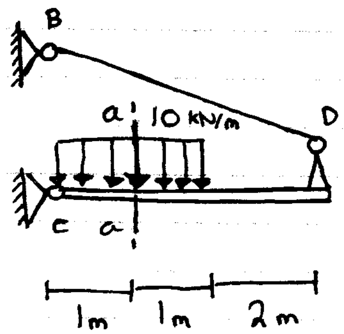
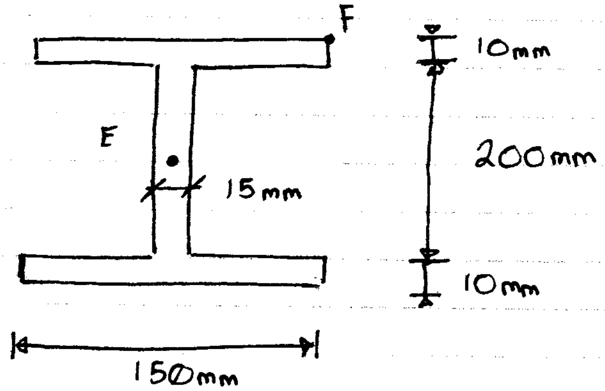
Assignment #1 due Jan. 23RD in class.
Chapter 8: 6, 9, 16, 20, 38, 49

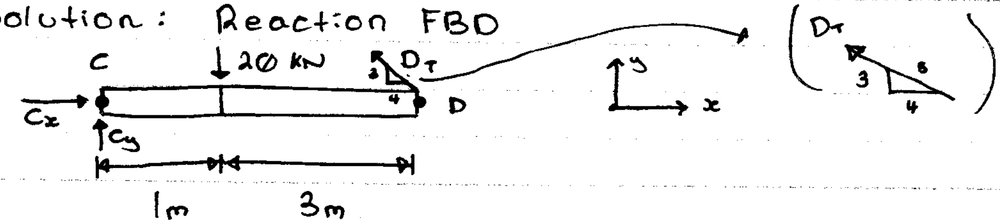
EXAMPLE:





Determine the state of stress at points E and F at section a-a.

Solution: Reaction FBD



$\Sigma M_c = 0$

$-20\text{ kN}(1\text{ m}) + (3/5)D_T(4\text{ m}) + (4/5)D_T(0.3\text{m}) = 0$

$D_T\left[(3/5)(0.3) + (4/5)(4)\right] = 20$

$D_T = 7.576\text{ kN}$

$\overset{+}{\rightarrow}\Sigma F_x = 0$

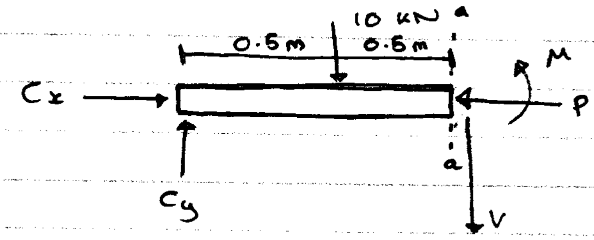
$-(4/5)D_T + C_x$

$C_x = 6.0606\text{ kN}$

$+\uparrow \Sigma F_y = 0$

$+(3/5)D_T + C_y$

$C_y = 15.4545\text{ kN}$



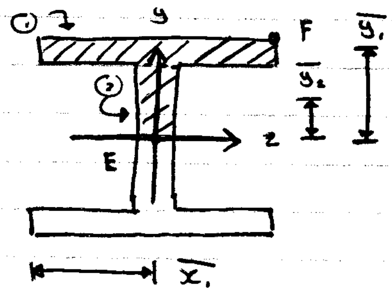
$P = 6.0606$ kN
$V_o = 5.4545$ kN
$M = 10.4545$ kN·m

Geometric properties of section a-a:

E is the centroid of the section.

$A = (150 \times 10 \times 2) + (200 \times 15) = 6000 \text{ mm}^2$
$= 6000 \times 10^{-6} \text{ m}^2 = 6 \times 10^{-3} \text{ m}^2$

$I = (1/12)(15)(200)^3 + 2\left[(1/12)(150)(10)^3 + (150)(10)(105)^2\right]$
$= 43.1 \times 10^6 \text{ mm}^4$
$= 43.1 \times 10^{-6} \text{ m}^4$



$Q_E = A_1\bar{y}_1 + A_2\bar{y}_2$
$= (150)(10)(105) + (100)(15)(50)$
$= 0.2325\,(10^6) \text{ mm}^3$
$= 0.2325 \times 10^{-3} \text{ m}^3$

$Q_F = 0$

At E:

Normal Stress

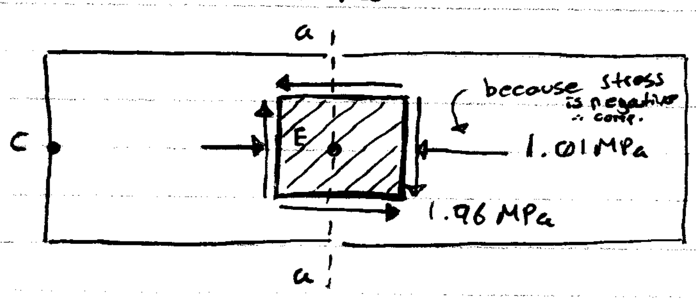
$$\sigma_E = \frac{-6.0606 \text{ kN}}{6 \times 10^{-3} \text{m}^2} + \frac{10.4545 \text{ kN}}{43.1 \times 10^{-6}}(0)$$

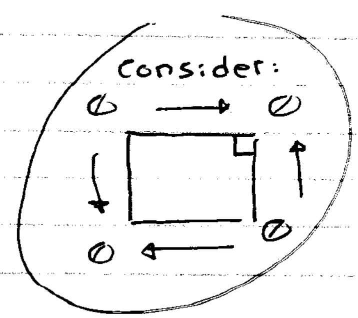
$\sigma_E = -1.0101$ MPa

Shear Stress

$$\tau_E = \frac{VQ_E}{It} = \frac{(5.4545 \text{ kN})(0.2325 \times 10^{-3} \text{ m}^3)}{(43.1 \times 10^{-6} \text{ m}^4)(15 \times 10^{-3} \text{ m})}$$

$\tau_E = 1.96$ MPa





At F:

Normal Stress

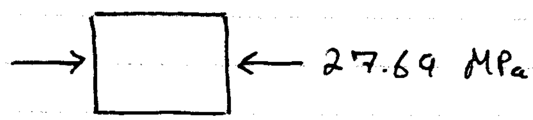
$$\sigma_F = \frac{-6.0606\ \text{kN}}{6 \times 10^{-3}\ \text{m}^2} - \frac{10.4545\ \text{kN}}{43.1 \times 10^{-6}\ \text{m}^4}(0.11\text{m})$$

$$\sigma_F = -27.69\ \text{MPa}$$

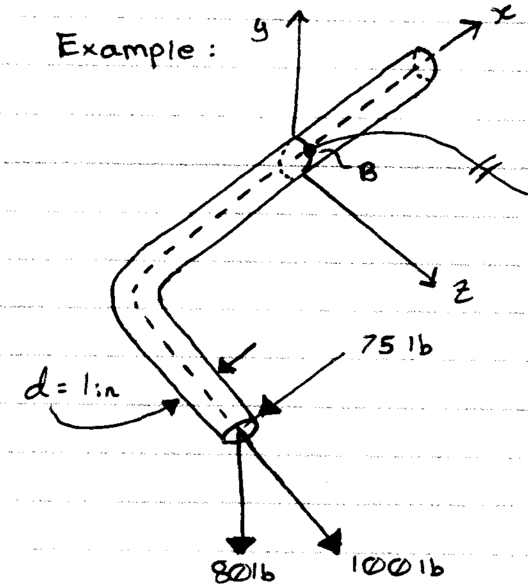
Shear stress

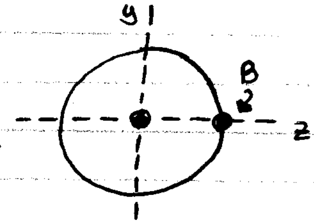
$$\tau_F = \frac{VQ_F}{It} = 0$$

State of stress



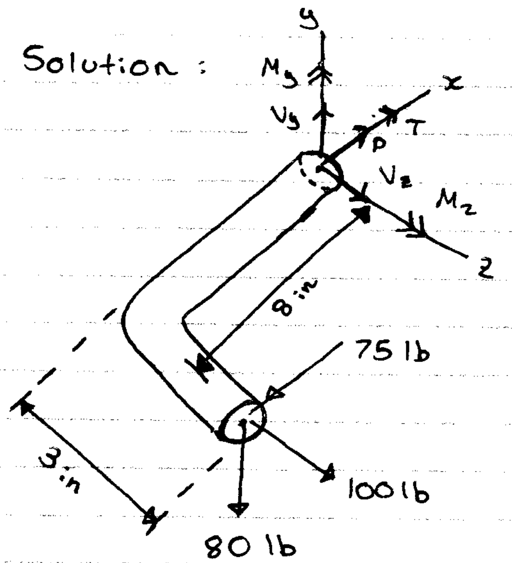
Example:





Determine the state of stress at B.

Solution:



$\Sigma F_x = 0 \qquad P - 75 = 0 \qquad P = 75\,lb$

$\Sigma F_y = 0 \qquad V_y - 80 = 0 \qquad V_y = 80\,lb$

$\Sigma F_z = 0 \qquad V_z + 100 = 0 \qquad V_z = -100\,lb$

$\Sigma M_x = 0$

$0 = T + (80\,lb)(3\,in)$

$T = -240\,lb \cdot in$

$\Sigma M_y = 0$

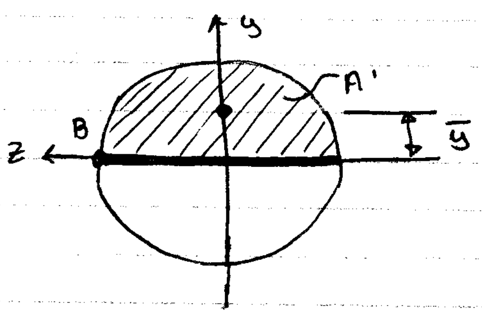
$0 = M_y - (75\,lb)(3\,in) + (100\,lb)(8\,in)$

$M_y = -575\,lb \cdot in$

$\Sigma M_z = 0$

$0 = M_z + (80\,lb)(8\,in)$

$M_z = -640\,lb \cdot in$



$I_y = I_z = (1/4)\pi r^4$

$= (1/4)\pi(0.5)^4$

$= 0.049087$

$J_o = I_y + I_z = \frac{1}{2}\pi(r)^4$

$= 0.098174$

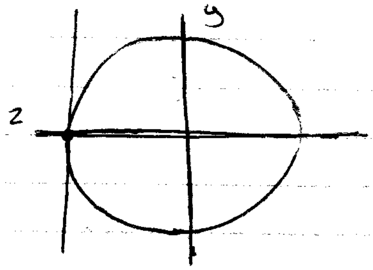
$A = \pi r^2 = \pi(0.5)^2$

$= 0.78540$

$Q_{B_z} = A'\bar{y}$

$= (1/2)\pi r^2 \times \frac{4r}{3\pi}$

$= 0.083333$



$$Q_{B1y} = A'\bar{z} = \emptyset$$

<u>Normal:</u>

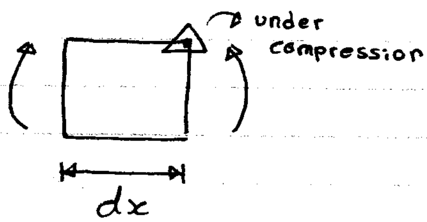
Axial Force $\sigma_{B1} = \dfrac{P}{A} = \dfrac{75}{0.78540}$

$= 95.493$

Bending moment M_y

$$\sigma_{B2} = -\left| \frac{M_y}{I_y} z \right| \Rightarrow -\left| \frac{-575}{0.04908} \cdot (0.5) \right|$$

$= -5846.9$



Bending moment M_z

distance from neutral axis

$$\sigma_{B3} = \left| \frac{M_z}{I_z} y \right| = \emptyset$$

$\therefore \ \sigma_B = \sigma_{B1} + \sigma_{B2} + \sigma_{B3}$

$= 95.493 - 5846.8 = -5761.5$ psi

<u>Shear:</u>

Torque $T = -240$ lb·in

$$\tau_{B1} = \frac{T}{J}\rho = \frac{240}{0.098174} \cdot (0.5)$$

$= 1223.3$ ↑

Shear Force V_y

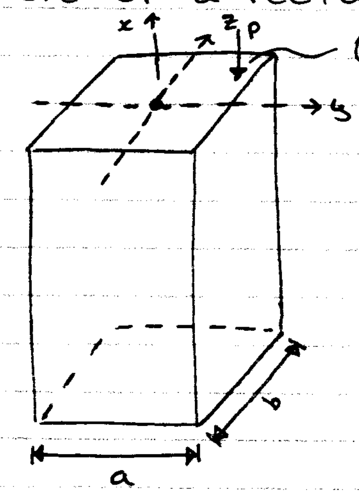
$$\tau_{B2} = \frac{V_y Q_{Bz}}{I_z t} = \frac{(80)\cdot(0.683333)}{(0.049087)\cdot(1)} = 135.8$$

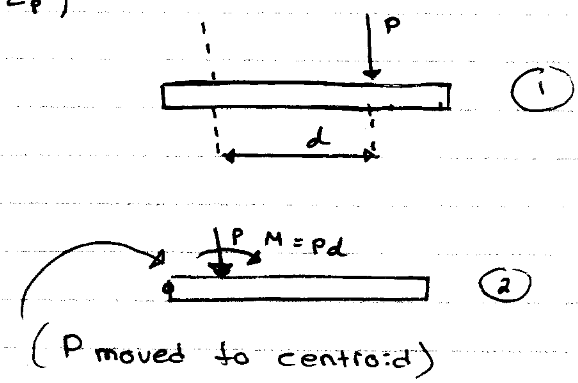
Shear force V_z

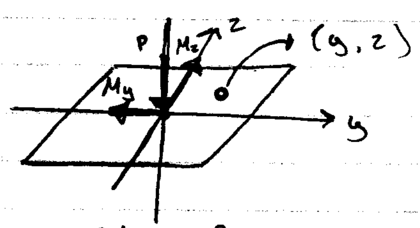
$$\tau_{B3} = \frac{V_z Q_{By}}{I_y t} = 0$$

$$\therefore \quad \tau_B = \tau_{B1} + \tau_{B2} + \tau_{B3} = 1359.1 \text{ psi}$$

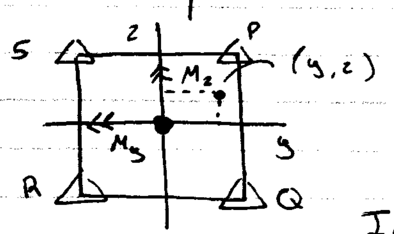
Core of a rectangular cross section







$M_y = P z_p$
$M_z = P \cdot y_p$



Normal stress:

$$\sigma = -\frac{P}{A} - \frac{M_y}{I_y} z - \frac{M_z}{I_z} y$$

$$I_y = (1/12)\,ab^3 = (1/12)\,b^2 \times ab = (1/12)\,A b^2$$

$$I_z = (1/12)\,a^3 b = (1/12)\,A a^2$$

$$\therefore \sigma = -P/A - \frac{P y_p}{1/12\, A b^2} z - \frac{P z_p}{1/12\, A a^2}$$

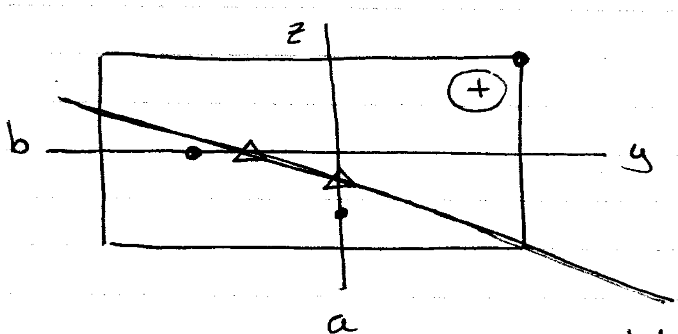
$$= -P/A\left(1 + \frac{12 y_p}{b^2} z + \frac{12 z_p}{a^2} y\right) \le 0$$

$$\Rightarrow \quad 1 + \frac{12 y_p}{b^2} z + \frac{12 z_p}{a^2} y \geq 0$$

At P: $\quad y = \tfrac{1}{2} a, \quad z = \tfrac{1}{2} b$

$$\Rightarrow 1 + \frac{12 y_p}{b^2} \times \frac{1}{2} b + \frac{12 z_p}{a^2} \times \frac{1}{2} a \geq 0$$

$$\Rightarrow 1 + \frac{6}{b} y_p + \frac{6}{a} z_p \geq 0$$



$$1 + \frac{6}{b} y_p + \frac{6}{a} z_p = 0$$