute max shear stress.

Solution: Since

$\sigma_1 = 32$ MPa

$\sigma_2 = 16$ MPa

$\sigma_1 > \sigma_2 > 0$

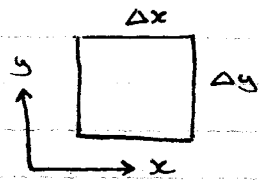
$$\therefore \quad \tau_{\substack{abs\\max}} = \frac{\sigma_1}{2} = \frac{32}{2} = 16 \text{ MPa}$$

and

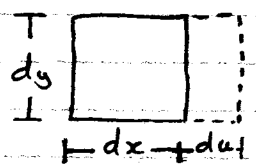
$$\tau_{\substack{max\\\text{in-plane}}} = \frac{\sigma_1 - \sigma_2}{2} = \frac{32-16}{2} = 8 \text{ MPa}$$

Ch. 10 - Strain Transformation

10.1 Plane Strain

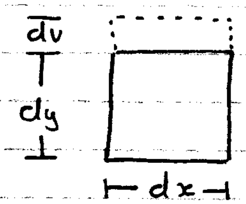


Sign convention notes:
elongation : (+)
compression : (−)

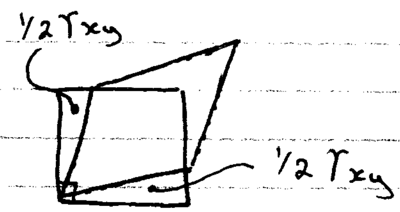


$\epsilon_x = \dfrac{du}{dx}$ = normal strain of a line segment in x-direction

3.72
4.43
6.62
7.3



$\epsilon_y = \dfrac{dv}{dy}$ = normal strain of a line segment in y-direction

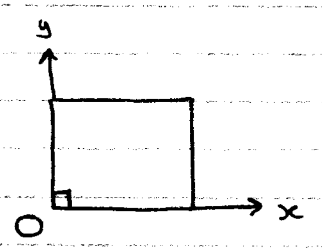


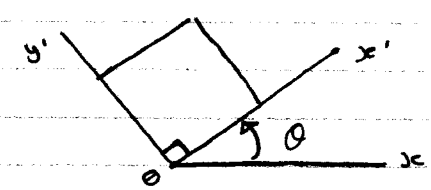
γ_{xy} = the shear strain

10.2 General Equations of plane-strain transformation

Sign convention :

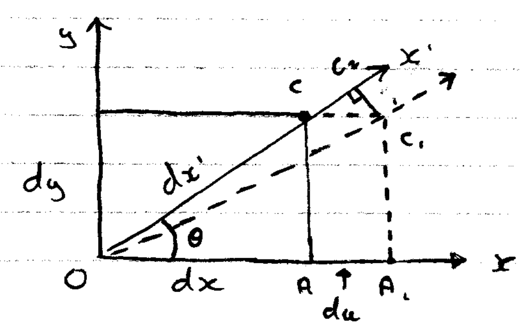
Normal strain : positive if elongation
Shear strain : positive if the deformed angle is less than 90°





$\epsilon_x, \epsilon_y, \gamma_{xy}$

$\epsilon_{x'}, \epsilon_{y'}, \gamma_{x'y'}$



$$\begin{cases} dx = \cos\theta \cdot dx' \\ dy = \sin\theta \cdot dx' \end{cases}$$

$$\epsilon_x = \frac{du}{dx}$$

$\underline{\Delta CC_1C_2}$: $\angle C_1CC_2 = \theta \qquad CC_1 = AA_1 = du$

$$\therefore \quad CC_2 = CC_1 \cos\theta = \cos\theta \, du$$
$$C_1C_2 = CC_1 \sin\theta = \sin\theta \, du$$

Normal Strain of OC:

$$\epsilon_{x'} = \frac{CC_2}{OC} = \frac{\cos\theta \, du}{dx'}$$

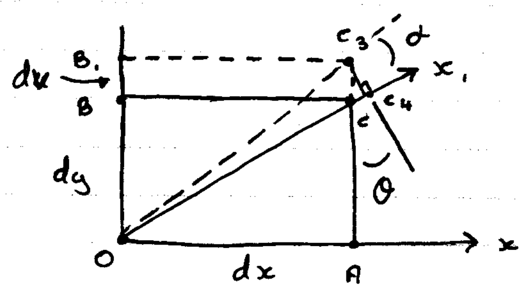
$\underline{\Delta OC_1C_2}$:

angle is clockwise

$$\tan\alpha = -\frac{C_1C_2}{OC_1} = -\frac{C_1C_2}{OC + CC_2} = -\frac{C_1C_2}{OC}$$

Since α is small:

$$\alpha \approx \tan\alpha = -\frac{C_1C_2}{OC} = -\frac{\sin\theta \, du}{dx'}$$

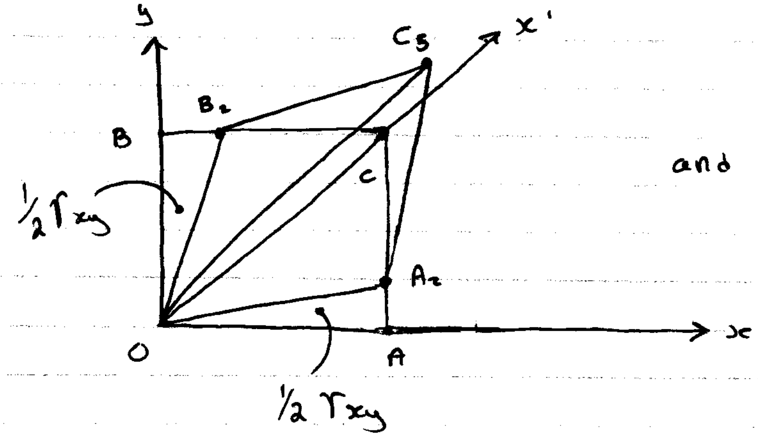


ΔCC_3C_4

$$CC_4 = CC_3 \sin\theta = \sin\theta \, dv$$
$$C_3C_4 = CC_3 \cos\theta = \cos\theta \, dv$$

$$\epsilon_{x'} = \frac{CC_4}{OC} = \frac{\sin\theta \, dv}{dx'}, \quad \overset{\alpha}{\cancel{\beta}} = \frac{C_3C_4}{OC} = \frac{\cos\theta \, dv}{dx'}$$

$$\epsilon_y = \frac{dv}{dy}$$



Horizontal displacement:

$$BB_2 = \tfrac{1}{2}\gamma_{xy} \, dy$$

and Vertical displacement:

$$AA_2 = \tfrac{1}{2}\gamma_{xy} \, dx$$

I

Normal Strain of OC

$$\epsilon_{x'} = \frac{\cos\theta}{dx'} \cdot \tfrac{1}{2}\gamma_{xy}\,dy + \frac{\sin\theta}{dx'} \cdot \tfrac{1}{2}\gamma_{xy}\,dx$$

Rotation

$$\alpha = \frac{-\sin\theta}{dx'} \cdot \tfrac{1}{2}\gamma_{xy}\,dy + \frac{\cos\theta}{dx'} \cdot \tfrac{1}{2}\gamma_{xy}\,dx$$

Total normal strain

$$\epsilon_{x'} = \frac{\cos\theta}{dx'}\,du + \frac{\sin\theta}{dx'}\,dv + \frac{\cos\theta}{dx'}\,\tfrac{1}{2}\gamma_{xy}\,dy + \frac{\sin\theta}{dx'}\,\tfrac{1}{2}\gamma_{xy}\,dx$$

Since $\epsilon_x = \dfrac{du}{dx} \Rightarrow \boxed{du = \epsilon_x\,dx}$

$\epsilon_y = \dfrac{dv}{dy} \Rightarrow \boxed{dv = \epsilon_y\,dy}$

$$\Rightarrow \epsilon_{x'} = \cos\theta \cdot \epsilon_x \frac{dx}{dx'} + \sin\theta\,\epsilon_y \frac{dy}{dy'} + \tfrac{1}{2}\gamma_{xy}\cos\theta \frac{dy}{dx'} + \tfrac{1}{2}\gamma_{xy}\sin\theta\frac{dx}{dx'}$$

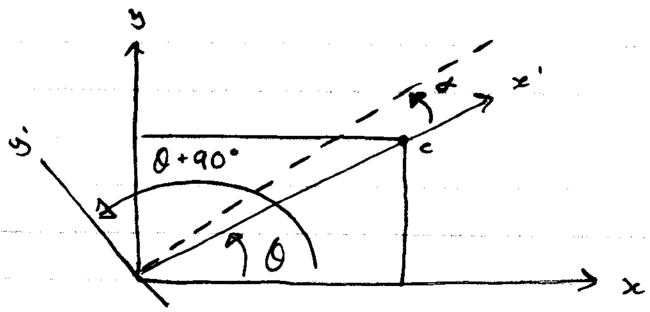
$$\frac{dx}{dx'} = \cos\theta \qquad \frac{dy}{dy'} = \sin\theta$$

$$\boxed{\epsilon_{x'} = \epsilon_x\cos^2\theta + \epsilon_y\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta}$$

Rotation of x' axis (OC)

$$\alpha = -\sin\theta\frac{du}{dx'} + \cos\theta\frac{dv}{dx'} - \tfrac{1}{2}\gamma_{xy}\sin\theta\frac{dy}{dx'} + \tfrac{1}{2}\gamma_{xy}\cos\theta\frac{dx}{dx'}$$

$$= (\epsilon_y - \epsilon_x)\sin\theta\cos\theta + \tfrac{1}{2}\gamma_{xy}(\cos^2\theta - \sin^2\theta)$$



$\epsilon_x, \epsilon_y, \gamma_{xy}$

$$\therefore \epsilon_{y'} = \epsilon_x \cos^2(\theta+90°) + \epsilon_y \sin^2(\theta+90°) + \gamma_{xy} \sin(\theta+90°)\cos(\theta+90°)$$
$$= \epsilon_x \sin^2\theta + \epsilon_y \cos^2\theta - \gamma_{xy} \sin\theta\cos\theta$$

$$\beta = (\epsilon_y - \epsilon_x)\sin(\theta+90°)\cos(\theta+90°) + \tfrac{1}{2}\gamma_{xy}\left(\cos^2(\theta+90°) - \sin^2(\theta+90°)\right)$$
$$= -(\epsilon_y - \epsilon_x)\sin\theta\cos\theta + \tfrac{1}{2}\gamma_{xy}(\sin^2\theta - \cos^2\theta)$$

$$90° \rightarrow 90° - \alpha + \beta$$

The shear strain

$$\gamma_{x'y'} = 90° - (90° - \alpha + \beta)$$
$$= \alpha - \beta$$
$$\boxed{\gamma_{x'y'} = 2(\epsilon_y - \epsilon_x)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)}$$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \tfrac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \tfrac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{(\epsilon_x - \epsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

Stress $\longleftrightarrow$ strain

$\sigma_x \longleftrightarrow \epsilon_x$ $\qquad\qquad \tau_{xy} \longleftrightarrow \tfrac{1}{2}\gamma_{xy}$

$\sigma_y \longleftrightarrow \epsilon_y$