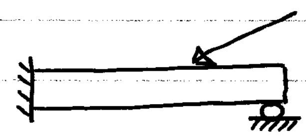
12.6 Statically Indeterminate Beams and Shafts

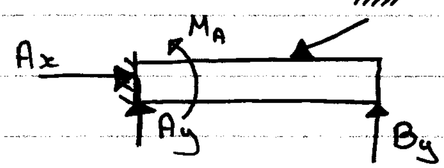
A member of unknown reactions is said to be statically indeterminate if the number of unknown reactions exceeds the number of equilibrium eqn's. The additional support reactions on the beam are called redundants.

The number of redundants is referred to as the degree of indeterminancy.

Example :



FBD:

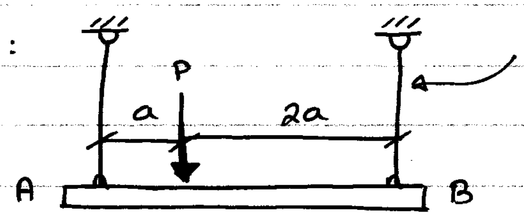


4 unknowns : A_x, A_y, B_y, M_A

3 equilibrium equations

$4 - 3 = 1$.

Example :

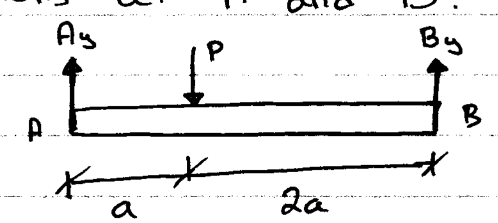


weightless rigid

weightless rigid bar

Reactions at A and B.

FBD.



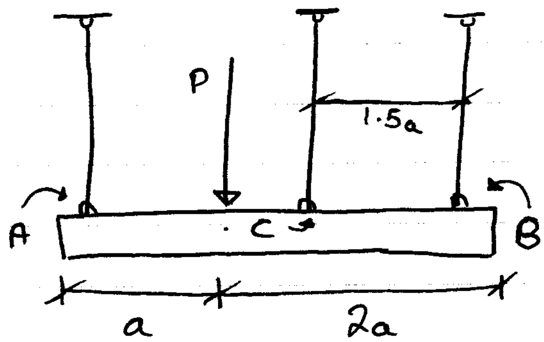
$$\Sigma M_A = 0$$

$$B_y \cdot 3a - P \cdot a = 0$$

$$B_y = \tfrac{1}{3}P$$

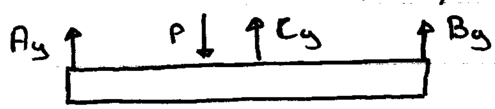
$$\therefore A_y = \tfrac{2}{3}P$$

EXAMPLE :



Find reactions at A, B, and C

FBD :

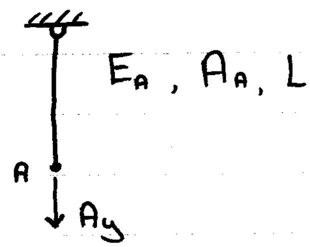


$$\Sigma F_y = 0 : \quad A_y + B_y + C_y - P = 0$$

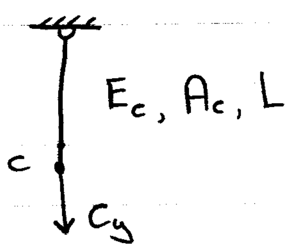
$$\Sigma M_A = 0 : \quad B_y \cdot 3a + C_y \cdot 1.5a - P \cdot a = 0$$

$$\Rightarrow \quad 3B_y + 1.5C_y - P = 0$$

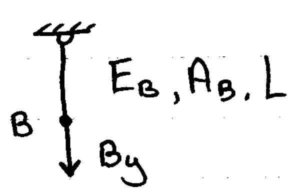
3 bars : rigid bar $\longrightarrow$ elastic bar



$$\delta_A = \frac{PL}{AE} \Rightarrow \frac{A_y L}{A_A E_A}$$

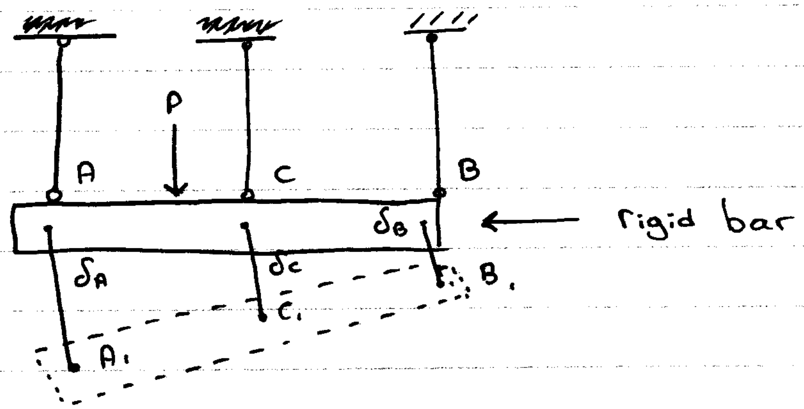


$$\delta_C = \frac{C_y L}{A_C E_C}$$



$$\delta_B = \frac{B_y L}{A_B E_B}$$

(this is an A)



$$\Rightarrow \quad CC_1 = \frac{AA_1 + BB_1}{2}$$

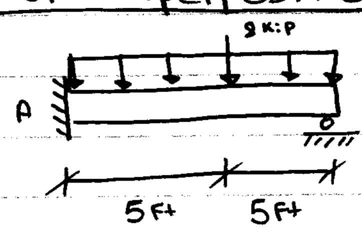
$$\Rightarrow \quad \delta_C = \frac{\delta_A + \delta_B}{2}$$

$$\Rightarrow \quad \delta_A + \delta_B - 2\delta_C = 0$$

$$\Rightarrow \quad \frac{A_y L}{A_A E_A} + \frac{B_y L}{A_B E_B} - \frac{2 C_y L}{A_C E_C} = 0$$

Method of Superposition

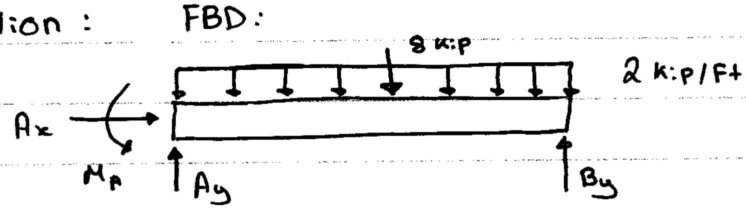
Example :



EI = const.

Find the reaction of the roller support at B.

Solution : FBD:



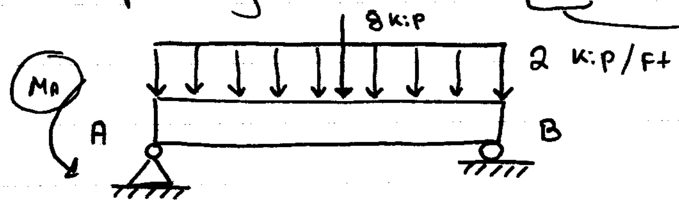
4 unknowns, 3 eqn's.

The degree of indeterminacy is 1.

M_A as the redundant Force

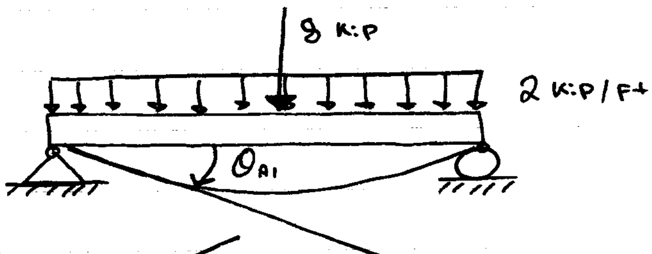
Remove the corresponding support,

the primary beam => S.S. beam (simplified structure)

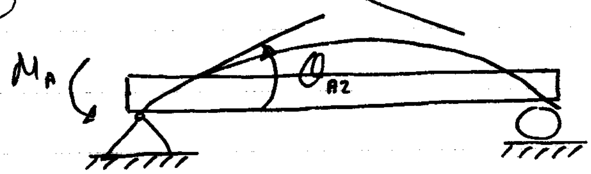


+

$$\boxed{\theta_A = 0} \Leftarrow$$



+



Superposition

$$\theta_A = \theta_{A1} + \theta_{A2} = 0$$

$$\theta_{A1} = \frac{-PL^2}{16EI} - \frac{\omega L^3}{24EI}$$

$$= -\frac{8(10)^2}{16EI} - \frac{2(10)^3}{24EI}$$

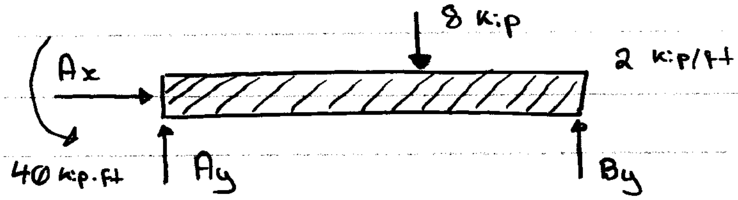
$$= \frac{-133.33}{EI}$$

$$\theta_{A2} = \frac{M_o L}{3EI} = \frac{M_A(10)}{3EI}$$

$$\Rightarrow -\frac{133.33}{EI} + \frac{10M_A}{3EI} = 0$$

$$\Rightarrow M_A = 40 \text{ kip}\cdot\text{ft}$$

FBD

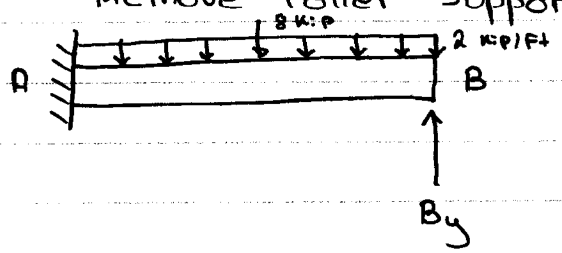


$$\Sigma M_A = 0 : \quad B_y \times 10 + 40 - 8 \times 5 - 20 \times 5 = 0$$

$$B_y = 10 \text{ kip}$$

Method #2 : (B_y as the redundant Force)

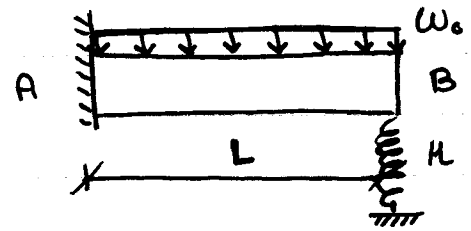
- Remove roller support at B



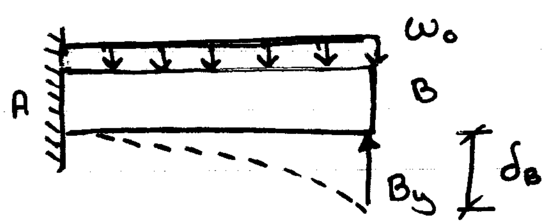
+

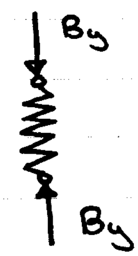
$$\boxed{v_B = 0}$$

Example : Determine the Force in the spring.
EI = Const.



Solution :





Beam : $\delta_B \downarrow$

Spring : $\dfrac{B_y}{k}$

$$\Rightarrow \boxed{\delta_B = \frac{B_y}{k}}$$

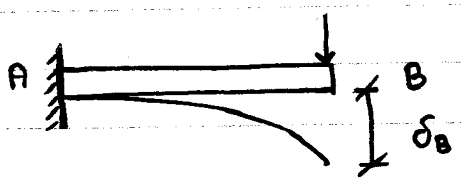
$$\delta_B = \frac{\omega_o L}{8EI} - \frac{B_y L^3}{3EI}$$

$$\Rightarrow \frac{\omega_o L^4}{8EI} - \frac{B_y L^3}{3EI} = \frac{B_y}{k}$$

$$\Rightarrow \frac{B_y}{k} + \frac{B_y}{\frac{3EI}{L^3}} = \frac{\omega_o L^4}{8EI}$$

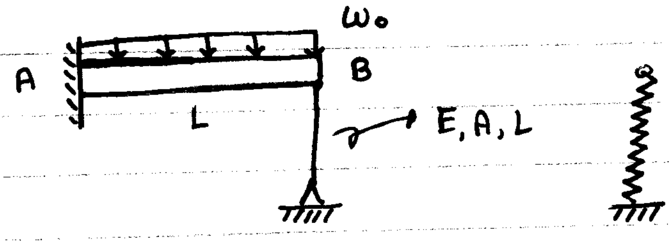
$$\Rightarrow B_y = \frac{\frac{\omega_o L}{8EI}}{1/k + 1/(3EI/L^3)}$$

$$\boxed{\delta_B = \frac{PL^3}{3EI} = \frac{P}{k}.}$$

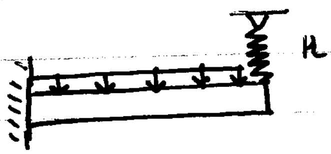


etc.

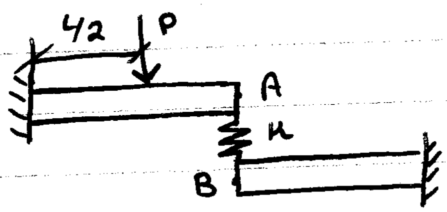
Here $k_1 = \frac{3EI}{L^3}$



$k = \frac{EA}{L_1}$



EXAMPLE:

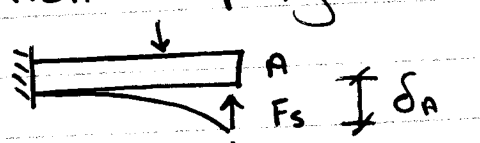


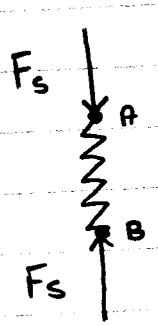
Determine the Force in the spring

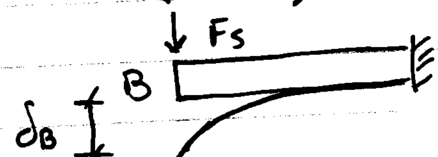
Given $P = 45000$ N, $k = 9 \times 10^5$ N/m

$E = 200$ GPa $\quad L = 3$m

Solution : Spring force is the redundant Force







$$\boxed{\delta_A - \delta_B = \frac{F_s}{k}}$$

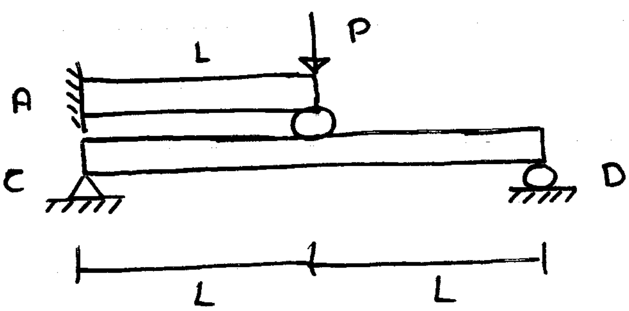
$$\delta_A = \frac{5L^3}{48EI} - \frac{F_s L^3}{3EI}$$

$$\delta_B = \frac{F_s L^3}{3EI}$$

$$\Rightarrow \quad \frac{5PL^3}{48EI} - \frac{F_s L^3}{3EI} - \frac{F_s L^3}{3EI} = \frac{F_s}{k}$$

$$\Rightarrow \quad F_s = 2577\ \text{N}$$

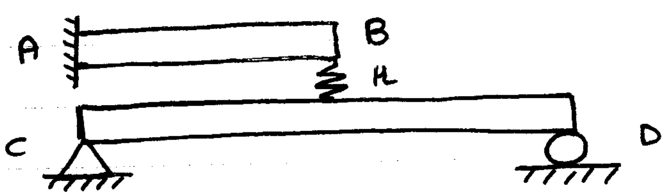
Example:



EI = const.

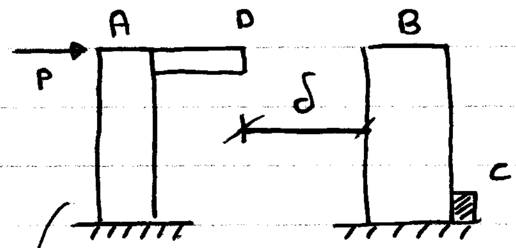
Find reactions at A, C, and D

$\delta_{B,\,cant.} - \delta_{B,\,ss.}$



$$\delta_{B,\,cant} - \delta_{B,\,ss} = \frac{F_s}{k}$$

EXAMPLE :



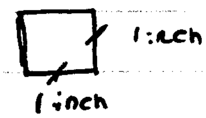
$\epsilon_c = -0.0004$

$\delta = 0.3$ in

$L = 5$ Ft

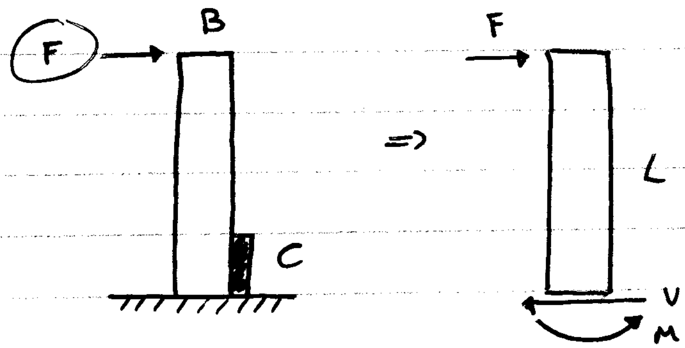
$E = 30 \times 10^6$ psi

Cross section of each beam :



Find the Force P :

Solution :



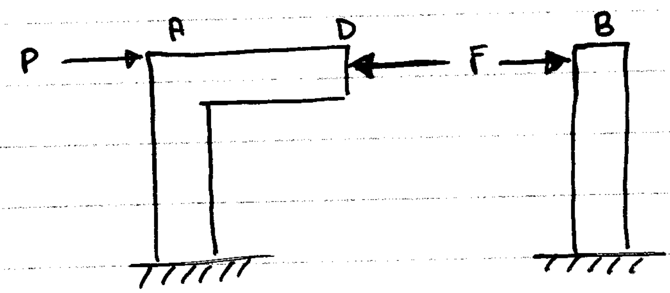
FBD: $M = FL$

$$|\sigma_c| = \frac{My}{I}$$

$$|\sigma_c| = |E\epsilon_c|$$

$$\Rightarrow F = \frac{M}{L} = \frac{EI \cdot (0.0004)}{0.5L}$$

$M \cdot (0.5)$



$$\delta_D = \frac{(P-F)L^3}{3EI}$$

$$\delta_B = \frac{FL^3}{3EI}$$

$$\delta_D - \delta_B = \delta$$

$$\frac{(P-F)L^3}{3EI} - \frac{FL^3}{3EI} = \delta$$

$\Rightarrow P = ?\ldots$