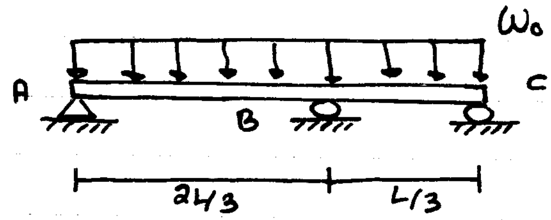
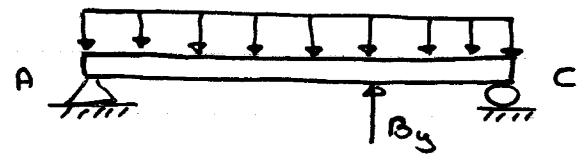
EXAMPLE :

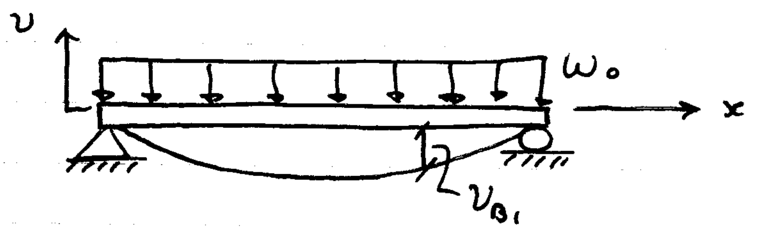


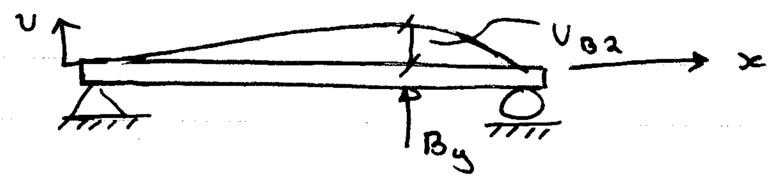

EI = const.

SOLUTION :




+ $\boxed{\delta_B = 0}$




$$\delta_B = 0 = v_{B1} + v_{B2}$$

and $$v_{B1} = \frac{-\omega x}{24EI}\left(x^3 - 2Lx^2 + L^3\right)\Big|_{x = \frac{2L}{3}}$$

$$= -\frac{\omega\left(\frac{2L}{3}\right)}{24EI}\left[\left(\frac{2L}{3}\right)^3 - 2L\left(\frac{2L}{3}\right)^2 + L^3\right]$$

$$= -0.01132\,\frac{\omega L^4}{EI}$$

and $$v_{B2} = \frac{-Pab}{6EIL}\left(L^2 - b^2 - a^2\right)$$

$$= -\frac{(-B_y)(2L/3)(L/3)}{6EIL}\left(L^2 - (L/3)^2 - (2L/3)^2\right)$$

$$= 0.01646\,\frac{B_y L^3}{EI}$$

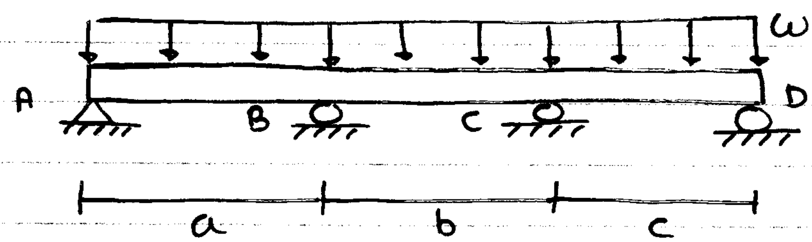
2

$$\therefore \quad \delta_B = v_{B1} + v_{B2}$$

$$= -0.01132 \frac{wL^4}{EI} + 0.01646 \frac{B_y L^3}{EI}$$

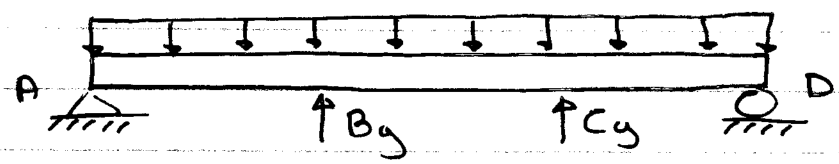
$$= 0 \quad , \quad B_y = 0.688\, wL \quad \text{(or something similar)}$$

EXAMPLE

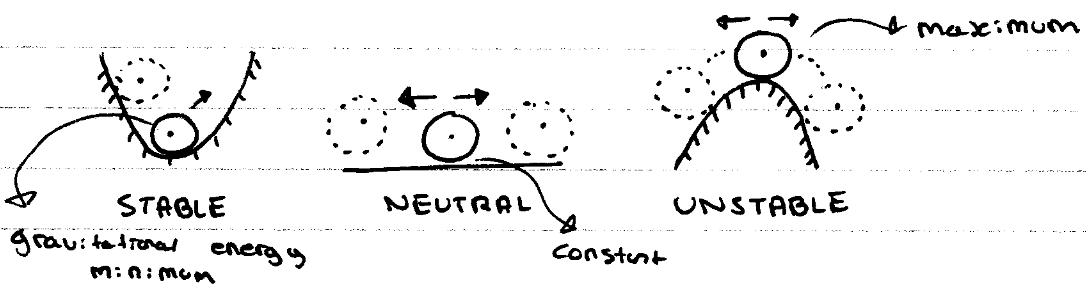



$EI = \text{const.}$

$L = a + b + c$



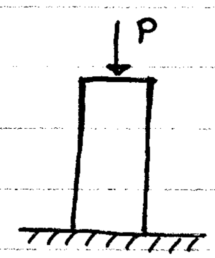

$$\delta_B = 0 \quad , \quad \delta_C = 0$$




## Ch. 13 - BUCKLING OF COLUMNS

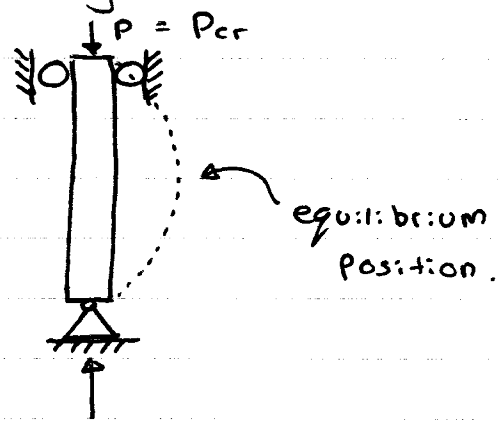
### 13.1 CRITICAL LOAD

COLUMNS are long slender members subjected to an axial compressive force.

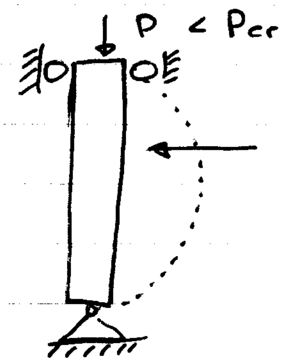



The lateral deflection that occurs in the column is called buckling.

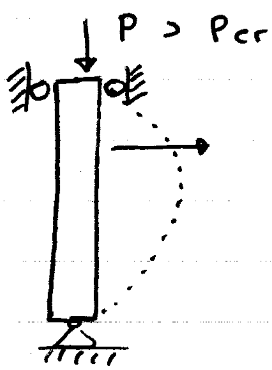
The maximum axial load that a column can support when it is on the verge of buckling is called the critical load, $P_{cr}$.




NEUTRAL



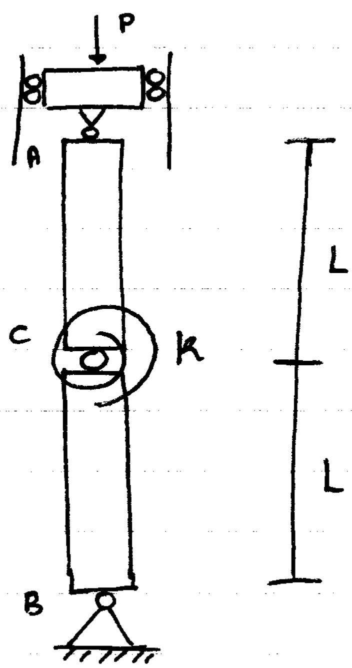

STABLE




UNSTABLE

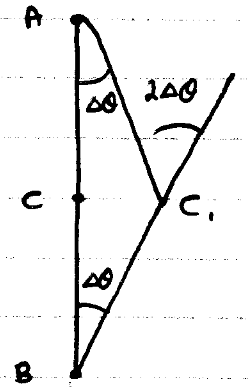
EXAMPLE :

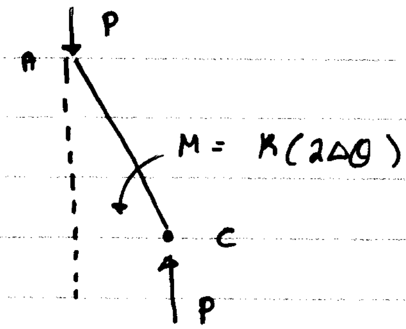
Find $P_{cr}$



Solution $P = P_{cr}$, many equilibrium positions

AC and BC a small rotation $\Delta\theta$




$$\sum M_c = 0 ;$$

$$P \cdot L \sin\Delta\theta - K \cdot 2\Delta\theta = 0$$

$\Delta\theta$ is small. $|\Delta\theta| \ll 1$

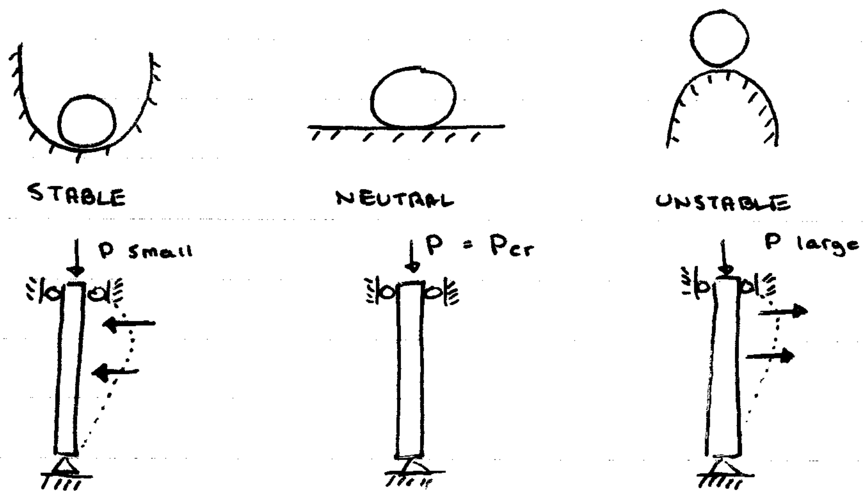
$$\sin\Delta\theta \approx \Delta\theta$$

$$\Rightarrow PL\Delta\theta - 2K\Delta\theta = 0$$

$$\Rightarrow \boxed{P = \frac{2K}{L}}$$

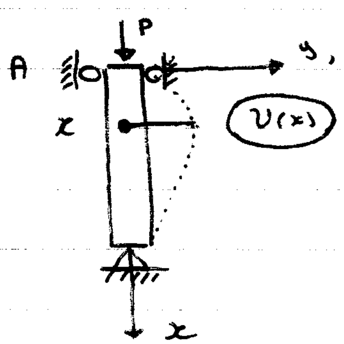
$$P_{cr} = \frac{2K}{L}$$

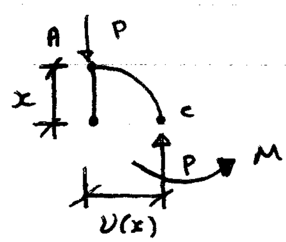
## 13.2 Ideal Column With Pin Supports




Assumptions:

1 - The column is perfectly straight before loading
2 - homogeneous, isotropic material
3 - axial load is applied through the centroid of the cross-section
4 - Linear relationship between the stress and the strain
5 - The column buckles or bends in a single plane.




$\Sigma M_c = 0$ :

$$Pv + M = 0$$

$$\boxed{M = -Pv}$$

Elastic curve:

$$EI\frac{d^2v}{dx^2} = M = -Pv$$

$$EI\frac{d^2v}{dx^2} - Pv = 0$$

$$\frac{d^2 v}{dx^2} + \frac{P}{EI} v = 0$$

Solution

$$v(x) = C_1 \sin\left(\sqrt{\tfrac{P}{EI}}\, x\right) + C_2 \cos\left(\sqrt{\tfrac{P}{EI}}\, x\right)$$

Boundary Conditions

At A, $x = 0$, $v = 0$

At B, $x = L$, $v = 0$

$\Rightarrow$ At A : $v = 0 = C_2$

$$v(x) = C_1 \sin\left(\sqrt{\frac{P}{EI}}\, x\right)$$

$\Rightarrow$ At B: $x = L$, $v = 0$

$$0 = C_1 \sin\left(\sqrt{\frac{P}{EI}}\, L\right)$$

$\Rightarrow C_1 = 0 \Rightarrow v(x) = 0$ —— TRIVIAL.

$$\sin\left(\sqrt{\frac{P}{EI}}\, L\right) = 0$$

$$\sqrt{\frac{P}{EI}}\, L = n\pi, \quad n = 1, 2, 3, \ldots$$

$$\Rightarrow P = EI\left(\frac{n\pi}{L}\right)^2, \quad n = 1, 2, 3, \ldots$$

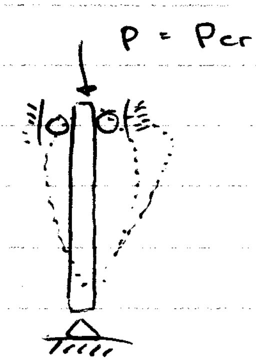
$\Rightarrow$ The smallest value of P is the so-called critical load, $P_{cr}$:

$$\Rightarrow \boxed{P_{cr} = EI\left(\pi/L\right)^2 = \frac{\pi^2 EI}{L^2}}$$

When $P = P_{cr}$

$$v(x) = C_1 \sin\left(\sqrt{P/EI}\, x\right)$$

$$v(x) = C_1 \sin\left(\frac{\pi x}{L}\right)$$



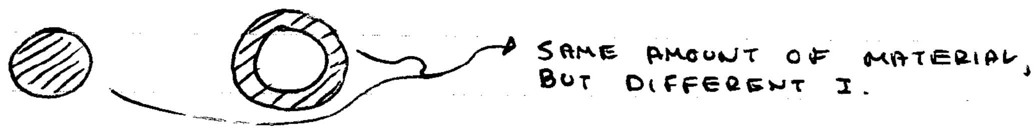
1# if $P < P_{cr}$ Stable

$P = P_{cr}$ Neutral

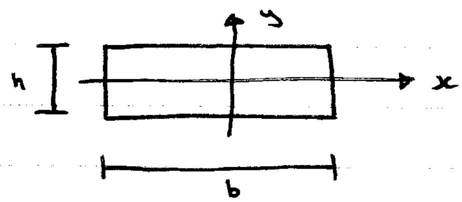
$P > P_{cr}$ Unstable

2# $P_{cr}$ is independent of the strength of the material.

3# $P_{cr}$ is proportional to the moment of inertia $I$




4#




$$I_x = \frac{1}{12} bh^3$$

$$I_y = \frac{1}{12} hb^3$$

$b > h$ : $I_y > I_x$

The bucking of a column will occur about the axis having a smaller moment of inertia.

5# $$\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}}$$

Define $r = \sqrt{I/A}$ (radius of gyration)

$$I = Ar^2$$

$$P_{cr} = \frac{\pi E A r^2}{L^2}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2}$$

Critical Stress

6# - A typical structural steel

$\sigma_y = 36$ ksi, $E = 29000$ ksi

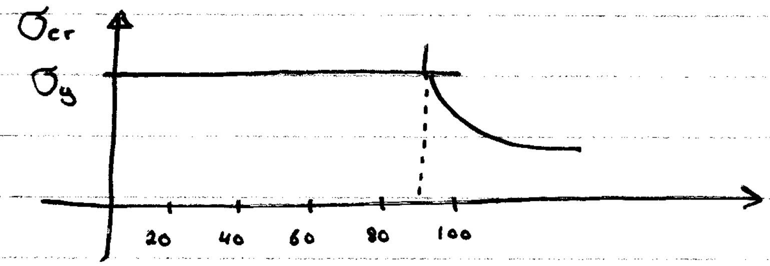
Using $\sigma_{cr} = \dfrac{\pi^2 E}{(L/r)^2}$

When $\sigma_{cr} = \sigma_y$

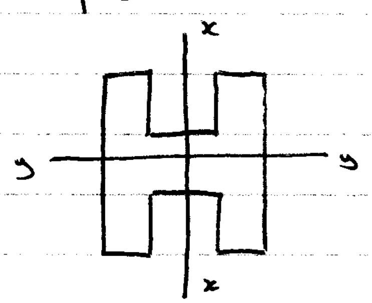
$$\Rightarrow \frac{\pi^2 E}{(L/r)^2} = \sigma_y$$

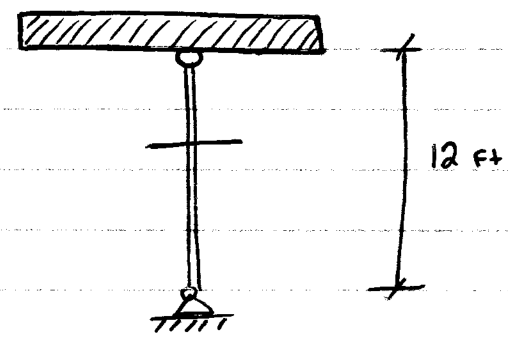
$$\Rightarrow \frac{L}{r} = \pi\sqrt{\frac{E}{\sigma_y}} = \pi\sqrt{\frac{29000}{36}} = 89$$

$$\begin{cases} L/r > 89 & \text{buckling first} \\ L/r < 89 & \text{strength} \end{cases}$$




Example:




$A = 9.13\ \text{in}^2$

$I_x = 110\ \text{in}^4$ $\quad I_y = 37.1\ \text{in}^4$

Determine the largest axial force the column can support before it either begins to buckle or the steel yields.

Solution : $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

Buckling will occur about y-y axis

Critical load ($P_{cr}$) $= \dfrac{\pi^2 (29000 \times 10^3)(37.1)}{(12 \times 12)^2}$

$= 512 \times 10^3$ lb

Critical Stress ($\sigma_{cr}$) $= \dfrac{P_{cr}}{A} = \dfrac{(512 \times 10^3)}{(9.13 \text{ in}^2)} = 56.1 \times 10^3$ psi

56.1 ksi $> \sigma_y = 36$ ksi

The column will yield first.

$\therefore P_{max} = A\sigma_y$

$= (9.13)(36)$

$= 329$ kip