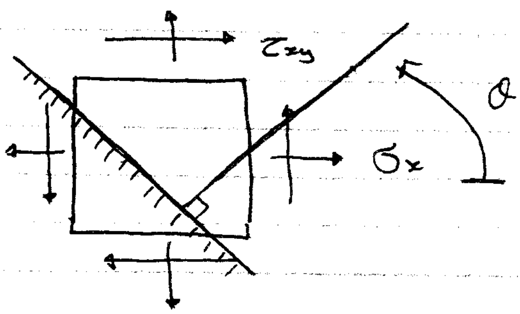
JAN. 30TH/17

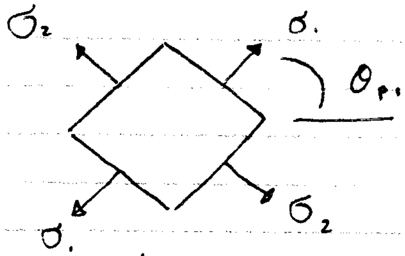


$$\sigma_{x'} = \sigma_{AVG} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

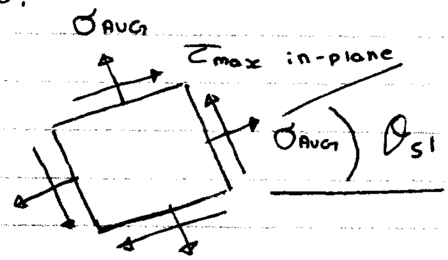
$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Inclined surface (θ):

$\sigma_{x'}(\theta)$ and $\tau_{x'y'}(\theta)$

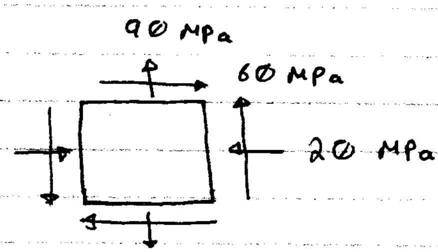


$$\sigma_{AVG} = \frac{\sigma_x + \sigma_y}{2}$$



$$[\theta_{p1} - \theta_{s1}] = 45°$$

Example:



a) Represent the state of stress in terms of principal stresses

b) Represent the state of stress in terms of max in-plane shear stress associated normal stress

Solution: $\sigma_x = -20$ MPa

$\sigma_y = 90$ MPa

$\tau_{xy} = 60$ MPa

a) $= \tan 2\theta_p = \dfrac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$

$= \dfrac{60}{(-20-90)/2} = -1.0909$

$$2\theta_p = -47.49^\circ, \quad 180^\circ + (-47.49^\circ)$$
$$\theta_p = -23.75^\circ, \quad 66.26^\circ$$

$$\sigma_{AVG} = \frac{\sigma_x + \sigma_y}{2} = \frac{-20 + 90}{2} = 35$$

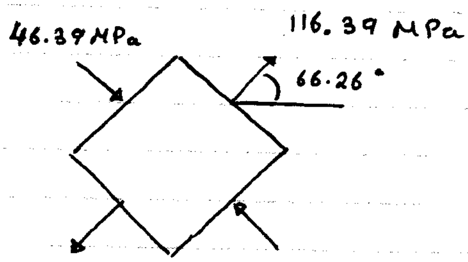
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{\left(\frac{-20 - 90}{20}\right)^2 + 60^2}$$
$$= 81.39$$

corrected

$$\sigma_1 = \sigma_{AVG} + R = 35 + 81.39 = \boxed{116.39\ \text{MPa}}$$
$$\sigma_2 = \sigma_{AVG} - R = 35 - 81.39 = -46.39\ \text{MPa}$$

when $\theta = 66.26^\circ$

$$\sigma_{x'} = \sigma_{AVG} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= 35 + \left(\frac{-20 - 90}{2}\right)\cos(2 \times 66.26^\circ) \ldots$$
$$\ldots + 60 \sin(2 \times 66.26^\circ) = 116.39 = \sigma_1$$



b) $$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$
$$= -\frac{(-20-90)/2}{60}$$
$$= 0.91667$$

$$2\theta_s = 42.51^\circ, \quad 180^\circ + 42.51$$
$$\theta_s = 21.26^\circ, \quad 111.26^\circ$$

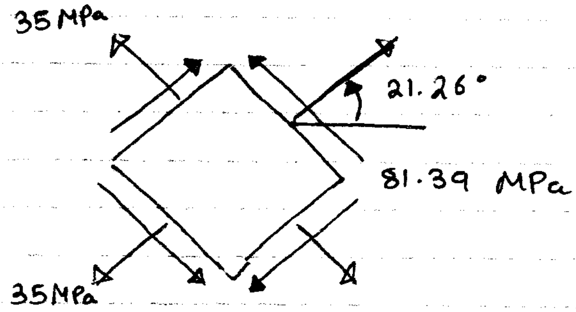
$$\tau_{\substack{max \\ \text{in-plane}}} = R \left(= \frac{\sigma_1 - \sigma_2}{2}\right)$$
$$= 81.39\ \text{MPa}$$

When $\theta = 21.26$

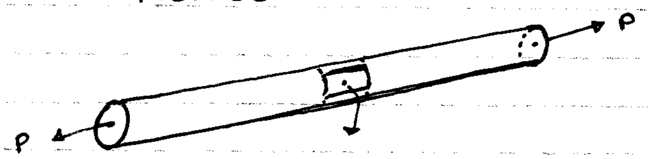
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

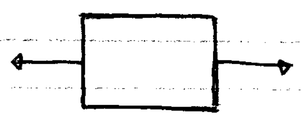
$$= -\frac{-20-90}{2}\sin(42.51^\circ) + 60\cos(42.51^\circ)$$

$$= 81.39 \text{ MPa}$$



Axial Force member

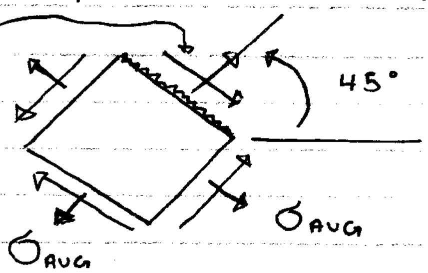




$\sigma = P/A$

$\therefore \quad \sigma_1 = P/A \quad , \quad \sigma_2 = 0$

$$\tau_{\substack{max \\ \text{in-plane}}} = R = \frac{\sigma_1 - \sigma_2}{2} = \frac{P}{2A}$$



$\theta = 45°$

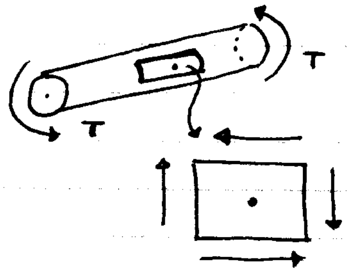
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{P/A - 0}{2}\sin 90^\circ + 0$$

$$= -\frac{P/A}{2A}$$

(∴ stress is negative + rest of arrows can be drawn in.)

Torsion:



$$\tau = \frac{T\rho}{GJ}$$

$$\sigma_x = \sigma_y = 0, \qquad \tau_{xy} = -\tau$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \infty$$

$$2\theta_p = 90°, \quad 180° + 90°$$

$$\theta_p = 45°, \quad 135°$$

$$\sigma_{AVG} = 0, \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \tau$$

$$\therefore \sigma_1 = \sigma_{AVG} + R = \tau$$

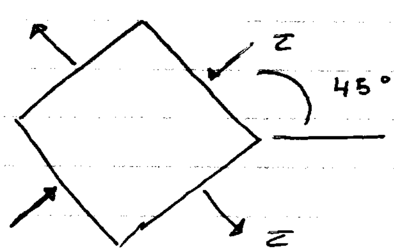
$$\sigma_2 = \sigma_{AVG} - R = -\tau$$

$$\theta = 45°$$

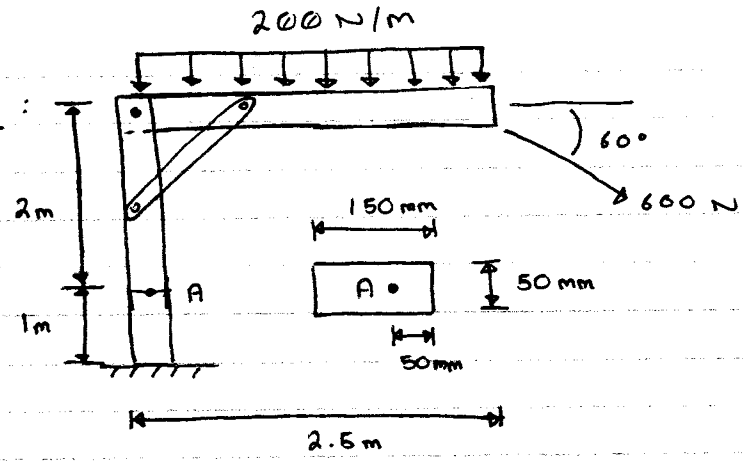
$$\sigma_{x_1} = \sigma_{AVG} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + (-\tau)\sin 90°$$

$$= -\tau$$

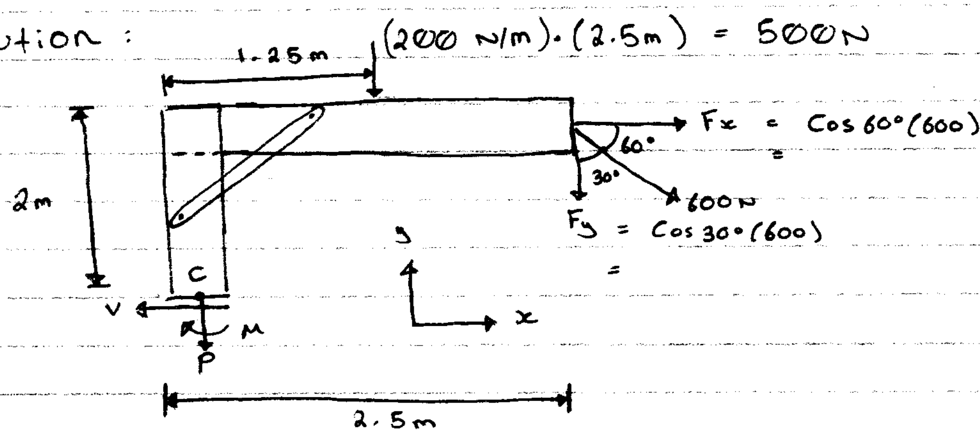


Example :



Determine the principal stresses and the max in-plane shear stress at A.

Solution :



$\Sigma F_x = 0$

$600\cos 60° - V = 0$

$V = 300 N$

$\Sigma F_y = 0$

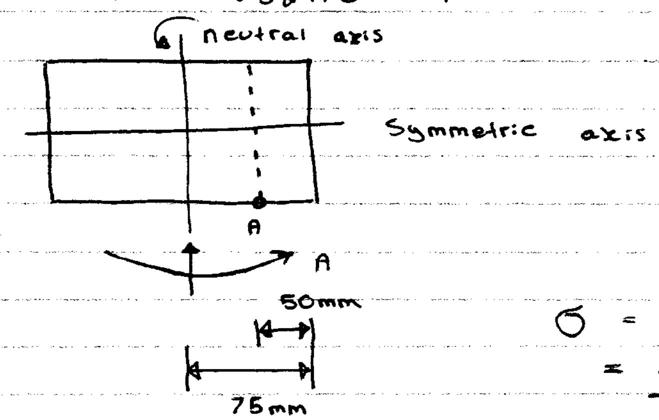
$-P - 500 - 600\cos 30° = 0$

$P = 1019.6\ N$

$\Sigma M_c = 0$

$0 = -M - 500 \times 1.25 - 600 \times \cos 30° \times 2.5 - 600 \cdot \cos 60° \times 2$

$M = -2524.0\ N.m$



$A = (150 \times 50)$

$I = (1/12) b h^3$

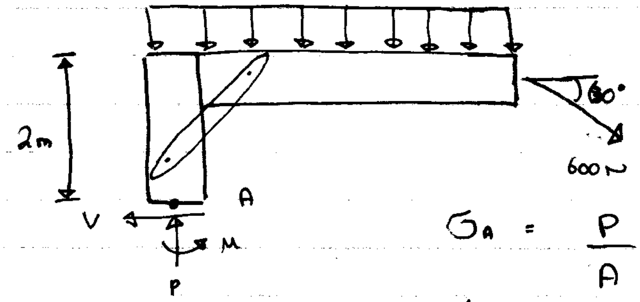
$= (1/12)(50)(150)^3$

$y = 25 mm$

$$\sigma = P/A + My/I$$

$$= \frac{-1019.6}{150 \times 50} - \frac{2524 \times 25 \times 10^3}{1/12\,(50)(150)^3}$$

$$= -4.623\ N/mm^2$$

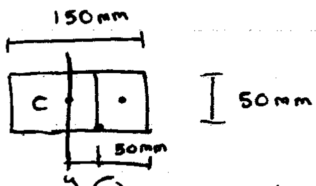


$P = 1019.6\,N$, $V = 300\,N$

$M = 2524.0 \times 10^3\ N \cdot mm$

$$\sigma_A = \frac{P}{A} + \frac{My}{I}$$

$$\sigma_A = \frac{1019.6}{(150 \times 50)} - \frac{2524(10^3)(25)}{(1/12)(50)(150)^3}$$

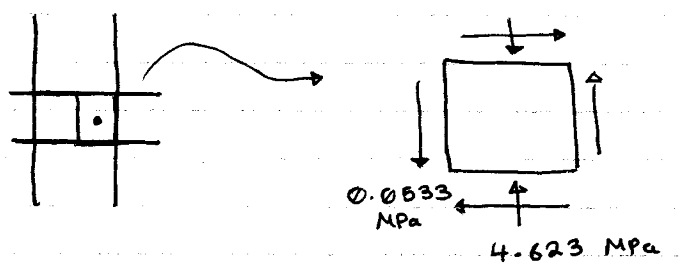


$$\sigma_A = -4.623\ MPa$$

$$Q = A'\bar{y}' = 50 \times 50 \times 50$$

$$\tau_A = \frac{VQ}{It} = \frac{300 \times 50 \times 50 \times 50}{(1/12) \times 50 \times 150^3 \times 50}$$

$$\tau_A = 0.0533\ MPa$$



At A :

$\sigma_x = 0$, $\sigma_y = -4.623$, $\tau_{xy} = 0.0533$

Principal Stresses

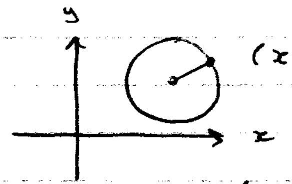
$$\sigma_{AVG} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 - 4.623}{2} = -2.3115$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - \tau_{xy}^2} = 2.3121$$

$\sigma_1 = 0.0006\ MPa$

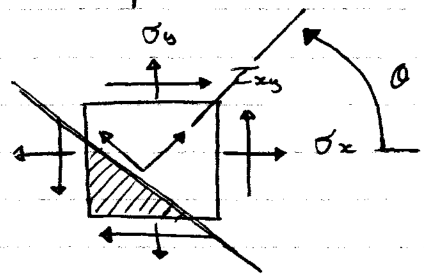
$\sigma_2 = -4.6236\ MPa$

9.4 Mohr's Circle — Plane Stress



(x_0, y_0), R

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$



$$\begin{cases} \sigma_{x'} = \sigma_{AVG} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_{xy} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \end{cases}$$

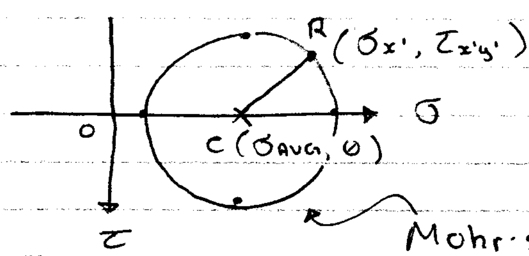
$$\Rightarrow (\sigma_{x'} - \sigma_{AVG})^2 + \tau_{x'y'}^2$$

$$= \left(\frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)^2 + \left(-\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)^2$$

$$= \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

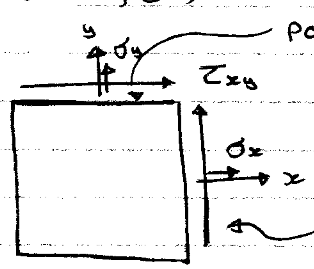
$$= R^2$$

$$(\sigma_{x'} - \sigma_{AVG})^2 + \tau_{x'y'}^2 = R^2$$



Mohr's Circle

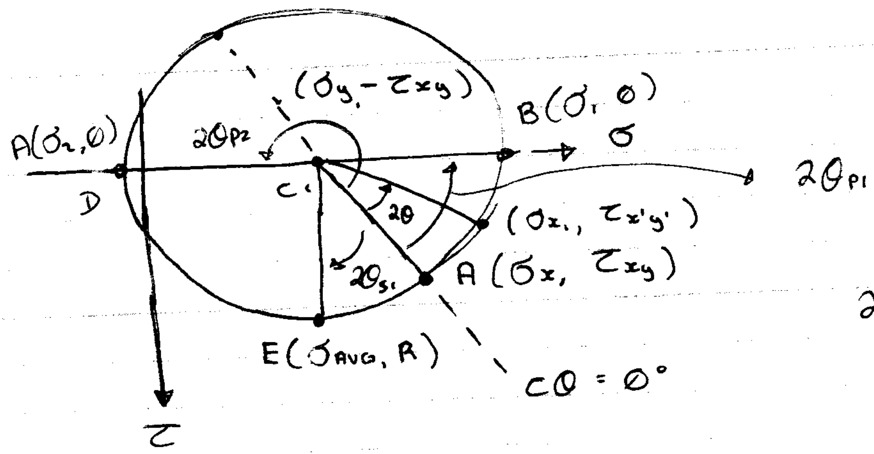
$(\sigma_{AVG}, 0) \leftarrow$ Centre, $R \leftarrow$ radius



Positive y-Face $\rightarrow \theta = 90° \rightarrow (\sigma_y - \tau_{xy})$

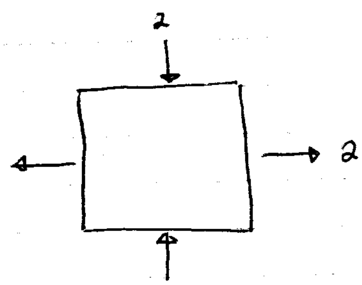
$\theta = 0°$

Positive x-Face



$2\theta_{P2} - 2\theta_{P1} = 180°$
& difference between two angles is 90°

1° → Centre $(\sigma_{AVG}, 0)$
2° → Reference Point $A\ (\sigma_x, \tau_{xy})$

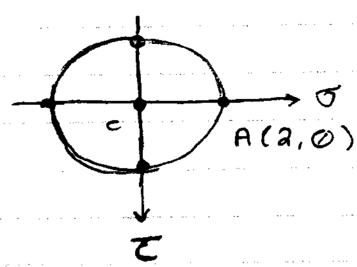


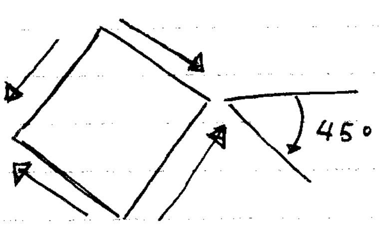
$\sigma_x = 2 \quad \sigma_y = -2$
$\tau_{xy} = 0$

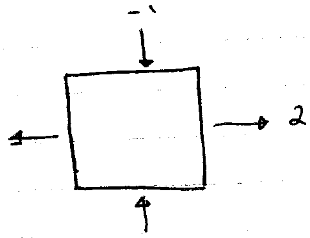
$$\sigma_{AVG} = \frac{\sigma_x + \sigma_y}{2} = \frac{2-2}{0} = 0$$

Centre $C\ (\sigma_{AVG}, 0) = (0, 0)$

$A\ (\sigma_x, \tau_{xy}) = A(2, 0)$

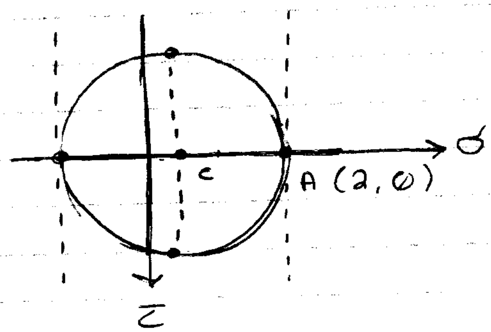






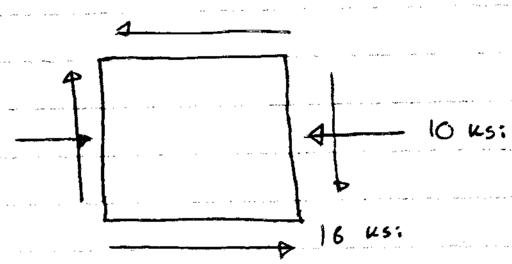
$$\sigma_{AVG} = \frac{\sigma_x + \sigma_y}{2} = 1/2$$

C (1/2, 0)
A (2, 0)



$R = |AC|$
$= 1.5$

Example:

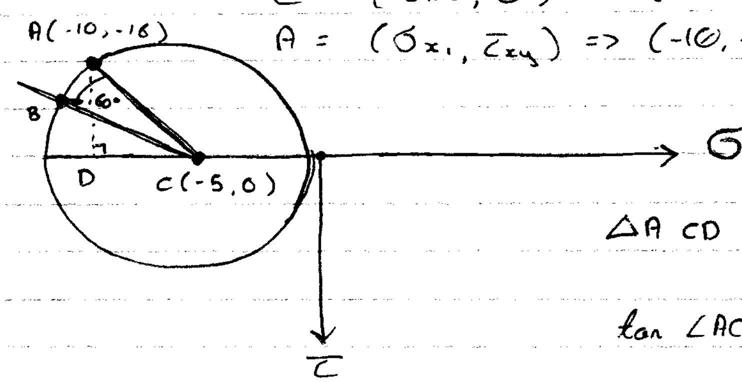


Find the equivalent state of stress on an element if it is oriented 30° CCW from the element shown.

Solution: $\sigma_x = -10$, $\sigma_y = 0$, $\tau_{xy} = -16$

$$\sigma_{AVG} = \frac{\sigma_x + \sigma_y}{2} = \frac{-10 + 0}{2} = -5$$

$\therefore$ $C = (\sigma_{AVG}, 0) \Rightarrow (-5, 0)$
$A = (\sigma_x, \tau_{xy}) \Rightarrow (-10, -16)$



$\triangle ACD$: $AD = 16$
$CD = |-5-(-10)| = 5$

$$\tan \angle ACD = \frac{AD}{CD} = \frac{16}{5} = 3.2$$

$\angle ACD = 72.647°$

$\angle BCD = \angle ACD - 2(30°)$
$= 72.646° - 60°$
$= 12.646°$